WAGNER THEORY OF STEEP WAVE IMPACT

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ABSTRACT

The plane unsteady problem of wave impact onto a vertical wall is considered. The stage of violent interaction between the wave and the wall is distinguished and treated analytically within the Wagner approach. It is assumed that the flow has been simulated numerically or recorded in an experiment and both the flow region and velocity field are known at the beginning of the impact stage. The Wagner approach provides the pressure distribution along the wall and the jet velocity. The evolution of the impact pressures is determined in quadratures. The analytical solution provides the initial data for numerical simulation of the flow after the impact.

1 Introduction

The approach originally derived by Wagner (1932) to evaluate impact hydrodynamic loads on a body entering calm water, is applied to the problem of wave impact onto a vertical wall. Three types of wave impact can be distinguished. The first type (Wagner type) corresponds to the case where the wave front is inclined from the wall. In the simplest model it is possible to approximate the wave profile with an infinite liquid wedge, which hits the wall at a constant speed. In this case both the finite depths of the water in front of the wave front and behind it are neglected. The problem of a blunt liquid-wedge impact can be reduced to the problem of equivalent rigid-wedge entry into an initially calm liquid half-plane, the solution of which is well-known. The second type (Bagnold type) corresponds to the case where the wave front is inclined towards the wall. Air pocket is closed when the top of the wave contacts the wall. The air in the cavity is compressed by the hydrodynamic pressure thereafter. The third type (Transition type) corresponds to the case where the wave front is almost vertical with a shallow depression. The air outflow from the gap between the wall and approaching it wave front may be of importance. Small variations of the wave shape may essentially change the impact characteristics. The liquid flow in this case is extremely unstable and the highest hydrodynamic loads are expected. In the present study the first type of the wave impact is considered. Presence of air in the gap between the wall and the wave front, as well as the surface tension, are not taken into account.

2 Formulation of the problem

Two-dimensional unsteady liquid flow in the presence of gravity is governed by the following equations with respect to the velocity potential $\Phi(x, y, t)$ and the position of the liquid free surface $F(x, y, t) = 0$:

$$\Delta\Phi = 0 \qquad \text{in } \Omega(t),$$

$$2\Phi_t + 2gy + (\nabla\Phi)^2 = 0 \qquad F_t + \nabla\Phi\cdot\nabla F = 0 \qquad \text{on } \partial\Omega_f(t),$$
$$\Phi_x = w_t(y,t) \qquad \text{on } \partial\Omega_v(t), \qquad \Phi_y = 0 \qquad \text{on } \partial\Omega_H,$$
$$\Phi(x,y,0) = \Phi_0(x,y), \qquad F(x,y,0) = F_0(x,y).$$

Here $\Omega(t)$ is the flow region at time instant t. The liquid boundary $\partial\Omega$ consists of the three parts $\partial\Omega = \partial\Omega_f \cup \partial\Omega_v \cup \partial\Omega_H$. Dynamic and kinematic conditions are satisfied on the liquid free surface $\partial\Omega_f(t)$, where g is the acceleration due to gravity. Equation $F(x,y,t) = 0$ describes the free surface position. On the wetted part of the wall $\partial\Omega_v(t)$, horizontal velocity of the flow is equal to the normal velocity of the wall which can be elastic. Vertical velocity is equal to zero along the horizontal rigid bottom $\partial\Omega_H$. At $t = 0$ velocity field of the flow $\nabla\Phi_0(x,y)$ and position of the free surface, $F_0(x,y) = 0$, are given.

When a wave front with almost vertical face approaches vertical wall, the curvature of the free surface at the water line increases. Usually numerical methods fail to simulate properly the liquid flow near such points and analytical study is required. The stage of violent interaction between the wall and the wave is short but this is the stage during which the hydrodynamic pressures take their maximal values. This is the stage which is under consideration below.

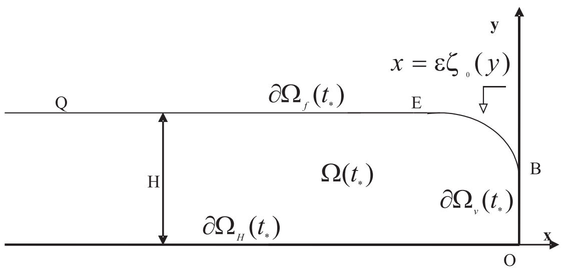

Figure 1. Scheme of the flow before the impact.

