Using generalized modes for time domain seakeeping calculations

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Introduction

For calculation of whipping, springing and multi-body interaction more than the standard six degrees of freedom are needed. By generalizing all degrees of freedom to flexural modes it is possible to create a code which can be used for whipping, springing, single and multi body problems.This approach is described in [1] and [2] for frequency domain calculations.

Usually the hydrodynamic coefficients are calculated using pre-defined displacements of every panel for the six degrees of freedom. Using the general modes approach, the displacements of the panels is an additional input for the hydrodynamic calculation. For example: a heave mode is created by a shape vector with $(0, 0, 1)$ for all panels. The shape vector as calculated by e.g. a beam or 3D FEM is used for modes for whipping or springing calculations.

In the frequency domain, rigid bodies and the bending modes can be treated exactly equal. After the hydrodynamic coefficients are calculated the system of unknowns displacements can be solved and the response of all modes is known. Due to the non-linear terms in the time domain calculation, it is not possible to solve rigid body and bending modes in a similar way. It is necessary to account for the rigid (multi) body dynamics.

Example

The response of two coupled barges, see figure 1, is calculated to illustrate the use of generalized modes in time domain. Springs in all directions are used to couple the barges.

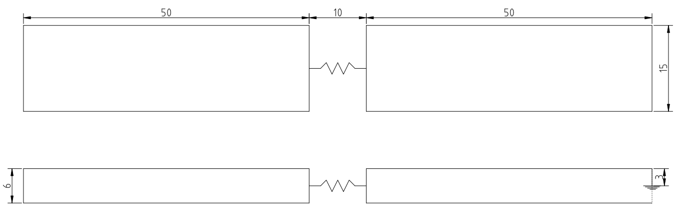

Figure 1: Barge

The response is calculated using two approaches. For the first approach the barges are two bodies coupled with springs. In this case the first six DoFs are the rigid body displacements of the first barge and the second six DoFs are the rigid body displacements of the second barge. For the second approach the two barges are considered to be one flexible body. The first six DoFs are the rigid body motions of the two barges together and the other six are flexural modes between the barges. The heave and pitch motions of the two approaches are shown in figure 2.

In frequency domain both approaches result in exactly the same response. In the time domain the one flexible body approach will be incorrect when the relative angles between the barges are large. When for the two bodies the (Euler) rotations are correctly taken into account the shape of the bodies will still be correct with large rotations. The flexural approach will result in a distorted shape if the angles are large. The difference between the two approaches is shown in figure 3. This difference shows also the need to account for the rigid body dynamics of all bodies.

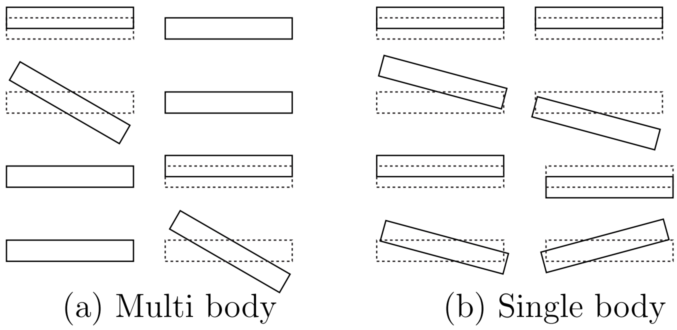

Figure 2: Degrees of freedom for heave and pitch

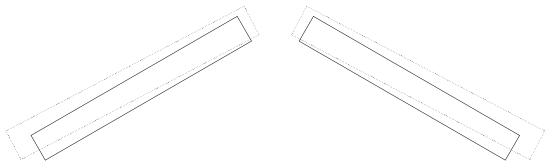

Figure 3: Shape of 30 degree pitch for single body (dotted) and double body

Time domain

The response of the modes are calculated using the procedure proposed by Cummings [3]:

$$(\mathbf{m} + \mathbf{m}_{\mathbf{a}}^{\infty}) \cdot \bar{Y}_a + \int_0^t \mathbf{K}(t-\tau) \cdot \bar{Y}_v \, d\tau + \mathbf{k} \cdot \bar{Y}_d = \bar{F}(t) \tag{1}$$

Vectors $\bar{Y}_a, \bar{Y}_v$ and $\bar{Y}_d$ are the actual acceleration, velocity and displacements of all modes. The 4th order Runge Kutta method is used to integrate this equation of motion.

Figure 4 shows the different coordinate systems that are used. Every body has his own ship-fixed system.

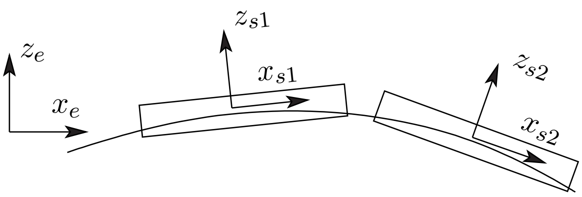

Figure 4: Coordinate systems

Modal description

The mode shapes are described by the modal shape vector $\bar{h_i}$. This vector is created for all points: panels and masses. The same vector is also used for the frequency domain calculation. For the non-linear time domain calculation an additional matrix **S** is introduced which transfers the modes to the six rigid body DoFs of the individual bodies. Matrix **S** has a number of columns which is equal to the number of modes and the number of rows is equal the number of bodies times six. For a normal single body calculation this matrix will be the identity matrix. For two body calculation with only heave and pitch modes, matrix **S** will be a 12 by 4 matrix.

