

SECOND ORDER NEAR-TRAPPING FOR MULTI-COLUMN STRUCTURES AND NEAR-FLAT QTFS

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INTRODUCTION

Arrays of vertical surface piercing columns in an otherwise empty ocean exhibit near-trapped modes – which may be defined as modes of strong local free surface vibration that ‘fit within the array’ but which are damped by radiation to infinity. Such modes may be excited by incident free waves with the necessary frequency, or through forcing by nonlinear combinations of freely incident linear waves. The most well-known example of this nonlinearity is second-order near trapping as described by Malenica, Eatock Taylor and Huang (1999). The excitation at second order is local to the cylinder array, through products of pairs of waves with component frequencies (f_1, f_2) , the vigorous response being at the sum frequency.

This paper presents a discussion of the nature of the quadratic sum response for pairs of input frequencies with the same output frequency sum, giving rise to a large 2nd order contribution to the free-surface elevation in the array $\eta^{(2+)} = QTF(x, y) A_1 A_2 \exp i 2\pi(f_1 + f_2)t$. The spatial structure of the response is implicit within the complex QTF coefficient.

An apparently new observation is the general form of the sum QTF when near-trapping is significant. Ordinarily the QTF is viewed as having 2 arguments, being the frequencies (f_1, f_2) of the input wave pair. Instead, an informative way of treating these arguments is as the sum output frequency $\Omega = 2\pi(f_1 + f_2)$ and the frequency difference $\Delta\omega = 2\pi(f_1 - f_2)$, this being the distance away from the leading diagonal. In some cases the sum QTF in this form is a strong function of the output frequency (as expected) and *virtually independent* of the distance away from the leading diagonal. This observation has the same empirical form as the Newman approximation (1974) for difference frequency force QTFs for vessels in irregular waves but here we claim that it can apply to surface elevation sum QTFs for cylinder arrays.

If this result can be justified, it would be very useful in an engineering design context since calculation of the full $(N \times N)$ QTF matrix for N linear input components requires considerable computational effort using codes such as WAMIT or DIFFRACT. We suggest that the whole matrix might be approximated using only the leading diagonal.

We conclude with a realistic practical example showing that 2nd order near-trapping may be significant in the context of offshore engineering design.

LINEAR RESPONSE

We consider a 4-column concrete gravity base structure with a caisson of height 15m standing in water of depth 53m, as shown in Figure 1. Plane waves are incident from the left. Figure 2 shows the linear transfer function for surface elevation for this structure at a point on the center-line between the front two legs ($x = -35\text{m}$, $y = 0\text{m}$).

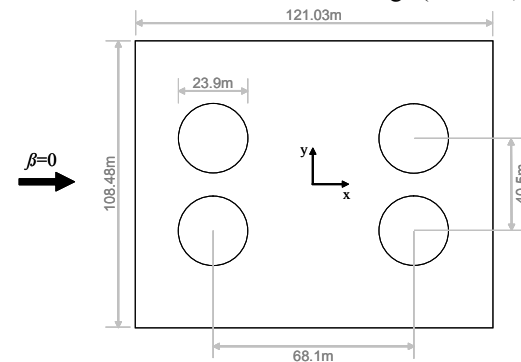


FIGURE 1. A four column structure on a caisson

Figures 3 and 4 show the surface elevation around the structure for an input frequency of $f_1 = 0.126\text{Hz}$, the lowest response peak. Note the regions of very vigorous surface response, between the front legs and close to the rear legs.

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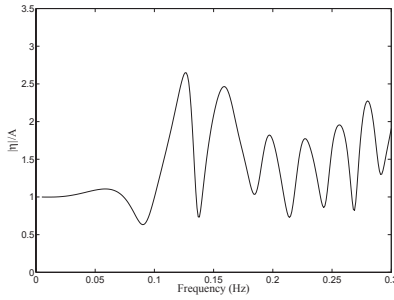


FIGURE 2. Linear surface elevation amplification between the front two legs

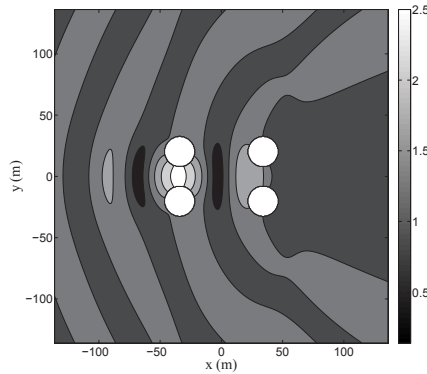


FIGURE 3. Contour map of linear surface motion for waves at $f_I=0.126\text{Hz}$.

