# Simple analytical approximation to a ship bow wave

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## 1. Bow-wave height

A simple analytical expression for the height (above the mean free-surface plane) $Z_b$ of the bow wave generated by a ship that advances at constant speed $U$ in calm water is given in [ 1 ]. This expression directly defines $z_b = Z_b\, g/U^2$, where $g$ is the acceleration of gravity, in terms of the ship speed $U$, draft $D$ and waterline entrance angle $2\,\alpha_E$ as

$$z_b = \frac{Z_b\, g}{U^2} \approx \frac{C^Z}{1+F_D}\,\frac{\tan\alpha_E}{\cos\alpha_E} \quad \text{with} \quad F_D = \frac{U}{\sqrt{g\,D}} \quad \text{and} \quad C^Z \approx 2.2 \tag{1}$$

This expression is based on elementary fundamental theoretical considerations (dimensional analysis, and asymptotic behaviors in limits $\alpha_E \to 0$, $D \to 0$ and $D \to \infty$) and experimental data (to determine $C^Z$). The simple analytical expression (1) is in excellent agreement with experimental measurements for wedge-shaped ship bows. Expression (1) is also in good agreement with measurements for the Wigley hull and the Series 60 model, and similar ship-bow forms, especially if the simple procedure given in [ 1 ] is used to define an effective draft $D$ and an effective waterline entrance angle $2\,\alpha_E$. Experimental measurements and theoretical predictions are compared in Fig.1 , where the normalized bow-wave height $(Z_b\, g/U^2)\cos\alpha_E\,/\tan\alpha_E$ is represented as a function of the draft-based Froude number $F_D = U/\sqrt{g\,D}$. The left side of Fig.1 shows the experimental data, for nine ship hulls, considered in [ 1 ] and the right side of Fig.1 reports measurements performed at the École Centrale de Nantes for a rectangular flat plate towed at speed $U = 1.25\,, 1.5\,, 1.75$ and $2\,m/s$, draft $D = 0.3\,m$, incidence angle $\alpha_E = 10^\circ, 15^\circ, 20^\circ$ and flare angle $\gamma = 0^\circ, 10^\circ, 15^\circ, 20^\circ$. The solid line in Fig.1 is the approximation (1) . Expression (1) for a bow-wave height and the Bernoulli steady-flow constraint $Z_b\, g/U^2 \le 1/2$ are used in [ 2 ] to obtain a simple analytical criterion that predicts when a ship bow wave is necessarily unsteady. This criterion is in good agreement with experimental observations of bow waves generated by a flat plate [ 2 ] .

## 2. Location of bow-wave crest

A simple analytical expression for the location of a ship bow-wave crest, defined as the distance $t_b = T_b\, g/U^2$ between a ship stem and bow-wave crest, is given in [ 1 ] . This expression, obtained using fundamental theoretical considerations (dimensional analysis, limits $D \to 0$ and $D \to \infty$) and experimental data (to determine $C^X$), is

$$t_b = t_b(F_D) = C^X/(1+F_D) \quad \text{with} \quad C^X \approx 1.1 \tag{2}$$

Experimental measurements and theoretical predictions are compared in Fig.2 . The left side of Fig.2 shows the experimental data, for five ship hulls, considered in [ 1 ] and the right side of Fig.2 reports measurements performed at the École Centrale de Nantes for a flat plate towed at speed $U$, draft $D$, incidence angle $\alpha_E$ and flare angle $\gamma$. The solid line in Fig.2 is the approximation (2) . The flat-plate data lie mostly below the theoretical line (2) .

