Steep wave impact onto a complex 3D structure

Ten I.[1], Korobkin A.[2], Malenica Š.[1], De Lauzon J.[1] & Mravak Z.[1]

(1) BUREAU VERITAS - DR, Paris, France (igor.ten@bureauveritas.com)
(2) School of Mathematics, University of East Anglia, Norwich, UK

Introduction

The hydroelastic interactions during the impact of steep wave onto a complex structures are discussed. This problem is relevant for sloshing impacts apearing in the tanks of the LNG ships. The overall problem of sloshing impacts being extremely complex, some rational approximations were presented in [2], and this work represents the specific part related to the steep wave impact. First results for this type of impact were presented in [1] for purely 2D case and here we extend it to the impact onto a 3D complex structures, such as the NO96 boxes of the containement system of the LNG tanks. Indeed, even if the fluid flow can reasonably be approximated as a 2D, the complexity of the containement system structure need to be considered fully 3D and solved by complex numerical solvers such as Abaqus, Nastran ... The method which is presented here uses the so called hybrid approach which means that the fluid flow is modelled by the 2D strip approach, while the structural behavior is solved using the 3D FEM model.

Mathematical formulation

The basic configuration before impact is shown in Figure 1. The fluid of height H and width L occupies a region $x < 0$, $0 < y < L$, and $0 < z < H$, where the plane $z = 0$ corresponds to the flat rigid bottom, and the vertical z-axis is directed upward. Before the impact, $t < 0$, a part of the liquid boundary $x = 0$, $0 < z < H - H_w$ is in contact with the vertical wall. The boundary part $x = 0$, $H - H_w < z < H$ corresponds to the vertical face of the wave (hydraulic jump), which approaches the wall at constant speed U and hits the wall at $t = 0$. Only one part of the vertical wall is elastic, $S \equiv [y_1, y_2] \times [z_1, z_2]$, and the rest is rigid.
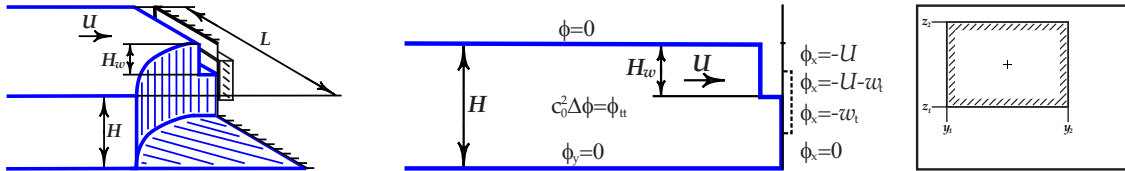

Figure 1. Formulation of the problem

The fluid flow is studied within the acoustic approximation. At the initial stage of the impact, which is of short duration, the problem is linearized and the boundary value problem shown in Figure 1 is formulated. The problem is solved in non-dimensional variables which relate to dimensional one, denoted by a prime, as follow

$$x' = Hx, \ y' = Hy, \ z' = Hz, \ \varphi' = UH\varphi, \ H_w = Hh_w, \ t' = \frac{H}{c_0}t, \ p' = \rho U c_0 p, \ w' = \frac{HU}{c_0}w, \ L = Hl. \ (1)$$

where, c_0, ρ, φ, p, and w are sound speed in the fluid at rest, the fluid density, velocity potential, pressure distribution, and deflection of the structure, respectively.

Fluid flow

Within the acoustic approximation the flow is potential. Velocity potential $\varphi(x, y, z, t)$ satisfies in non-dimensional variables (1) the wave equation

$$\Delta \varphi = \varphi_{tt} \qquad (x, y, z) \in \Omega \equiv (-\infty, 0] \times [0, l] \times [0, 1], \qquad (2)$$

