Numerical Solution of Body-Exact Problem in the Time Domain with a Panel-Free Method

W. Qiu* and H. Peng**
*Memorial University of Newfoundland, St. John's, Canada A1B 3X5
**Oceanic Consulting Corporation, St. John's, Canada A1B 2X5
Email: qiuw@engr.mun.ca

Introduction

Most of computations for wave-body interactions in the time domain are based on the panel methods. For body-exact problems, repanelization of the instantaneous wetted body surface is required at each time step. Based on the work of Qiu and Hsiung (2002) and Qiu *et al.* (2004), the body-exact problem has been solved in the time domain with the panel-free method and exact geometry. In the present study, the body boundary condition is imposed on the instantaneous wetted surface exactly at each time step. The free surface boundary is assumed linear so that the time-domain Green function can be applied. The body geometry is represented by Non-Uniform Rational B-Spline (NURBS) surfaces. At each time step, the instantaneous wetted surface is obtained by trimming the entire body surface. With the panel-free method, the body-exact problems are solved without involving repanelization of the wetted hull surface at each time step.

For illustration, hydrodynamic forces on a submerged sphere undergoing large amplitude motion were computed and compared with analytical solutions. The computation was also extended to a vertical cylinder under prescribed motion.

Mathematical Formulation

It is assumed that the fluid is incompressible, inviscid and free of surface tension and that the flow is irrotational. The velocity potential, $\phi(P(x,y,z);t)$, satisfies the following governing equation, boundary conditions, far-field conditions and initial conditions:

$$\nabla^2\phi = 0 \qquad \text{in } \Omega \tag{1}$$

$$\begin{aligned}
\frac{\partial^2\phi}{\partial t^2} + g\frac{\partial\phi}{\partial z} &= 0 && \text{on } Z=0,\ t>0\\
\nabla\phi\cdot\mathbf{n} &= V_n - \frac{\partial\phi_I}{\partial n} && \text{on } S_b(t),\ t>0\\
\nabla\phi,\ \frac{\partial\phi}{\partial t} &\to 0 && \text{at } S_\infty,\ t>0\\
\phi = 0,\ \frac{\partial\phi}{\partial t} &= 0 && t=0
\end{aligned}$$

where $\phi_I(P(x,y,z);t)$ is the incident wave potential, g is the gravitational acceleration, Ω is the computational domain, $S_b(t)$ is the instantaneous wetted body surface, S_∞ is the infinite boundary, t is time, and $\mathbf{n}$ is the unit inner normal vector pointing into the body surface from the water.

Since the linear free surface boundary condition is satisfied, the transient Green function can be applied for the initial boundary value problem.

Based on the work of Qiu and Hsiung (2002), the velocity potential, $\phi(P;t)$, can be obtained from the desingularized integrals in terms of source distribution as follows:

$$\begin{aligned}
\phi(P;t) &= \int_{S_b(t)} G_1\left[\sigma(Q;t) - \gamma(Q;t)\frac{\sigma(P;t)}{\gamma(P;t)}\right]dS\\
&+ 2\int_{S_b(t)} \sigma(Q;t)G_2 dS + \phi_0(t)\frac{\sigma(P;t)}{\gamma(P;t)}\\
&+ \int_0^t d\tau\int_{S_b(\tau)} \sigma(Q;\tau)F dS\\
&- \frac{1}{g}\int_0^t d\tau\int_{\Gamma(\tau)} \sigma(Q;\tau)FV_{\Gamma n}V_n d\Gamma
\end{aligned} \tag{2}$$

where $G_1(P,Q) = -\frac{1}{4\pi}(\frac{1}{r}+\frac{1}{r_1})$, $G_2(P,Q) = \frac{1}{4\pi}\frac{1}{r_1}$, $F(P,Q;t-\tau)$ is the wave term of the Green function, $V_{\Gamma n}$ is the velocity of the instantaneous waterline, $\Gamma(\tau)$, in the direction of $\mathbf{n}_\Gamma$, $\mathbf{n}_\Gamma$ is the unit normal of $\Gamma(\tau)$ on the free surface, $\gamma(P;t)$ is the source distribution on the instantaneous wetted surface $S_b(t)$ which makes the instantaneous body surface an equipotential surface of potential $\phi_0(t)$. It can be seen that the first integral in the right hand side (RHS) of Eq. (2) is zero when P coincides with Q. Therefore, the singularity in the integral equation due to the Rankine term is removed.