An alternative, fully analytical, simple expression for $t_b$, obtained in [ 2 ] using an elementary Lagrangian analysis of the motion of a fluid particle that passes through a ship stem, is

$$t_b = t_b(z_b) = \sqrt{2\, z_b\,(1 - 2\, z_b)} \tag{3}$$

This expression presumes $z_b \le 1/2$, i.e. a steady bow wave [ 2 ] . The experimental measurements already considered in Fig.2 are compared to the analytical expression (3) in Fig.3 . Although considerable scatter of experimental data points can be observed (notably on the left side of Fig.3 for the Series 60 model), the simple analytical approximation (3) appears to agree reasonably well with measurements. The (fully analytical) approximation (3) seems preferable to the (semi-analytical) approximation (2) in the steady ship-bow-wave regime, for which one has $z_b < 1/2$.

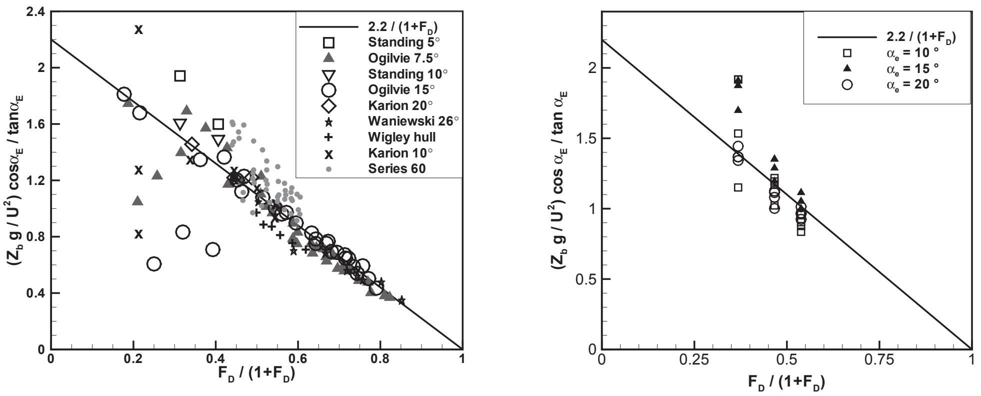


**Fig.1** Normalized bow-wave height $(Z_b\, g/U^2)\cos\alpha_E\,/\tan\alpha_E$ for nine ship hulls (left) and a flat plate towed at speed $U$, draft $D$, incidence angle $\alpha_E$ and flare angle $\gamma$ (right). The straight line is the approximation $2.2/(1+F_D)$ given by (1).

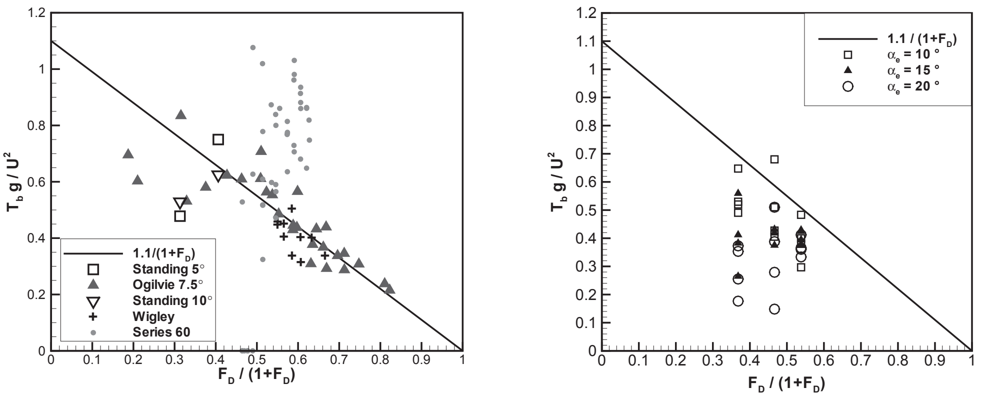


**Fig.2** Nondimensional distance $t_b = T_b\, g/U^2$ between a ship stem and bow-wave crest for five ship hulls (left) and a flat plate towed at an incidence angle $\alpha_E$ and a flare angle $\gamma$ (right). The straight line is the approximation $1.1/(1+F_D)$ given by (2).