We shall determine the liquid flow and the pressure distribution during the impact stage under the following assumptions: (i) at $t = 0$ the liquid region near the wall can be approximated as a half-strip D with depth H; (ii) normal deviation of the boundary $\partial\Omega(0)$ from ∂D is of the order of ϵH, where ϵ is small non-dimensional parameter; (iii) liquid is ideal and incompressible; (iv) normal velocity $\partial\Phi/\partial n$ of the free surface $\partial\Omega_f$ is finite and $\mid \partial\Phi/\partial n \mid < V$ along the front of the wave; (v) duration of the impact stage is of the order $\epsilon H/V$; (vi) the pressure on the free surface is constant.

We introduce non-dimensional variables and unknown functions as

$$x = Hx', \quad y = Hy', \quad t = \epsilon\frac{H}{V}t', \quad \Phi = VH\Phi', \quad p = \rho V^2\epsilon^{-1}p',$$

where a prime stands for non-dimensional quantities, ρ is the liquid density and p the hydrodynamic pressure. In the new variables

$$p' = -\frac{\partial\Phi'}{\partial t'} - \epsilon\left(\frac{1}{2}(\nabla'\Phi')^2 + y'\frac{gh}{V^2}\right).$$

Assumption (i) indicates that the position of the wave front can be described by the equation $x' = \epsilon\zeta'(y',t')$, where $\zeta'(y',t')$ is a smooth non-dimensional function and both $\zeta'(y',0)$ and $\zeta'_t(y',0)$ are given (see Figure 1). The kinematic condition on the wave front has the form

$$\frac{\partial\zeta'}{\partial t'} = \frac{\partial\Phi'}{\partial x'} - \epsilon\frac{\partial\zeta'}{\partial y'}\frac{\partial\Phi'}{\partial y'}.$$

When the wave front approaches the wall, the water in front of the wave is accelerated upwards. Even for very short initial stage, which is referred to as the impact stage, the increase of the wetted part of the wall can be significant. This conclusion comes from the assumption that the initial gap between the wave front and the wall is narrow and its thickness decreases in time. At $t' = 0$ the wetted part of the wall corresponds to the interval $[0, b'(0)]$. Water level on the wall moves upwards with the length of the wetted part of the wall being $b'(t')$. The function $b'(t')$ is unknown in advance and has to be determined together with the liquid flow and the pressure distribution.

Taking formally $\epsilon = 0$ in the original equations written in the non-dimensional variables, omitting primes and introducing new unknown functions $\phi(x, y, t)$ and $h(y, t)$ as

$$\Phi(x, y, t) = \Phi_0(x, y) + \phi(x, y, t), \quad \zeta(y, t) = \zeta_0(y) - v(y)t + h(y, t),$$

where $v(y) = -(\partial\Phi_0/\partial x)(0, y)$, we arrive at the following initial boundary-value problem

$$\partial^2\phi/\partial x^2 + \partial^2\phi/\partial y^2 = 0 \qquad (x > 0, 0 < y < 1), \tag{1}$$

$$\phi = 0 \qquad (x = 0,\ b(t) < y < 1 \text{ and } x > 0,\ y = 1), \tag{2}$$

$$\partial\phi/\partial x = w_t(y, t) \quad (x = 0,\ 0 < y < b(0)), \qquad \partial\phi/\partial x = w_t(y, t) + v(y) \quad (x = 0,\ b(0) < y < b(t)), \tag{3}$$

$$\partial\phi/\partial y = 0 \quad (y = 0,\ x > 0), \qquad \phi \to 0 \qquad (x \to +\infty), \tag{4}$$

$$\partial h/\partial t = (\partial\phi/\partial x)(0, y, t) \qquad (b(t) < y < 1), \qquad h(y, 0) = 0 \qquad (b(0) < y < 1), \tag{5}$$

$$h(y, t) > w(y, t) - \zeta_0(y) + v(y)t \qquad (b(t) < y \leq 1), \qquad h[b(t), t] = w[b(t), t] - \zeta_0[b(t)] + v[b(t)]t. \tag{6}$$

This boundary value problem is illustrated in Figure 2 for rigid wall, $w(y, t) = 0$, in the dimensional variables.