The rows of the **S**-matrix for bending modes for whipping and springing are filled with zeros because these are not rigid body motions. It is assumed that all modes are either rigid or pure bending without a rigid component.

Dynamics

The accelerations for all DoFs are solved in the ship-fixed frames. The motions are integrated in a earth-fixed frame. Before the acceleration can be integrated, all accelerations of rigid body modes have to be transfered to the earth-fixed frame using the Euler transformation matrix.

Vector $\bar{y}_{a,sf}$ are the ship-fixed accelerations of all modes. The ship-fixed accelerations of rigid body modes are obtained by:

$$\bar{y}_{a,r,sf} = \mathbf{S} \cdot \bar{y}_{a,sf} \tag{2}$$

The same transformation is applied to obtain the rigid body motions:

$$\bar{y}_{d,r} = \mathbf{S} \cdot \bar{Y}_d \tag{3}$$

For every individual body, n, the ship-fixed accelerations are transfered to earth-fixed accelerations:

$$\bar{y}_{a,r,e,(6n-5:6n)} = \mathbf{T}_{se}(\bar{y}_{d,r,(6n-2:6n)}) \cdot \bar{y}_{a,r,sf,(6n-5:6n)} \tag{4}$$

Where $\mathbf{T}_{se}$ is the Euler transformation matrix from ship-fixed to earth-fixed. The earth-fixed acceleration vector $\bar{Y}_a$ is obtained by:

$$\bar{Y}_a = \mathbf{S}^T \cdot \bar{y}_{a,r,e} \tag{5}$$

For the non-rigid body modes there are no differences between the earth and ship fixed coordinate systems:

$$\bar{Y}_{a,(i)} = \bar{y}_{a,r,e,(i)}, \quad \text{where } \mathbf{S}_{(n,:)} \equiv 0 \tag{6}$$

The earth-fixed acceleration vector $\bar{Y}_a$ is used for integration of the motions.

Forces

The pre-calculated hydrodynamic coefficients are used to calculate the diffraction and radiated forces. These coefficients are obtained using the mode shape vectors, therefore the coefficients will give the correct excitation for all the modes.

The radiation force is calculated using retardation functions. The hydrodynamic coefficients are considered to be earth-fixed, therefore the radiation force can be obtained by the history of the $\bar{Y}_v$ vector.

The diffraction force is calculated using the frequency domain RAOs. The actual location of the different bodies is used to calculate the diffraction force for all modes of the bodies.

The same kind of transformation as used in equations (2) to (6) are used to transfer the earth-fixed force to ship-fixed.

The pressure of the incoming wave and the hydrostatic pressure are calculated for every panel. This pressure is dependent on the earth-fixed location of the panel. The earth-fixed location is the earth-fixed displacement of the CoG of the body to which the panel belongs plus the earth-fixed distance between the panel and the CoG. The non-rigid modes displace in the ship-fixed frame. The total ship-fixed distance between the CoG and the panel is:

$$x_{st} = x_s + \sum_{i=1}^{\mathrm{ndof}} \bar{Y}_{d,(i)} \cdot h_i, \text{where } \mathbf{S}_{(i,:)} \equiv 0 \tag{7}$$

Where is x_s the ship-fixed coordinate of the panel and h the mode shape vector. i_s is the number of the body to which the panel is attached. The earth-fixed location of the panel, x_e is equal to:

$$x_e = \bar{y}_{d,r,(6i_s-5:6i_s-3)} + \mathbf{T}_{es}(\bar{y}_{d,r,(6i_s-2:6i_s)}) \cdot x_s \tag{8}$$

The force by the pressure at the panel, $\bar{f}_i$ will give an excitation force at the modes, F_i of:

$$F_i = h_i \cdot \bar{f}_i \tag{9}$$

After all forces are known the accelerations are calculated and the next time step is calculated.

Results

The motion of the coupled barges are calculated for an irregular head sea with a significant wave height of 8 meter. Figure 5 shows the relative pitch angle between the barges using both approaches in the time domain calculation. The results are almost identical. This shows both approaches are valid in time domain. In the case the springs between the barges are much weaker or when there is no coupling between the barges only the multi body approach will be correct because the flexural modes can not describe large rotation correct.

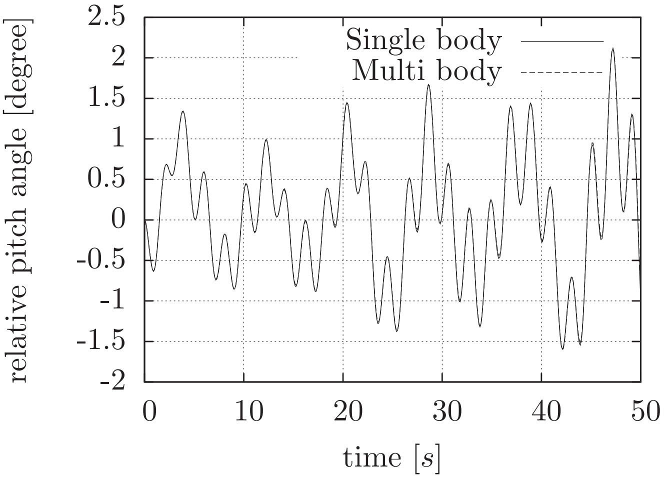

Figure 5: Relative pitch motion between barges

By introducing generalized modes for the hydrodynamic calculation it is possible to calculate whipping, springing and multi body interaction. The (multi) body dynamics should be accounted for if generalized modes are used in time domain.

References

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