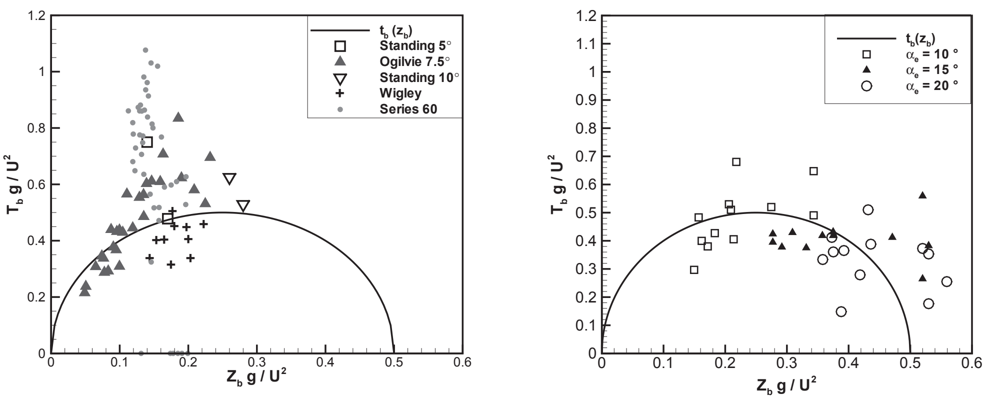


**Fig.3** Nondimensional distance $t_b = T_b\, g/U^2$ between a ship stem and bow-wave crest for five ship hulls (left) and a flat plate towed at an incidence angle $\alpha_E$ and a flare angle $\gamma$ (right). The distance $t_b$ is depicted as a function of the nondimensional wave height $z_b = Z_b\, g/U^2$, and the circle is the approximation $t_b = \sqrt{2\, z_b\,(1 - 2\, z_b)}$ given by (3).

**3. Parabolic (nonlinear) bow-wave front and sinusoidal (linear) bow wave aft of wave crest**

The elementary Lagrangian flow-analysis used to obtain (3) also yields the simple parabolic bow-wave approximation

$$z = z_b(1 - t_0^2/t_b^2) \tag{4}$$

Here, the origin $t_0 = 0$ is taken at the bow-wave crest. The parabolic wave profile (4) and expressions (3) and (1) define a family of parabolic ship bow waves that is entirely defined in terms of the height $z_b$ of the bow wave. These expressions are based on a nonlinear analysis (dimensional analysis and Newton's equations of motions), although this analysis is highly simplified. Specifically, a fluid particle that passes through a ship stem is presumed to follow a path determined by Newton's equations. Thus, interactions among water particles are ignored in this elementary Lagrangian flow analysis.

The parabolic bow waves defined by (4) with (3) and (1) are compared in [ 2 ; Fig.9 ] with experimental measurements of bow waves due to a flat plate immersed at a draft $D = 0.3\,m$ and towed at incidence angles $\alpha_E = 10°$ or $\alpha_E = 20°$, flare angles $\gamma = 0°$ , $10°$, $15°$, $20°$, and speeds $U = 2\,m/s$ or $1.5\,m/s$ ($F_D = 1.17$ or $0.87$) . This comparison shows that the parabolic bow-wave approximation is in fairly good agreement with experimental measurements for the front of the bow waves, i.e. between the leading edge of the plate and the crest of the bow wave. However, Fig.9 in [ 2 ] also shows that the simple analytical bow waves given by (4) with (3) and (1) are not in good agreement with experimental measurements beyond the bow-wave crest.

These experimental observations suggest that interactions among fluid particles, ignored in the elementary Lagrangian analysis used in [ 2 ] , are more important in the "recovery zone" past a wave crest than in the "build-up zone" between a ship stem and bow-wave crest. Thus, the parabolic bow wave defined by (4) with (3) may be used for $-t_b \le t_0 \le 0$ but not for $0 < t_0$ . The ship bow wave is then considered here aft the wave crest, i.e. for $0 < t_0$ . An obvious analytical approximation for a bow wave aft the crest is an elementary wave with wavelength $2\pi\,U^2/2$ , i.e. $z = z_b\,\cos t_0$ for $0 \le t_0$ . Here, $t_0 = 0$ at the crest of the bow wave and $t_0 = Tg/U^2$.