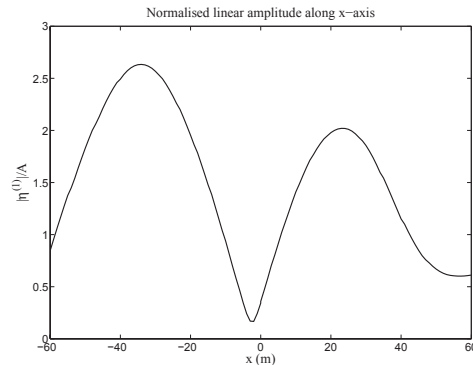


FIGURE 4. Linear amplitude response profile along the center-line of the structure at $f_I=0.126\text{Hz}$.

SECOND ORDER EXCITATION OF ENHANCED SURFACE RESPONSE

Significant linear amplification only occurs for short waves with periods of $\sim 8\text{s}$ and less. Severe winter storms with large significant wave heights typically have periods rather greater than this, so the question arises as to whether a large second order sum response might occur for a $\sim 16\text{s}$ incident wave. Thus, we consider the structure

excited by steep regular waves with a frequency $f_I=0.063\text{Hz}$, chosen such that twice this frequency is $2f_I=0.126\text{Hz}$, a frequency that we already know the structure responds at vigorously. We use the Oxford 2nd order diffraction program DIFFRACT (Eatock Taylor and Chau 1992).

Figure 5 shows the amplitude of the 2nd order sum response along the centre-line of the structure for this monochromatic incident wave. Two obvious remarks can be made: the peak values of the non-dimensionalised response are large: $\eta^{(2+)}/(kA^2) \sim 10$, and the geometric shape of the profile is remarkably similar to that of the linear response function. Notice the amplitude of this 2nd order sum local response is an order of magnitude larger than the bound 2nd order sum term in the incident wave, and even for a modest wave steepness the 2nd order sum response within the cylinder array can be as large as the linear incident wave. In Figure 5, the modulus of the total 2nd order response is shown as the solid line, the dashed line is the 2nd order potential part and the dotted line the 2nd order quadratic part, which shows no enhanced near-trapping behaviour.

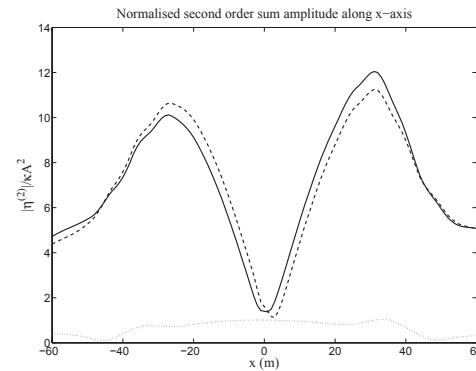


FIGURE 5. 2nd order sum amplitude profile along the center-line of the structure at $2f_I=0.126\text{Hz}$.

QTFs OFF THE LEADING DIAGONAL

We now discuss the general form of the 2nd order quadratic transfer functions for a pair of incident linear waves with frequencies (f_1, f_2) and amplitudes (A_1, A_2) . The QTFs are now dimensional, having units of $1/\text{m}$, and are defined in terms of the 2nd order surface as:

$$\eta^{(2+)} = QTF(x, y) A_1 A_2 \exp i 2\pi(f_1 + f_2)t$$

We present the QTF surface in Figure 6 as 3-D views of the real and imaginary parts, viewed so that the 1:1 frequency leading diagonal runs left to right and the frequency difference runs towards and away from the viewpoint. The geometry and wave approach direction

are still as shown in Figure 1. The input range of frequencies is chosen such that output frequencies $\Omega = 2\pi(f_1 + f_2)$ cover the two lowest frequency peaks shown in Figure 2.

These particular views are for the free-surface QTF at a point close to one of the rear legs, but the observation that the QTFs are virtually independent of the frequency difference $\Delta\omega = 2\pi(f_1 - f_2)$ with $\Omega = 2\pi(f_1 + f_2)$ fixed is true for all free-surface points within the array that we have examined. Notice also the strong variation of the QTFs in both amplitude and phase in the 1:1 frequency $\Omega = 2\pi(f_1 + f_2)$ direction, akin to the variation of the linear transfer function with frequency.