boundary conditions

$$\varphi_y = 0, \qquad y = 0 \text{ and } y = l, \tag{3}$$

$$\varphi = 0, \qquad z = 1, \tag{4}$$

$$\varphi_z = 0, \qquad z = 0, \tag{5}$$

$$\varphi_x = -\chi_1 + w_t\chi_2, \qquad x = 0, \tag{6}$$

where

$$\chi_1(y, z) = \begin{cases} 1, & 0 < y < L, \ 1 - h_w = h_1 < z < 1, \\ 0, & 0 < y < L, \ 0 < z < h_1 = 1 - h_w, \end{cases} \qquad \chi_2(y, z) = \begin{cases} 1, & (y, z) \in S, \\ 0, & (y, z) \notin S, \end{cases} \tag{7}$$

and initial conditions

$$\varphi(x, y, z, 0) = \varphi_t(x, y, z, 0) = 0. \tag{8}$$

The hydrodynamic pressure acting on the wall is given by the linearized Cauchy-Lagrange integral

$$p(y, z, t) = -\varphi_t(0, y, z, t). \tag{9}$$

The solution of the problem (2)-(8) is sought in the form of eigenfunction expansion:

$$\varphi(x, y, z, t) = \sum_{n=1}^{\infty} X_n(x, t) V_n(y, z). \tag{10}$$

$$V_n = \frac{2}{\sqrt{l}} \cos[\lambda_{i(n)} y] \cos[\mu_{j(n)} z], \quad \lambda_i = \frac{\pi i}{l} \ (i = 0, 1, 2, ...), \quad \mu_j = \frac{\pi}{2}(2j - 1) \ (j = 1, 2, ...), \tag{11}$$

$\delta_{nm} = 1$ if $n = m$ and zero otherwise. The eigenfunctions $V_n(y, z)$ satisfy the boundary conditions (3)-(5). Substituting (10) into (2) we obtain equations for the coefficients $X_n(x, t)$ which form boundary-value problem with initial conditions provided by (8). The obtained problem is solved with the help of Laplace transform. Finally, after few algebra, the solution at $x = 0$ can be written as the following

$$X_n(0, t) = \int_0^t F_n(\tau) J_0(r_n(t - \tau)) d\tau, \quad F_n(t) = \int_0^1 \int_0^l [-\chi_1(y, z) + w_t(y, z, t)\chi_2(y, z)] V_n(y, z) dy dz, \tag{12}$$

where J_0 is the Bessel function.

The pressure distribution $p(y, z, t)$ on the wall, $x = 0$, is :

$$p(y, z, t) = -\sum_{n=1}^{\infty} \left[\frac{d}{dt} \int_0^t F_n(\tau) J_0(r_n(t - \tau)) d\tau \right] V_n(y, z). \tag{13}$$

Structural dynamics

The structural deflection at the wall $w(y, z, t)$ is the solution of the structural dynamic equation subjected to the following initial conditions:

$$w(y, z, 0) = w_t(y, z, 0) = 0. \tag{14}$$

Regardless of the method (analytical or numerical) which is employed to solve the structural dynamics, the wall deflection is developed in the following series:

$$w(y, z, t) = \sum_{n=1}^{\infty} a_n(t) \Psi_n(y, z), \tag{15}$$

where $\Psi_n(y, z)$ are the adequate shape functions.

The most natural choice for the shape functions are the structural eigenmodes because they satisfy the boundary conditions by definition, and in addition they are orthogonal. The orthogonal property of the eigenmodes allows the reduction of the structural dynamic problem to the evolution equation for the modal amplitudes $a_n(t)$:

$$\alpha_n \frac{d^2 a_n}{dt^2} + d_n \left(a_n + \gamma_n \frac{da_n}{dt} \right) = \int_S p(y, z, t) \Psi_n(y, z) dS, \tag{16}$$

with the following initial conditions:

$$a_n(0) = 0, \qquad \dot{a}_n(0) = 0. \tag{17}$$

The coefficients α_n, d_n and γ_n are the normalized mass, stiffness and damping coefficients respectively. In the case of simple elastic structures (uniform beam, plate, ...) these coefficients can be obtained analytically, but in the general case the numerical methods are usually employed. The most common method is the finite element method (FEM) which will be used here in the context of the commercial code Abaqus.