**4. Simple composite analytical approximation to bow wave**

The change of variable $t_0 = t_b\sqrt{1-\sigma_s} - t$ in the foregoing complementary parabolic and sinusoidal approximations yields the composite bow wave

$$\left.\begin{array}{ll} z = z_b\,(\sigma_s + 2\sqrt{1-\sigma_s}\,t/t_b - t^2/t_b^2) & \text{for } 0 \le t \le t_b\sqrt{1-\sigma_s} \\ z = z_b\cos(t - t_b\sqrt{1-\sigma_s}) & \text{for } t_b\sqrt{1-\sigma_s} \le t \\ \text{with} \quad t_b = \sqrt{2\,z_b\,(1-2\,z_b)} & \text{and} \quad 0 \le \sigma_s = z_s/z_b < 1 \end{array}\right\} \tag{5}$$

Here, $\sigma_s = z_s/z_b$ defines the ratio of the elevation $z_s$ of the free surface at the ship stem over the height $z_b$ of the wave crest. Expressions (5) yield $z = z_s$ at a ship stem $t = 0$ , and $z = z_b$ at a bow-wave crest $t = t_b\sqrt{1-\sigma_s}$ . Expressions (5) and expression (1) for the nondimensional bow-wave height $z_b$ provide a simple ab-initio (without calculations) analytical approximation to a ship bow wave (in the steady bow-wave flow regime). This elementary approximation represents a ship bow wave in terms of two complementary pieces: a (nonlinear) parabolic wave front (ahead of the bow-wave crest) followed by a (linear) sinusoidal elementary wave, with wavelength $2\pi\,U^2/2$ , aft of the wave crest. The analytical bow wave defined by (5), with $\sigma_s = 0$ , and (1) is compared to experimental bow-wave measurements (made at the École Centrale de Nantes) for a towed rectangular flat plate in Fig.4 . This comparison shows that the analytical approximation given by (5) and (1) agrees reasonably well with experimental observations, in spite of the highly simplified analysis that underlie these analytical approximations.