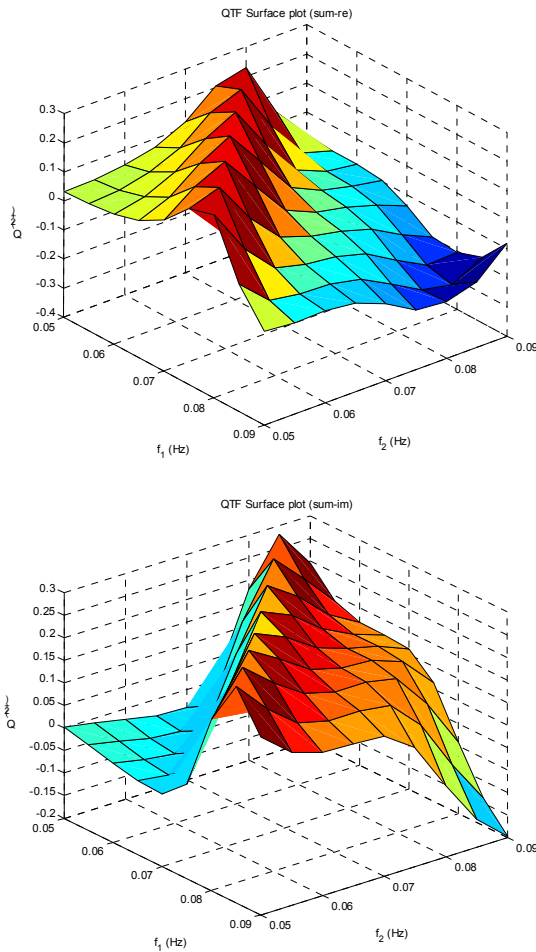


FIGURE 6. Form of the QTF matrix with (f_1, f_2) .
Top real part, bottom imaginary.

WHY THE QTFs ARE NEAR FLAT?

The vigorous second order sum responses of a cylinder

array are dominated by the second order sum potential. Although there is a quadratic contribution, this is small ($\sim O(1)kA^2$, see Figure 2). The second order potential response can be itself be separated into 2 parts: the scattering of the incoming bound second order sum potential and the field driven by the free-surface integral.

We consider the free-surface integral contribution first. This can be written as the solution to a linear wave-like equation:

$$\frac{\partial \phi^{(2)}}{\partial z} - \frac{\Omega^2}{g} \phi^{(2)} = f(x, y)$$

The term $\Omega = 2\pi(f_1 + f_2)$ is the sum output frequency at second-order. The source $f(x, y)$ is simply a set of products of linear wave components. This equation has an integral solution

$$2\pi\phi^{(2)} + \int \phi^{(2)} \frac{\partial G}{\partial n} dS_B - \int G \frac{\partial \phi^{(2)}}{\partial n} dS_B = \int G f dS_F,$$

the first 2 integrals being over the rigid body surface, the 3rd integral being over the free-surface.

$G(\Omega)$ is a *linear* Green's function for the potential $\phi^{(2)}$. Its basic structure when combined with the 2 integrals over the rigid body surfaces captures the near resonant response of the multi-column system both spatially and temporally. Thus $G(\Omega)$ is completely unaffected by changes in the (f_1, f_2) pair as long as the sum $\Omega = 2\pi(f_1 + f_2)$ is unaltered. Any change in the QTF with Ω constant must arise because of variations in the source term $f(x, y)$ with the choice of the frequency pair.

What is the nature of this source term $f(x, y)$? It consists of simple products of linear components at frequencies (f_1, f_2) , each linearly diffracted by the cylinder array. At frequencies well below $f_1 = 0.1$ Hz, the linear response is not significantly modified from that of the incident field, see Figure 2. With $\Omega = 2f_1 = 0.126$ Hz, the 2nd order sum interactions drive the lowest near-resonant mode for the array. Thus, the array is compact in terms of linear diffraction for each of the linear components, pairs of which efficiently couple quadratically to the structure, because these linear waves are of the order of 4x longer. Thus, the product 'source terms' cannot vary significantly across the array depending on the precise values of the pair (f_1, f_2) . The only significant spatial variation in the linear waves will be the local distortion in the vicinity of each column. This will be similar to a steady potential flow past each column, dependent on the size of the columns rather than the wavelength.

