Similarity solution for wedge-shaped fluid/structure impact

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1. Introduction

Fluid/structure impact has important applications in naval architecture, ocean engineering and coastal engineering. In general, the problem is time dependent. There are cases, however, in which the time variable can be incorporated into the spatial variables. The problem then becomes self-similar. There are various solutions for these types of problems. The present work will consider a case in which the detailed analysis seems to have been missing so far, the oblique impact between an asymmetric water wedge and an asymmetric solid wedge.

2. Mathematical model

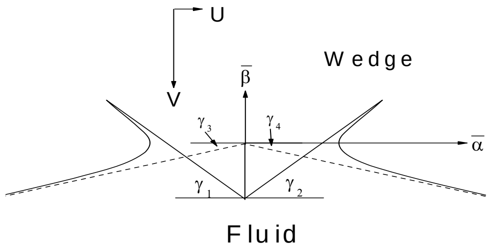

Fig.1 Sketch of the problem

We consider the two dimensional problem of the oblique collision of a water wedge with a solid wedge, as shown in Fig.1. In the Cartesian coordinate system Oxy, the velocity potential ϕ satisfies the Laplace equation

$$\nabla^2\phi = 0 \tag{1}$$

in the fluid domain. On the wedge surface S_0 we have

$$\frac{\partial\phi}{\partial n} = \vec{U}\cdot\vec{n} \tag{2}$$

where $\vec{U} = U\vec{i} - V\vec{j}$, and $\vec{n} = n_x\vec{i} + n_y\vec{j}$ is the normal vector of the body surface pointing out of the fluid domain, and $\vec{i}$ and $\vec{j}$ are unit vectors in the x and y directions respectively. The dynamic and kinematic boundary conditions on the free surface S_F or $y = \varsigma$ can be written as

$$\phi_t + \frac{1}{2}\nabla\phi\nabla\phi = 0 \tag{3}$$

$$\varsigma_t = \phi_y - \phi_x\varsigma_x \tag{4}$$

Assume the origin of the system is on the undisturbed free surface and the impact occurs at $t = 0$. The tip of the solid wedge is then at $(Ut, -Vt)$. The effect of gravity on the flow is ignored in Eq.(3), as we are concerned with only the initial stage of the impact at high speed. The solution is expected to be self similar, as there is no length scale. If we introduce

$$\alpha = x/Vt, \quad \beta = z/Vt, \quad \phi(x,z,t) = V^2 t\varphi(\alpha,\beta) \tag{5}$$

we have

$$\nabla^2\varphi = \frac{\partial^2\varphi}{\partial\alpha^2} + \frac{\partial^2\varphi}{\partial\beta^2} = 0 \tag{6}$$

$$\varphi - \alpha\varphi_\alpha - \beta\varphi_\beta + \frac{1}{2}(\varphi_\alpha^2 + \varphi_\beta^2) = 0 \tag{7}$$

$$\beta - \alpha\beta_\alpha = \varphi_\beta - \varphi_\alpha\beta_\alpha \tag{8}$$

on the free surface S_F, and

$$\frac{\partial\varphi}{\partial n} = \varepsilon n_\alpha - n_\beta \tag{9}$$

on body surface S_0, where $\varepsilon = U/V$.

The tip of the solid wedge is now at $(\varepsilon,-1)$ in the new system. For the convenience in the computation, we can further define $\alpha = \overline{\alpha} + \varepsilon$ and $\beta = \overline{\beta}$, which in fact means a coordinate system moving with the solid wedge with the horizontal speed. The tip of the solid wedge is then at $(0,-1)$. Eqs(7) and (8) can be rewritten as

$$\varphi - \overline{\alpha}\varphi_{\overline{\alpha}} - \overline{\beta}\varphi_{\overline{\beta}} - \varepsilon\varphi_{\overline{\alpha}} + \frac{1}{2}\nabla\varphi\nabla\varphi = 0 \tag{10}$$

$$\overline{\beta} - \overline{\alpha}\beta_{\overline{\alpha}} - \varepsilon\beta_{\overline{\alpha}} = \varphi_{\overline{\beta}} - \varphi_{\overline{\alpha}}\overline{\beta}_{\overline{\alpha}} \tag{11}$$