Coupling

In order to solve the coupled hydroelastic problem, we need to express the right-hand side in (16), in terms of the modal coefficients $a_n(t)$. After some algebra we arrive at the following system of integro-differential equations with the unknown coefficients $a_n(t)$:

$$\alpha_n \frac{d^2 a_n}{dt^2} + d_n \left(a_n + \gamma_n \frac{da_n}{dt} \right) = P_n(t) - \frac{d^2}{dt^2} \sum_{m=1}^{\infty} \int_0^t a_m(\tau) K_{nm}(t-\tau) d\tau. \tag{18}$$

Here

$$P_n(t) = -\sum_{k=1}^{\infty} v_k T_{kn} J_0(r_k t), \quad K_{nm}(t) = \sum_{k=1}^{\infty} T_{km} T_{kn} J_0(r_k t), \tag{19}$$

$$T_{kn} = \int_S V_k(y,z) \Psi_n(y,z) dS, \quad v_k \equiv -\int_{1-h_w}^1 \int_0^l V_k(y,z) dy dz. \tag{20}$$

Let us define new unknown function $b_n(t)$ as

$$b_n \equiv \alpha_n a_n + \sum_{m=1}^{\infty} \int_0^t a_m(\tau) K_{nm}(t-\tau) d\tau. \tag{21}$$

Then equation (18) takes the form

$$\frac{d^2 b_n}{dt^2} + d_n \left(a_n + \gamma_n \frac{da_n}{dt} \right) = P_n(t), \qquad n = 1, 2, \dots . \tag{22}$$

The system (21) and (22) is solved numerically with the following initial conditions ($n > 1$):

$$b_n(0) = 0, \qquad \dot{b}_n(0) = a_n(0) = 0. \tag{23}$$

Numerical Results

In order to validate the coupling procedure for FEM structural modelling, we chose the case of the uniform plate for which the analytical solution is available. In the case of the FEM method, the integral (20) must be evaluated numerically. This is done by fitting $\Psi_n^{(l)}$, where superscript l denotes the Ψ_n value at the l-th node, by B-Spline fitting surface $\tilde{\Psi}_n(y,z)$ using the IMSL standard Fortran subroutines.

We chose the case where the plate occupies the whole wall and the following basic parameters are used: sound speed $c_0 = 1500 m/s$, water density $\rho_w = 1000 kg/m^3$, impact velocity $U = 1m/s$, structural damping $\gamma = 0.001s$, water height $H = 2.0m$, wave height $H_w = 0.5m$ Poisson's ratio $\nu = 0.3$, plate density $\rho_b = 7800 kg/m^3$, Young's module $E = 0.207e12 N/m^2$, width of the wall $L = 2.0m$ and plate thickness $h = 0.02m$.

In Figure 2, the time history of the plate deflection at few representative points is presented. As we can see the agreement between two class of results is almost perfect which concludes the validation of the numerical model.