Thus, neither the Green's function nor the source term are strong functions of $\Delta\omega = 2\pi(f_1 - f_2)$ with $\Omega = 2\pi(f_1 + f_2)$ fixed. Hence the calculated QTFs, being the solution to the integral equation driven by the free-surface integral, cannot vary significantly with the distance away from the leading diagonal.

It should be stressed that this flatness of the contribution of the the free-surface integral to the overall second-order QTFs can only be expected to occur for output frequencies corresponding to the lowest first one or two near-trapped modes. Only when the corresponding individual linear waves within the 2nd order pair are both long compared to the scattering array but the 2nd order response 'fits' within the array will the simple argument work.

There is still the contribution to the QTFs from the scattering of the incident second order sum potential to consider. Since the sum frequency Ω is the same for all appropriate linear pairs and this frequency term which controls the efficiency of scattering, then the resultant scattered field is likely to be close to independent of the $\Delta\omega$ distance for the frequency pair. This requires that the array is relatively compact on a wavelength corresponding to $(k_1 + k_2)$, possibly a more stringent requirement than for the free-surface integral contribution.

The relative contributions of the free-surface integral and the second-order incident component to the overall sum potential will depend on water depth, and this is currently being examined. However, we believe that both contributions to the 2nd order sum potential term should satisfy our Newman-type approximation.

THE PHYSICAL SIGNIFICANCE OF LARGE QTFs AND NEAR-TRAPPING AT 2ND ORDER

We conclude by presenting the predicted surface elevation time history to 2nd order between the rear two legs for a focussed wave group of linear crest elevation of 11m incident on the structure. For this case, the peak of the Gaussian spectrum is located at $f_p = 0.07\text{Hz}$, close to one half of the second-order trapping frequency. The Gaussian spectrum for the wave group is chosen so it resembles the peak of a JONSWAP shape without the high frequency tail. For more details see Walker et al. 2007.

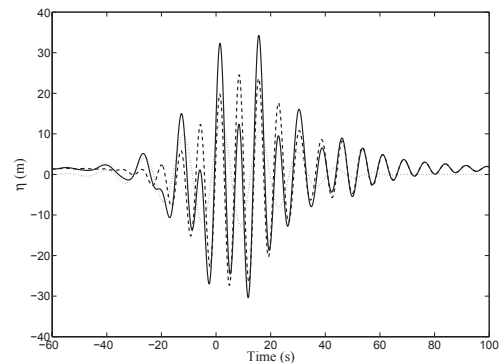


FIGURE 7. Predicted surface elevation between the rear legs of the structure for an 11m incident focused wave group : $\eta^{(1)}$, ----- $\eta^{(2+)}$, — $\eta^{(1+2)}$.

The incident wave is large but perhaps typical of those of interest in design. The free-surface is predicted to reach over 30m above mean sea level – 3× the amplitude of the incoming wave - an elevation of concern for water impact with the deck of a platform. Thus, second-order near-trapping seems to be important in terms of water projection to very high levels for structures with closely spaced large volume legs. The implications for setting deck elevations in design are clear.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the Engineering and Physical Sciences Research Council (EPSRC) studentship supporting Dan Walker, and use of the Oxford Supercomputing Centre.

REFERENCES

- EATOCK TAYLOR, R. and CHAU, F.P. (1992). Wave diffraction theory - some developments in linear and non-linear theory. Trans. ASME, J. Offshore Mech. and Arctic Eng. 114, 185-194.
- MALENICA, S., EATOCK TAYLOR, R. and HUANG, J.B. (1999) Second order water wave diffraction by an array of vertical cylinders. Journal of Fluid Mechanics, 390, 349-373.
- NEWMAN, J.N. (1974) Second order slowly varying forces on vessels in irregular wave. Paper 19, pages 182-186 in 'The Dynamics of Marine Vehicles and Structures in Waves', ed. Bishop, R.E. and Price W.G. Publ. By I.Mech. E, London
- WALKER, D.A.G., EATOCK TAYLOR, R., TAYLOR, P.H. and ZANG, J. (2007) Wave diffraction and near-trapping by a multi-column gravity base structure. Submitted to Ocean Eng.