The source strength can be solved from the following desingularized equation:

$$\begin{aligned}\frac{\partial \phi(P;t)}{\partial n_P} &= -\sigma(P;t) \\ &+ \int_{S_b(t)} \left[\sigma(Q;t)\frac{\partial G_1}{\partial n_P} - \sigma(P;t)\frac{\partial G_1}{\partial n_Q}\right] dS \\ &+ 2\int_{S_b(t)} \sigma(Q;t)\frac{\partial G_2}{\partial n_P} dS \\ &+ \int_0^t d\tau \int_{S_b(t)} \frac{\partial F}{\partial n_P}\sigma(Q;\tau) dS \\ &- \frac{1}{g}\int_0^t d\tau \int_{\Gamma(\tau)} \sigma(Q;\tau)\frac{\partial F}{\partial n_P} V_{\Gamma n} V_n d\Gamma \quad (3)\end{aligned}$$

Note that the second term in RHS of Eq. (3) is zero when P coincides with Q.

Since the desingularized integral equations, (2) and (3), can be discretized over the exact geometry with Gaussian quadrature, the next step is to accurately compute coordinates, normals and Jacobians of Gaussian points on the instantaneous wetted surface. The wetted surface $S_b(t)$ can be obtained by trimming the master body surface below the water plane ($Z = 0$) at each time step. In order to automate the trimming process, it is desirable to describe the master surface by mathematical representation. In this work, the body geometry is represented by NURBS surfaces.

With the representation of the exact master surface by NURBS, a surface/surface intersection problem will then be solved to find the instantaneous wetted portion. A bisection scheme is applied to determine the waterline in the uv-plane. It is assumed that there are no self-intersecting waterlines and the waterline intersects with the boundary of a patch only at two points.

The force on the body is computed by integrating the pressure over the instantaneous wetted surface $S_b(t)$. The temporal and spatial derivatives of ϕ_I can be obtained from the incident wave potential. The velocities, $\nabla\phi$, are computed based on the work of Bingham and Maniar (1996) from the velocity terms normal and tangential to the body surface, i.e, $\nabla\phi = \nabla\phi \cdot \mathbf{n} + \nabla\phi \cdot \mathbf{t}$, where $\mathbf{n}$ and $\mathbf{t}$ are the normal vector and the tangential vector of a point on the body surface, respectively. The temporal derivative, $\frac{\partial\phi}{\partial t}$, is evaluated based on the work of Dameier (1999) as

$$(\frac{\partial \phi}{\partial t})^n = \frac{\phi^n - \phi^{n-1}}{\Delta t} - \mathbf{U}\cdot\nabla\phi^n - \mathbf{u}_t^n \cdot \{\phi_{uv}^n\} \quad (4)$$

where the superscript n denotes the nth time step, $\mathbf{U}$ is the body moving speed, $\mathbf{u}_t = \{u_t, v_t\}$ denotes the velocity vector of a Gaussian point which can be determined by $u_t^n = (u^n - u^{n-1})/\Delta t$ and $v_t^n = (v^n - v^{n-1})/\Delta t$, and Δt is the time step.

After the temporal and spatial derivatives of ϕ are determined, the pressures on Gaussian points can calculated and forces on the body surface can be obtained.

Numerical Results

The hydrodynamic forces on a submerged sphere undergoing large amplitude motion were first computed and compared with the analytical solutions by Wu (1994). Forces on a vertical cylinder with prescribed heave motions are then presented and compared with numerical results based on the linear solution.

Submerged Sphere: The submerged sphere was described by eight NURBS surfaces. Each NURBS surface is formed by a 4×4 control net and B-Splines with degrees of 3 in the u- and v-directions. In the computation, the submerged depth, h, measured from the water surface to the centre of the sphere is three times of the radius of the sphere (R). The sphere is under pure heave motions. The time step was chosen as 0.1 seconds. The computed heave force, nondimensionalized as $F' = F/(\frac{4}{3}\rho\pi R^3)$, were compared with the analytical solutions by Wu (1994) for various motion amplitudes, η/R=0.5, 1.0 and 1.5, at $kR = 0.1$ and $kR = 1.0$, where ρ is the density of fluid, η is the heave motion amplitude, and k is the wave number.

Figure 1 shows the computed heave forces in comparison with the analytical solutions of Wu (1994) for η/R=1.0 at kR=0.1. Figures 2 and 3 present the results for η/R=0.5 and 1.0 at $kR = 1.0$. With the motion amplitude increased, greater discrepancies are shown between the numerical results and the analytical solutions. Note that the analytical solution of Wu (1994) was based on the multipole expansion and was obtained by truncating the infinite series. It is thought that the discrepancies are due to the truncated error in the analytical solution.