Now we chose more complex case of 3D structure representing the rectangular box of $1m$ width, $1m$ height and $0.3m$ depth. The center of the box is placed at $y = 1.0m$, $z = 1.5m$ and the channel width is $L = 2m$. All other parameters are the same. Few snapshots during the impact are presented in Figure 3. There is no results for comparisons in this case, and we can just mention that the calculations are stable both in space and in time.
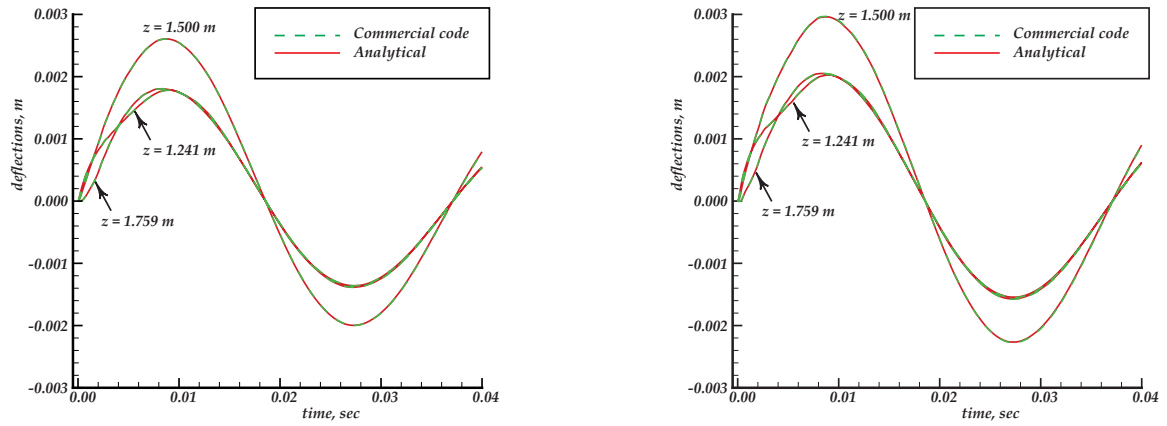
Figure 2. Time history of plate deflection for few representative points at $y = 1.0m$ (left) and $y = 1.5m$ (right).
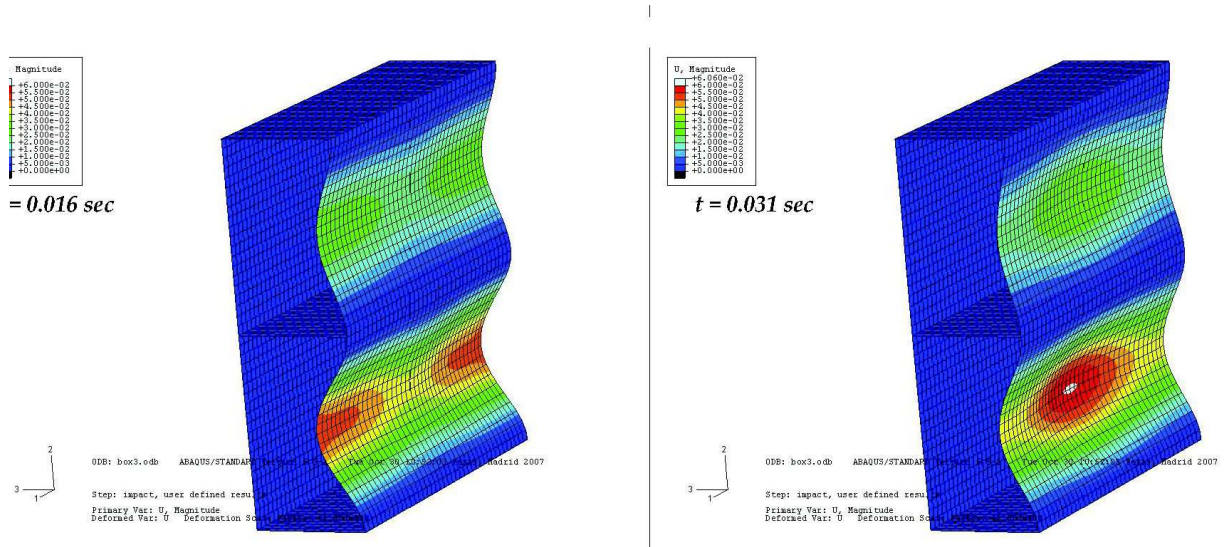

Figure 3. Snapshots during steep wave impact onto rectangular box.

Conclusions

We presented here the semi numerical method able to simulate the steep wave impact onto a complex structures modelled by the general 3D FEM numerical codes such as Abaqus. The model was validated on the case of elastic plate for which the analytical solution is available. The future work consist in applying and validating the method on the real NO96 boxes used in the tanks of LNG carriers.

References

[1] KOROBKIN, A. & MALENICA Š., 2007. : "Steep Wave Impact Onto Elastic Wall", 22nd IWWWFB, Plitvice, Croatia.

[2] MALENICA Š, KOROBKIN A.A., SCOLAN Y.M., GUERET R., DELAFOSSE V., GAZZOLA T., MRAVAK Z., CHEN X.B. & ZALAR M. 2006. : "Hydroelastic impacts in the tanks of LNG carriers.", 4th Int. Conference on Hydroelasticity, Wuxi, China.