To impose the boundary conditions more effectively in the numerical simulations, we rotate the system $\overline{\alpha} - o - \overline{\beta}$ clockwise by $\pi/2 - \gamma_2$ to form a new system $\xi - o - \eta$, when $\overline{\alpha} > 0$. Eqs.(10) and (11) on the right hand side of the wedge then become

$$\varphi - \xi\varphi_\xi - \eta\varphi_\eta - \varepsilon(\sin\gamma_2\varphi_\xi + \cos\gamma_2\varphi_\eta) + \frac{1}{2}(\varphi_\xi^2 + \varphi_\eta^2) = 0 \tag{12}$$

$$\eta - \xi\eta_\xi - \varepsilon(-\cos\gamma_2 + \eta_\xi\sin\gamma_2) = -\varphi_\xi\eta_\xi + \varphi_\eta \tag{13}$$

To improve the efficiency in iteration, we can rewrite these two equations in an integral form, following the procedure of Wu, Sun & He(2004). Thus we can write Eq(13) as

$$\eta - \xi\eta_\xi = \varepsilon(-\cos\gamma_2 + \eta_\xi\sin\gamma_2) - \varphi_\xi\eta_\xi + \varphi_\eta \tag{14}$$

If the right hand side of this is treated as known, the differential equation can be solved as

$$\eta = -\xi\left(\int_{\xi_0}^{\xi}\frac{\varphi_\eta - \varphi_\xi\eta_\xi + \varepsilon(-\cos\gamma_2 + \eta_\xi\sin\gamma_2)}{\xi^2}d\xi - \frac{\eta_0}{\xi_0}\right) \tag{15}$$

where the (ξ_0,η_0) is the intersection point between the free surface and a control surface S_C far away from the solid surface. Similarly, Eq.(12) can be written as

$$\varphi = \xi \int_{\xi_0}^{\xi} \frac{\varphi_\xi \varphi_\eta \eta_\xi + 0.5(\varphi_\xi^2 - \varphi_\eta^2) - \varepsilon \sin\gamma_2 \left(\varphi_\xi + \varphi_\eta \eta_\xi\right)}{\xi^2} d\xi \qquad (16)$$

in which $\varphi(\xi_0, \eta_0) = 0$ has been assumed. When $\bar{\alpha} < 0$, or on the left hand side of the solid wedge, a similar procedure can be followed. Once the solution is found, the pressure in the fluid can be obtained from

$$p = -\rho V^2 (\varphi - \bar{\alpha}\varphi_{\bar{\alpha}} - \bar{\beta}\varphi_{\bar{\beta}} - \varepsilon\varphi_{\bar{\alpha}} + \frac{1}{2}\nabla\varphi\nabla\varphi) \qquad (17)$$

where ρ is the density of the fluid.

To solve the boundary value problem, an integral equation along the boundary of the fluid is established based on Cauchy theorem for the complex velocity potential. The boundary is then divided into small elements along which linear variation of the potential is assumed. An iterative method is used to satisfy the nonlinear boundary conditions on the free surface which is unknown itself and is obtained through the solution.

3. Numerical results and discussion

In previous publications, following impact problems have been considered: (1) a symmetric solid wedge entering a flat water surface vertically (Dobrovol'skaya 1969, Zhao & Faltinsen 1993), (2) an asymmetric wedge entering a flat water surface vertically (Semonov & Iafrati 2006), (3) a symmetric (Wu 2007a) and an asymmetric (Cumberbatch 1960, Zhang, Yue & Tanazawa 1996), water wedge hitting on a flat wall, (4) a symmetric liquid wedge hitting a symmetric solid wedge (Wu 2007b). Here we shall undertake simulations and give results for the oblique impact of water wedges and solid wedges, which can be either symmetric or asymmetric. Based on the definitions given in Fig.1, Fig.2 gives results for a solid wedge entering calm water with $\gamma_1 = \gamma_2 = 45^0$ and with $\gamma_1 = 40^0$, $\gamma_2 = 20^0$ respectively. Fig.3 gives results for impact between a liquid wedge and a solid wedge with $\gamma_1 = \gamma_2 = 30^0$, $\gamma_3 = \gamma_4 = 30^0$, and with $\gamma_1 = 40^0, \gamma_2 = 20^0, \gamma_3 = 40^0, \gamma_4 = 50^0$ respectively. Detailed analysis and discussions will be given in the workshop.

