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A method for generating highly nonlinear periodic waves in physical wave basins

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This abstract describes a new method for generating nonlinear waves of constant form in physical wave basins. The idea is to combine fully dispersive linear wavemaker theory with nonlinear shallow water wave generation theory; and use an exact nonlinear theory as the target. We refer to the method as an ad-hoc unified wave generation theory, since there is no rigorous analysis behind the idea which is simply justified by the improved results obtained for the practical generation of steady nonlinear waves.

We seek a method to provide a time history of the piston wavemaker position $X(t)$ as a function of the depth-averaged horizontal fluid velocity under a wave $U(x,t)$. For nonlinear shallow water waves, the horizontal particle velocity is nearly uniform with depth and we can directly relate the two quantities by

$$\frac{dX^{sw}(t)}{dt} = U(X^{sw}(t), t) \tag{1}$$

as proposed by [3] who successfully used the technique to generate nonlinear waves with Cnoidal wave theory providing $U(x,t)$.

Linear potential theory can also be used to provide the wave paddle motion corresponding to a given linear wave condition in the far field. Assuming a periodic solution at frequency ω with far field surface elevation $\eta(x,t) = \Re\{A(\omega)\exp(\mathrm{i}(\omega t - kx))\}$, the required motion of a piston wavemaker is given by

$$X(t) = \Re\{X_a(\omega)\exp(\mathrm{i}\omega t)\} \Rightarrow \quad \mathrm{i}c_0 X_a(\omega) = A(\omega), \quad c_0 = \frac{4\sinh^2 kh}{2kh + \sinh 2kh} \tag{2}$$

where the linear dispersion relation $\omega = \sqrt{gk\tanh kh}$ relates the wave frequency to the wavenumber, h is the water depth and g the gravitational acceleration. This can be put in terms of the depth-averaged horizontal velocity at the paddle $U(0,t) = \Re\{B(\omega)\exp(\mathrm{i}\omega t)\}$ since $B(\omega) = \frac{\omega}{kh}A(\omega)$ to get

$$\mathrm{i}\omega X_a(\omega) = \Lambda\, B(\omega), \qquad \Lambda = \frac{kh}{c_0} = kh\frac{2kh + \sinh 2kh}{4\sinh^2 kh}. \tag{3}$$

The basis for the ad hoc unified wave generation theory is the observation that in the shallow water limit $(kh \to 0)$ $\Lambda \to 1$ and we can thus split (3) into

$$\mathrm{i}\omega X_a^{sw}(\omega) = B(\omega), \qquad X_a(\omega) = \Lambda\, X_a^{sw}(\omega). \tag{4}$$

By analogy with nonlinear shallow water wave generation theory, we combine the two ideas to get an ad hoc unified generation procedure as follows. First, compute a shallow water signal from

$$\frac{dX^{sw}(t)}{dt} + \omega_c X^{sw}(t) = U(X^{sw}(t), t) \tag{5}$$

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where a small term proportional to the wave paddle position has been added to act as a high pass filter with characteristic frequency ω_c. This is followed by a dispersion correction step using

$$X(t) = \mathcal{F}\{\Lambda(\omega)\mathcal{F}^{-1}\{(X^{sw}(t)\}\} \tag{6}$$

where $\mathcal{F}$ represents the Fourier transform which is evaluated in practise via a Fast Fourier Transform with a suitable cut-off frequency corresponding to the maximum response frequency of the wavemaker. Any theory, or even a numerical model, can be used to provide the depth-averaged velocity but for highly nonlinear periodic waves, stream function theory (see *e.g.* [1]) is a good choice. From continuity, we can write $U(x,t) = \frac{c\eta(x,t)}{h+\eta(x,t)}$ where the phase speed c and the elevation η are obtained from the theory.

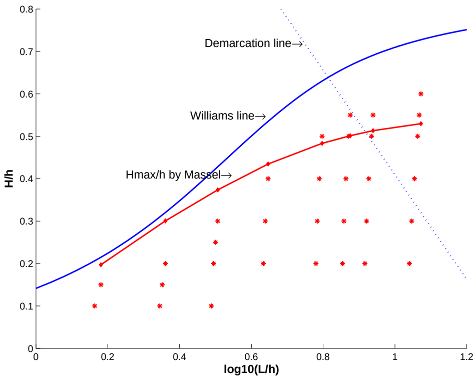

Figure 1: Experimental test conditions shown together with some proposed theoretical and experimental limits.

