2D and 3D problems of internal-wave radiation by a body oscillating in a uniformly stratified fluid

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Introduction. Internal waves play important role in the dynamics of atmosphere and ocean [1], being responsible for significant transport of momentum and energy through stratified fluids. Radiation and diffraction of internal waves significantly affect the motions of deep submersibles, submerged buoys and moored structures [2]. In the case of constant vertical density gradient (uniform stratification) theoretical description of internal-wave generation by an oscillating body is complicated by non-trivial dispersion relation. The energy radiated by a vibrating cylinder is known to be spread into four wave beams inclined at angle θ to the vertical, with the cylinder at the center of this pattern known as 'St. Andrew cross' wave [3]. In 3D case the internal waves are confined within a double cone. The dispersion relation for internal waves in a uniformly stratified fluid relates angle θ with the frequency of oscillations. Since the wave length does not appear in the dispersion relation, the spatial structure of internal waves has non-trivial dependence on the body geometry, direction and frequency of oscillations, and the fluid viscosity. Another complication is that linear theory of ideal uniformly stratified fluid predicts infinite displacements of fluid particles along the lines tangent to the oscillating body and inclined at angle θ to the vertical [4]. With fluid viscosity taken into account, realistic estimate of the cross-beam distribution of the displacements of fluid particles has been obtained in [5]. However, the solution [5] does not fulfill the no-slip condition on the body surface and therefore neglects the effect of the boundary layers. When the amplitude of oscillations and the thickness of the boundary layer are small compared to the size of the oscillating body, solution [5] has been confirmed experimentally in [6, 7]. Analysis of hydrodynamic loads [4, 5] has been experimentally confirmed in [8] and extended to the bodies of arbitrary shape in [9, 10].

When the amplitude of oscillations, the thickness of the boundary layer and the size of the oscillating body are comparable quantities, we may expect strong non-linear interaction between the boundary-generated vorticity and wave radiation and generation of secondary mass-transfer currents. This issue is addressed in the present study where in 2D case we consider large-amplitude horizontal, vertical and circular oscillations of a circular cylinder. Also, we consider the duration of the transient processes of internal-wave beams formation after the start-up of the motion. Theory [4, 5] considers only the steady-state harmonic oscillations of a body and yields no information on the transients. As the main experimental tool in 2D case we use synthetic schlieren technique [6].

In the 3D case syntetic schlieren data processing requires a tomographic inversion. In the case of axisymmetric waves generated by vertical oscillations of a sphere the technique is described in [11] and the extension to 3D cases is presented in [12]. In the present work we consider internal waves generated by horizontal oscillations of a sphere as a generic example of a 3D problem and compare the results of measurements with theoretical estimates.

Experimental setup. Experiments with circular cylinder were carried out at LIH in a test tank of size $160 \times 20 \times 50\,cm^3$. The tank was filled to the depth of $47\,cm$ with a uniformly stratified fluid. The value of the background buoyancy frequency was $N = [(-g/\rho)d\rho/dy] = 0.9 s^{-1}$, where g is the gravity acceleration, ρ is the fluid density, and y is the vertical coordinate. Internal waves were generated by a circular cylinder of diameter $D = 2cm$ undergoing horizontal, vertical or circular oscillations. In the case of circular oscillations the variation of coordinates of the center of the cylinder with time is described (for clockwise motion) by $x_c = a \sin \omega t$ and $y_c = a \cos \omega t$, where a is the radius of the cylinder trajectory, and ω is the oscillation frequency. The cases of rectilinear horizontal and vertical oscillations correspond to