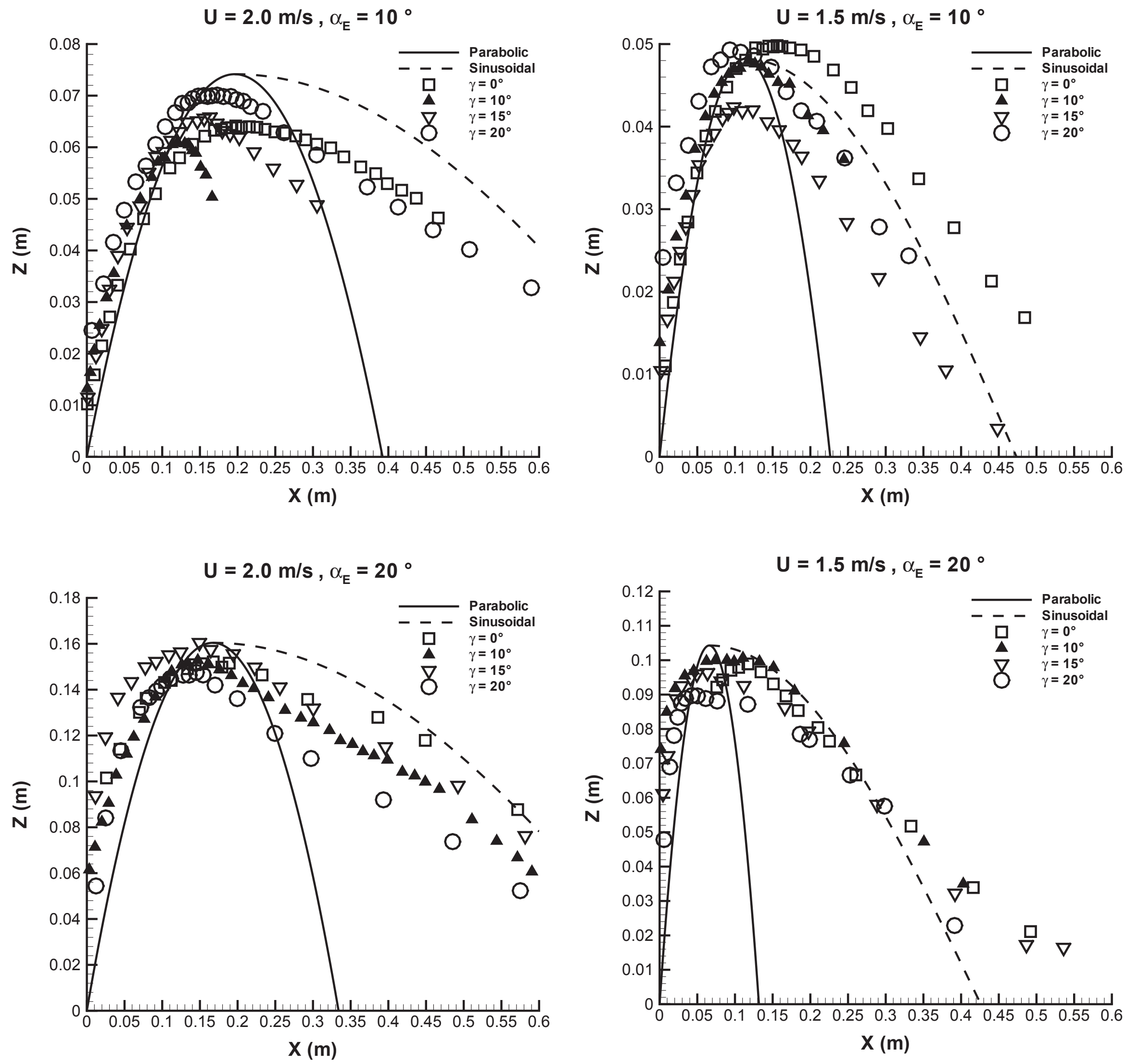


**Fig.4** Comparison of the simple analytical bow-wave approximation given by (5) and (1) with experimental measurements for a rectangular flat plate immersed at a depth $D = 0.3\,m$ and towed at flare angles $\gamma = 0°$ , $10°$, $15°$, $20°$ and incidence angles $\alpha_E = 10°$ (top half) or $\alpha_E = 20°$ (bottom) with speeds $U = 2\,m/s$ (left side) or $U = 1.5\,m/s$ (right) .

## 5. Additional results

Additional experimental measurements of wave profiles for a rectangular flat plate, immersed at a depth $D = 0.2\,m/s$ and towed at incidence angles $\alpha_E = 25°, 30°, 45°, 60°, 75°, 90°$ and flare angles $\gamma = 0°, 10°, 15°, 20°, 30°, 40°$ with speeds $1.25\,m/s \leq U \leq 2.5\,m/s$ , have recently been performed and are currently analyzed. These measurements are expected to yield an expression for the height $z_b$ of a ship bow wave that extends expression (1), which presumes a small waterline entrance angle $\alpha_E < 25°$ and a small flare angle $\gamma < 20°$. These experimental measurements and generalized expression for the height $z_b$ of a ship bow wave will be reported at the Workshop.

## References

[ 1 ] Noblesse F., Hendrix D., Faul L., Slutsky J. (2006) “Simple analytical expressions for the height, location, and steepness of a ship bow wave”, *J. Ship Research* **50**:360-370

[ 2 ] Noblesse F., Delhommeau G., Guilbaud M., Hendrix D., Karafiath G. (2006) “Analytical and experimental study of unsteady and overturning ship bow waves”, *26th Symp. on Naval Hydrodynamics*, Rome