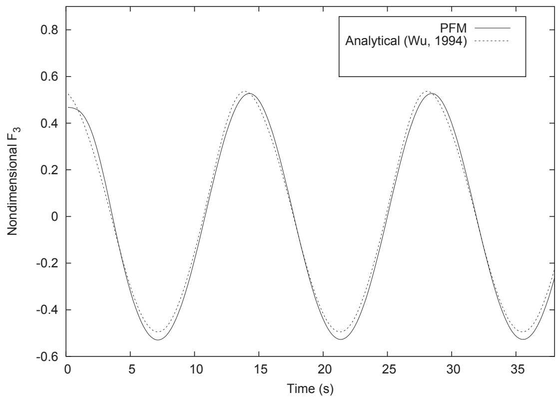

Figure 1: Heave force at η/R=1.0 and kR=0.1

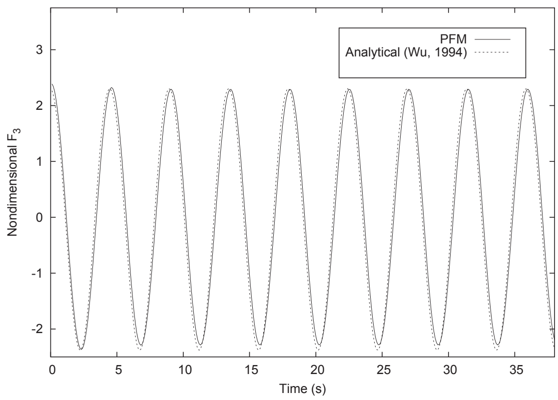

Figure 2: Heave force at η/R=0.5 and kR=1.0

Cylinder: The computation was then performed for a vertical cylinder under prescribed motion. The draft of the cylinder at rest is T. The ratio of radius R to T is given as 2.0. The vertical velocity of the cylinder is prescribed to be $V(t) = a\omega \sin(\omega t)$ with the body displacement of $z(t) = -a\cos(\omega t)$ where a is the heave motion amplitude. Computations were carried out for various amplitude ratios, a/T=0.05, 0.1 and 0.15. In the computations, 576 Gaussian points (192 on the bottom and 384 on the side surface) were automatically distributed on the instantaneous wetted surface of the cylinder at each time step. The time step was chosen as 0.05 seconds and $\omega = \pi/2$. The computed heave forces minus the hydrostatic forces are nondimensionalized as $F' = F/(\rho g R^2 a)$ and compared with the linear solution in the time domain. The linear results were computed from the impulse response function by the panel-free method (Qiu and Hsiung, 2002).

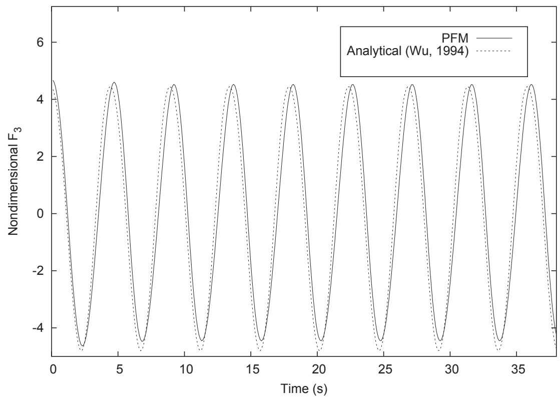

Figure 3: Heave force at η/R=1.0 and kR=1.0

In Fig. 4, the computed heave force is compared with the linear solution for $a/T = 0.05$. As we can see, the results follow closely with the linear solution for relatively small amplitude a. For larger amplitudes, $a/T = 0.1$ and $a/T = 0.15$, the heave forces are compared with the linear solutions in Fig. 5. With the amplitude a increased, differences at peaks and troughs tend to be larger. The nonlinear values of the heave forces by the body-exact solution are smaller than the linear solutions.

Figure 6 shows the components of the vertical force on the cylinder including hydrostatic, inertial (-$\partial\phi/\partial t$) and quadratic ($-|\nabla\phi|^2/2$) components for the case of $a/T = 0.10$. The quadratic term is primarily at the second harmonic. The total force along with the hydrostatic and inertial terms mainly oscillate at the forcing frequency.