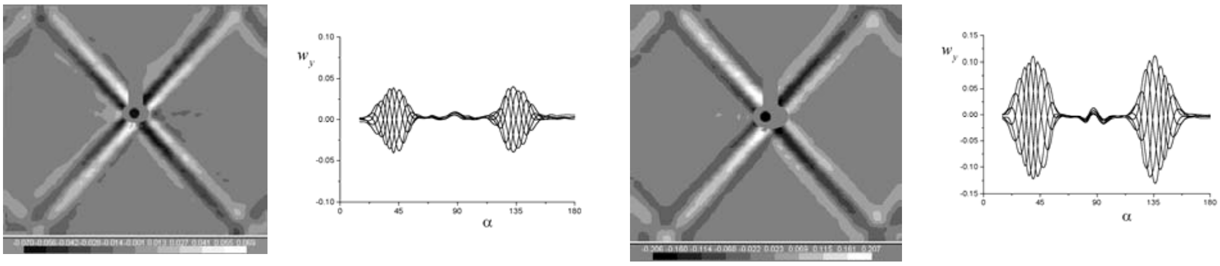
$y_c = 0$ and $x_c = 0$, respectively. The value of the densimetric Stokes number was $\beta = D^2 N/\nu = 260$.

The synthetic schlieren technique [6] is based on analysis of optical distortions due to internal waves in a continuously stratified fluid. An image of a screen with a contrast black and white pattern placed on one side of the test tank is recorded by a digital camera placed on the opposite side of the tank. The linear analysis [6] shows that the apparent vertical displacement of an element of the pattern δ_y from its initial position observed through the fluid at rest is directly proportional to the local change in the square of the buoyancy frequency so that $\Delta N^2 = -B\delta_y$ (in experiments $B = 3.34 s^{-2} cm^{-1}$). We use non-dimensional representation of the buoyancy frequency perturbations $w_y = -\Delta N^2/N^2$, where w_y has the physical sense of the derivative of the vertical displacements of fluid particles over vertical coordinate. Oscillation frequency is normalized as $\Omega = \omega/N$. For quantitative evaluation of δ_y we use a standard cross-correlation analysis performed with the Dantec Flowmanager software. A contrast pattern of randomly spaced black dots was printed with regard to recommendations presented in [13].

The experiments with horizontally oscillating spheres were performed at LEGI in a test tank of size $100 \times 100 \times 50 cm^3$. The fluid depth was $45 cm$. The value of the background buoyancy frequency was $N = 1.1 s^{-1}$. Experiments were performed with spheres of diameter $D = 6cm$ and $3cm$. Since the information on the density-gradient perturbations is integrated along the light ray path, the data processing in 3D case requires a tomographic inversion. In a general case the problem is ill-posed [12]. However, the problem is well-posed in a special case when the perturbation field can be expressed as $\Delta N^2 = f(R)s(\varphi)$ in the cylindrical coordinate system $yR\varphi$ (here y-axis is vertical, R is the horizontal distance from the y-axis, φ is the azimuthal angle) for each horizontal plane $y = const$. In the case of vertical oscillations of an axisymmetric body $s(\varphi) \equiv 1$. Theoretical analysis shows that in the far field in the case of horizontal oscillations of a sphere $s(\varphi) = \cos\varphi$ so that the tomographic inversion can be done similar to the axisymmetric case [11]. In the near field we use the technique of fluorescence layering to measure the internal waves.