To test the new wave generation method, an experimental program was initiated at DHI - Water & Environment. The test conditions in terms of relative wave height H/h (nonlinearity) and relative wavelength L/h (dispersion) are shown in Figure 1 together with several suggested limiting lines. The line labelled "Williams line" corresponds to the theoretical limit for the steepest stable wave [6] (see also [2]). The line labelled "Hmax/h by Massel" shows a collection of laboratory and field data sets by [5] as parametrised by [4] which has been suggested as a practical breaking limit for wave generation using linear theory. The line marked "Demarcation line" shows the proposed boundary between the regions of validity of the Stokes and Cnoidal wave theories from [2].

Figure 2 shows results from the steepest and shallowest point on the experimental program ($H/h = 0.6, L/h = 11.8$) and compares measured time series of surface elevation at several positions along the wave flume. The upper plot shows the result of using shallow water wave generation together with Cnoidal theory as the target. Clearly the generated wave in this case is not of constant form. The lower plot shows the result using the new unified generation theory together with stream function theory as the target and the wave is clearly very close to regular. On both plots the theoretical result from stream function theory is also shown.

Figure 3 shows an intermediate water depth case at $L/h = 7.5$ and $H/h = .55$ and the new generation method is in this case compared to Stokes 2nd-order wave generation theory, and again we can see that the new method leads to a substantial improvement in uniformity along the length of the flume.

Finally, Figure 4 shows a case of simultaneous wave generation and active wave absorption using the shallow water conditions of Figure 2. This demonstrates that the method can be used for simultaneous generation and active wave absorption, although this test in fact

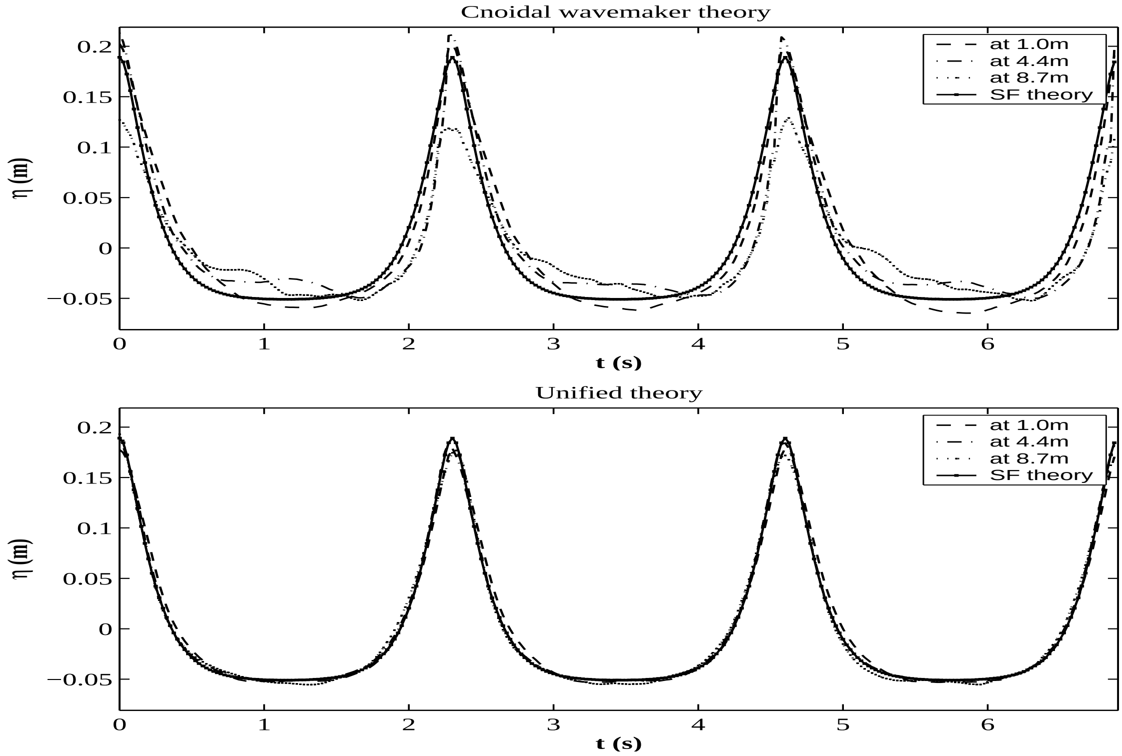

Figure 2: Steep shallow water wave generation with $L/h = 11.8$ $H/h = 0.6$.