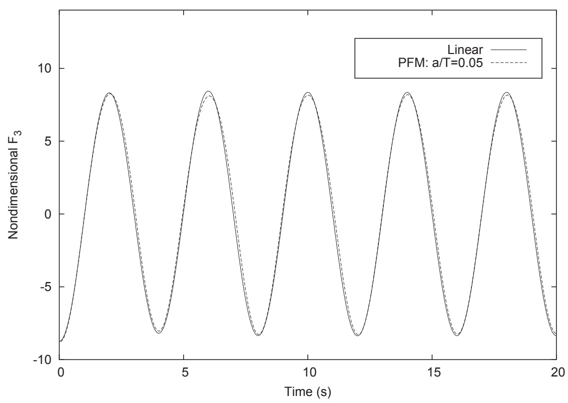

Figure 4: Heave force at a/T=0.05, $\omega = \pi/2$

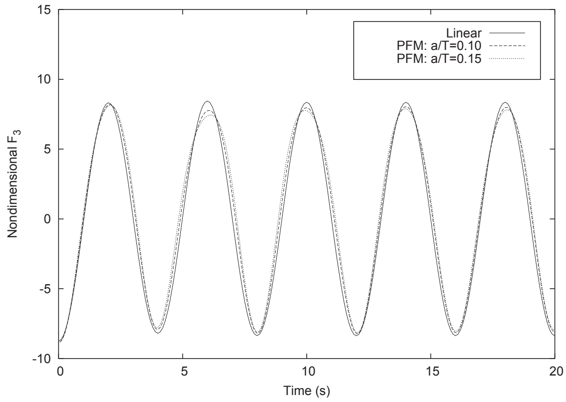

Figure 5: Heave forces at a/T=0.1 and 0.15, $\omega = \pi/2$

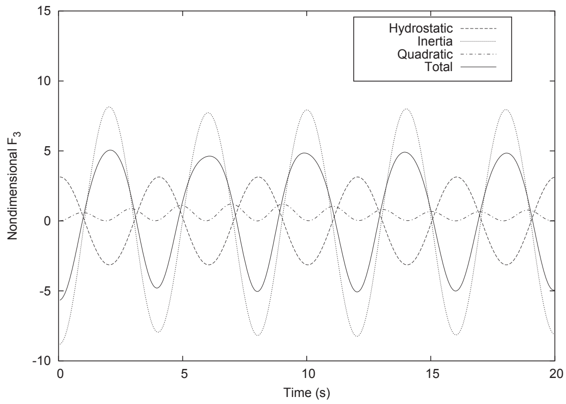

Figure 6: Heave force components at $a/T = 0.1$, $\omega = \pi/2$

Conclusions

The body-exact problem has been solved in the time domain by the panel-free method. In the present study, the body boundary condition is imposed on the instantaneous wetted surface exactly at each time step. The free surface boundary condition is linearized so that the time-domain Green function can be applied. The body geometry is represented by NURBS surfaces. At each time step, the instantaneous wetted surface is obtained by trimming the entire body surface. Gaussian points are automatically distributed over the wetted surface. With the panel-free method, the body-exact problem is solved without involving repanelization of the instantaneous wetted hull surface at each time step.

The computations have been carried out to a submerged sphere and a vertical cylinder under prescribed large-amplitude motion. The hydrodynamic forces on the submerged sphere agree well with the analytical solutions. The computed forces on the vertical cylinder are compared with numerical results from the impulse response function. For small amplitude motion, the body-exact solutions by the panel-free method follow closely with the linear solution. With the amplitude increased, the nonlinear values from the body-exact solution are smaller than the linear results.

Studies are being carried out to compute motions and forces on free floating bodies in waves with and without forward speed.

Acknowledgments

This work was supported by the Natural Sciences and Engineering Research Council of Canada. Useful discussions with Prof. G.X. Wu at University College London are very much appreciated.

References

Bingham, H.B. and Maniar, H.D. (1996). Computing the Double-Body m-terms Using a B-spline Based Panel Method. Proceedings of the 11th International Workshop on Water Waves and Floating Bodies, Hamburg.

Danmeier, D.G. (1999). A Higher-Order Panel Method for Large-Amplitude Simulation of Bodies in Waves. Ph.D. Thesis, Massachusetts Institute of Technology, Massachusetts.

Qiu, W.,Chuang, J.M. and Hsiung, C.C. (2004). Numerical Solution of Wave Diffraction Problem in the Time Domain with the Panel-Free Method. Journal of Offshore Mechanics and Arctic Engineering, Vol. 126, No. 1, pp. 1-8.

Qiu, W. and Hsiung, C.C. (2002). A Panel-Free Method for Time-Domain Analysis of Radiation Problem. Ocean Engineering, Vol. 29, No. 12, pp. 1555-1567.

Wu, G.X. (1994). Hydrodynamic Forces on a Submerged Sphere Undergoing Large Amplitude Motion. Journal of Ship Research, Vol. 38, No. 4, pp. 272-277.