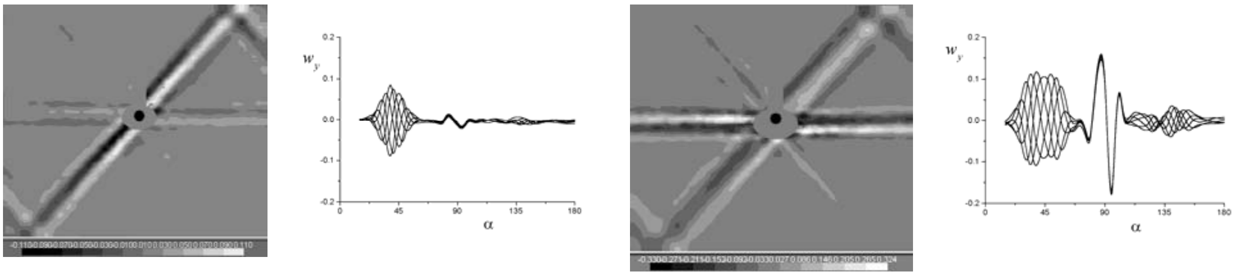
Results and discussion. Typical wave patterns and corresponding distributions of w_y along the angular coordinate α at distance $6D$ from the origin of the coordinate system xOy are shown in figure 1. The angular coordinate α is counted clockwise from the vertical. The profiles $w_y(\alpha)$ taken with the phase shift of $\pi/6$ correspond to the 20th period after the motion start-up. In the case of horizontal oscillations the 'St. Andrew cross' wave pattern has mirror symmetry with respect to the horizontal plane $y = 0$, and in the case of circular oscillations we observe symmetry with respect to rotation by 180°. For horizontal oscillations we observe the disturbances of equal intensity in all four quadrants of the Cartesian coordinate system. In the case of the clockwise circular oscillations the waves in the first and third quadrants have much higher intensities than those observed in the second and fourth quadrants. At small value of the motion amplitude the intensity of waves in the first quadrant in the case of circular oscillations is twice the intensity of the waves in the case of the horizontal or vertical oscillations. A parallel can be drawn with the well-known classic results [14, 15] for surface waves: circular clockwise oscillations of a submerged circular cylinder in homogeneous fluid generate only the surface waves propagating to the right. This property is related with symmetry and antisymmetry of wave patterns generated by vertical and horizontal oscillations.

As the radius of the orbital motion increases, the density-gradient disturbance at zero frequency increases markedly so that for $a/D = 0.45$ the maximum magnitudes of w_y in the horizontal stripe are higher than those observed in the main wave beams. The secondary wave radiation into the second and fourth quadrants markedly increases with a/D.



$a/D = 0.15$ $a/D = 0.45$

Horizontal oscillations



$a/D = 0.15$ $a/D = 0.45$

Circular oscillations

Figure 1: Wave patterns and corresponding distributions of w_y *along the angular coordinate* α *at distance* $6D$ *from the cylinder. The oscillation frequency is* $\Omega = 0.76$. *The field of view is 53 by 44 cm. Grey levels show the field of* w_y.

It is known that the formation of the wave beams is a complicated process with interrelated effects due to the size of the disturbance, the start-up of the motion and the viscosity of the fluid [16]. In the present work the time-evolutions of internal waves at a certain fixed distance from the origin have been analyzed via construction of spatiotemporal images. Both the waves in the secondary wave beams and the 'zero-frequency' disturbance tend to evolve at a time scale much longer than the period of oscillations T. In the horizontal stripe maximum positive and negative disturbances in the case of circular oscillations does not reach saturated values within the first 20 periods of oscillations. Since there is a continuous horizontal pumping of fluid due to orbital motion of the cylinder, there are reasons to believe that the steady state in the horizontal stripe cannot be reached. The orbital motion ultimately leads to formation of sharp horizontal density interfaces. In contrast, in the case of horizontal oscillations the 'zero-frequency' disturbance does not tend to increase during the studied span of time.

Conclusions. This report presents an experimental study of internal waves generated by rectilinear and circular oscillations of a circular cylinder in a uniformly stratified fluid. Synthetic schlieren technique is used for quantitative measurements of internal wave parameters.

It is shown that in the case of small oscillations of the circular cylinder the observed wave patterns are in good agreement with the linear scenario [4, 5, 17]. The possibility of concentrating wave energy in two wave beams instead of four in the case of circular oscillations is important in the context of design of effective internal-wave generators, and in relation to some oceanographic applications where the occurrence of single-beam internal-wave structures has been recently emphasized theoretically and experimentally [18, 19]

As the amplitude of circular oscillations increases, significant non-linear effects are observed. In particular, strong density-gradient disturbance is generated in the horizontal stripe due to nearly horizontal currents. The disturbance evolves at the time-scale much longer than the oscillation period and ultimately leads to formation of sharp interfaces. The drastically different magnitudes of the perturbation in the horizontal stripe in the cases of circular and rectilinear oscillations are related with non-zero and zero values of the mean total angular momentum of fluid, respectively.

Experimental data for the spheres oscillating horizontally in a uniformly stratified fluid are currently being processed. The comparison of experimental data and theoretical predictions will be presented at the workshop.

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