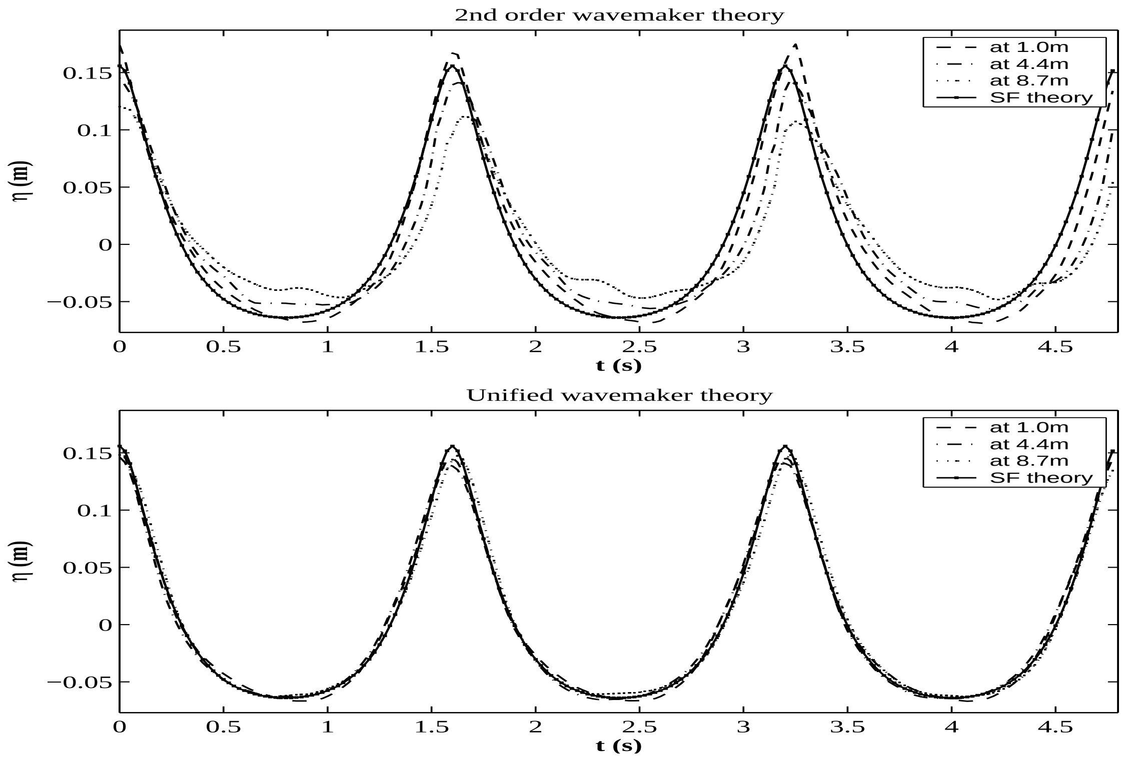

Figure 3: Steep intermediate depth case with $L/h = 7.5$ $H/h = 0.55$.

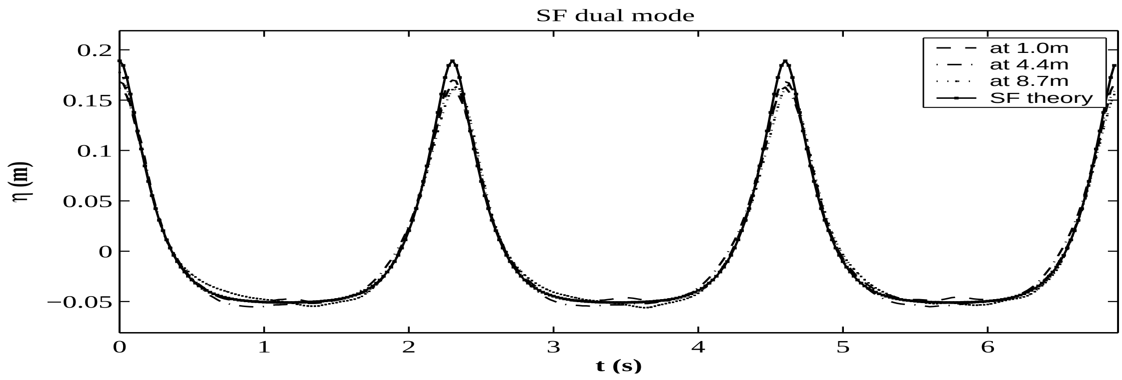

Figure 4: Steep shallow water waves $L/h = 11.8$ $H/h = 0.6$, with active absorption.

had no reflected waves to absorb. More details and results from the experimental program can be found in [8].

We note that this method was originally developed to provide an interface for a combined numerical/physical model, and that work including many experimental tests for both regular and irregular waves on uniform and variable bathymetries and in both two and three dimensions is described in [7].

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