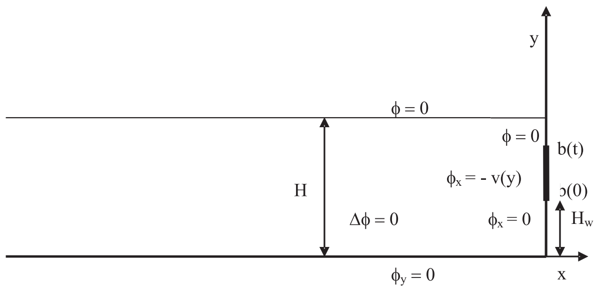

Figure 2. BVP with respect to the potential $\phi(x, y, t)$.

In present study we do not specify a model of the wall response. Some formulae are derived for $w(y, t) \neq 0$ but calculations are performed assuming that the wall deflection is known. If the function $b(t)$ is known as well, then the boundary-value problem (1)-(4) provides the unique solution, which makes it possible to analyze the liquid flow and the pressure field under the wave impact. The shape of the wave front can be found from the initial problem (5). Substituting $h(y, t)$ into (6), we obtain the equation with respect to the function $b(t)$. This is nonlinear equation which provides main difficulties in analysis of the wave impact problem. After the equation has been solved we need to verify that the wave front does not penetrate the vertical wall above the water level on the wall, $b(t) < y < 1$. This means that we have to check that the inequality in (6) is satisfied with the obtained function $h(y, t)$. If not, the impact model has to be modified to be able to simulate the wave impact with attached cavity.

The boundary-value problem (1)-(6) can be considered as water-entry problem for the blunt body, initial shape of which is given by the equation $x = -\zeta_0(y)$, penetrating the half-strip of the liquid, $0 < y < 1$, at the velocity $v(y)$. During the penetration the body shape can be changed according to the function $w(y,t)$. The velocity $v(y)$ is different, in general case, for different points of the body. This means that the body is deformable even if $w(y,t) = 0$ but its shape is prescribed in advance.

3 Solution of the wave impact problem

The boundary value problem (1) - (4) has been solved by using the displacement potential and the conformal mapping of the flow region onto the lower half-plane. Equation for the dimension of the wetted part of the wall, $b(t)$, was obtained as

$$t = \frac{1}{M(b)} \int_{b_0}^{b} \frac{[\zeta_0(y) - w(y,t)]\,\mathrm{d}y}{\sqrt{\cos\pi y - \cos\pi b}}, \quad M(b) = \int_{b_0}^{b} \frac{v(y)\,\mathrm{d}y}{\sqrt{\cos\pi y - \cos\pi b}} \tag{7}$$

and the pressure distribution along the wetted part of the wall, $0 < y < b(t)$, as

$$p(0,y,t) = \frac{\dot{b} M(b) \sin\pi b}{\sqrt{\cos\pi y - \cos\pi b}}. \tag{8}$$

Equation (8) shows that the pressure is square root singular at the contact point $y = b(t)$. Therefore, the inner solution close to the contact point can be obtained in the same way as it was done for water entry problem. Note that the pressure tends to zero as $b \to 1$.

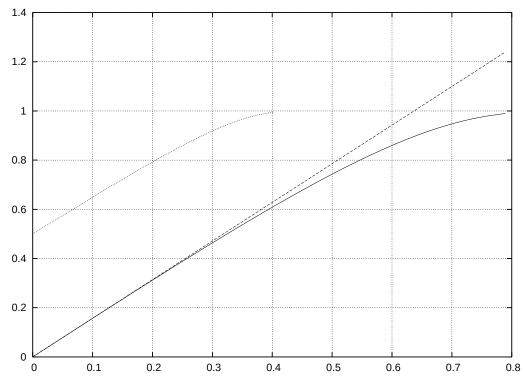

Figure 3. Position of the contact point as a function of time in non-dimensional variables.

Calculations of the contact point position were performed for two cases: (1) $b_0 = 0$, $\zeta_0(y) = y$, $v(y) = 1$; (2) $b_0 = 0.5$, $\zeta_0(y) = y - 0.5$, $v(y) = 1$. The first case corresponds to wedge-shaped jet impact onto dry wall. In the second case a half of the wall is wetted before the impact and the wave front is inclined from the wall at a constant angle. The solid line in Figure 3 corresponds to the contact point position in the first problem, dashed line corresponds to the solution of the infinite liquid wedge impact problem and the dotted line to the contact point position in the second problem. It is seen that in the first problem the motion of the contact point is weakly dependent on depth of the liquid for $0 < b(t) < 0.25$ and the velocity of the contact point is smaller than that for the infinite liquid wedge impact; this means, the hydrodynamic loads due to wave impact are smaller than those in the problem of liquid wedge impact.