4. Conclusions

The present work is for a more general case of solid wedge/water wedge impact problem. It is solved based on the self-similar flow. When the boundary surface has curvature, a curved liquid column (Wu 2007a) or a closed liquid droplet (Wu 2007b) for example, the time stepping method would have to be used. It is still however possible to use the present result as an initial solution in many cases, as adopted by Wu(2006) for the twin wedges problem.

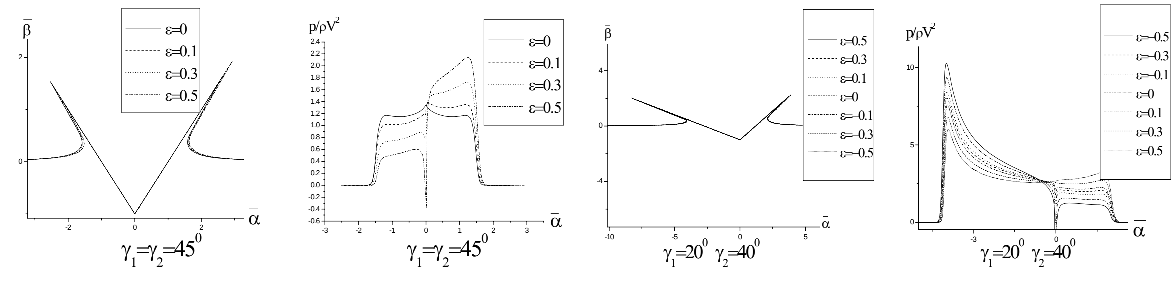

Fig.2 Oblique entry of a symmetrical/asymmetrical wedge

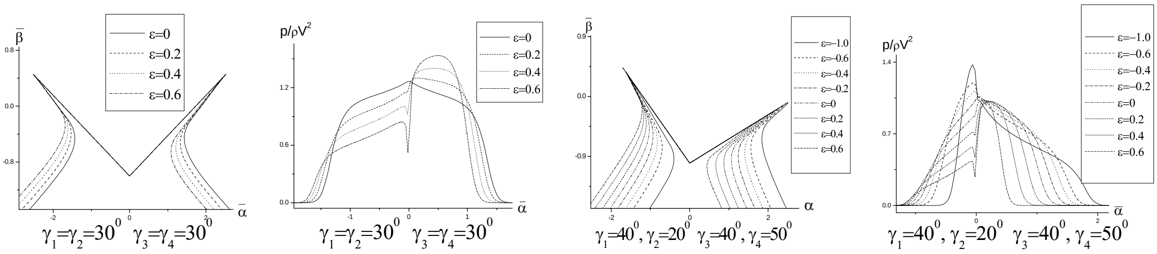

Fig.3 Oblique collision of water wedge and solid wedge

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References

Cumberbatch, E. 1960, The impact of a water wedge on the wall, J. Fluid Mech. 7, 353-374.

Dobrovol'skaya, Z. N. 1969, Some problems of similarity flow of fluid with a free surface. J. Fluid Mech. 36, 805-829.

Semenov, Yu. A., Iafrati, A. 2006, On the nonlinear water entry problem of asymmetric wedges J. Fluid Mech. 547, 231-256.

Wu, G.X., 2006. Numerical simulation of water entry of twin wedges. J. of Fluids and Structures 22, 99-108.

Wu, G.X. 2007a. Fluid impact on a solid boundary, J. of Fluids and Structures 23, 755-765.

Wu, G.X. 2007b. Liquid column and liquid droplet impact. Quart, J. of Mechanics and Applied Mathematics 60, 497-511

Wu, G.X., Sun, H., He, Y.S., 2004. Numerical simulation and experimental study of water entry of a wedge in free fall motion. J. of Fluids and Structures 19, 277-289.

Zhang,S., Yue,D.K.P., Tanizawa,K., 1996, Simulation of plunging wave impact on a vertical wall, J. of Fluid Mech., 327, 221-254

Zhao, R., Faltinsen, O. M. 1993 Water entry of two-dimensional bodies. J. Fluid Mech. 246,593-612.