Second-order diffraction by two concentric truncated cylinders

by

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1 General

Structures with moonpools have, or they could have, many useful applications in offshore technology. They can be used for wave trapping or for other offshore operations such as pipe laying or recovery of divers. Examples of recent works dealing with moonpools are those reported by Shipway and Evans (2002) and McIver and Newman (2003) who studied trapping structures with two moonpools. Newman (2003) analyzed the low frequency resonant modes for moonpools with variable cross section. Molin (2001) presented a study for the piston and sloshing modes in rectangular moonpools of large horizontal dimensions. Here, we present a solution for the second-order diffraction problem for a piston-like arrangement that consists of two cylindrical structures with an annular moonpool between them. The solution method is based on the semi-analytical formulation for the second-order diffraction potentials which was proposed by Huang and Eatock Taylor (1996) and which was extended recently by Mavrakos and Chatjigeorgiou (2006) for ring-shaped fluid domains that extend up to the free surface. The arrangement that is considered in this work was investigated recently by Mavrakos (2004, 2005) assuming linear potential theory. The main objective of this work is to investigate to a certain extent the contribution of the second-order potential to both the vertical exciting forces on the interior piston-like cylinder and the water motion in the annulus between the internal and external bodies.

2 Formulation and Solution

The main objective of the present work is the derivation of the second-order diffraction potentials in all fluid regions surrounding the bodies and through these, the calculation of the second-order forces and the nonlinear wave elevation around the bodies. The incident waves are considered to be monochromatic with amplitude $H/2$. The method that is adopted for the mathematical description of the potentials in the external fluid region **A** and the moonpool **C,** see Fig. 1, is based on the semi-analytical formulation proposed by Huang and Eatock Taylor (1996). As all boundary conditions involved in the hydrodynamic problems of the lower fluid regions **B** and **D** are homogeneous, the second-order diffraction potentials will be given by equations similar to those of the first-order problems. The corresponding formulation was taken from the work reported by Mavrakos (2005). Thus, the total second-order diffraction potentials in fields **B** and **D** will be given by:

$$\varphi_{2B} = -\mathrm{i}\omega(H/2)^2 \sum_{m=-\infty}^{\infty} \mathrm{i}^m \sum_{n=0}^{\infty} \varepsilon_n \left[R_{mn}^{(2B)}(r)F_{mn}^{(2B)} + \tilde{R}_{mn}^{(2B)}(r)\tilde{F}_{mn}^{(2B)}\right]\cos(n\pi z/h_2)e^{\mathrm{i}m\theta} \quad (1)$$

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$$\varphi_{2D} = -\mathrm{i}\omega (H/2)^2 \sum_{m=-\infty}^{\infty} \mathrm{i}^m \left\{ \sum_{n=0}^{\infty} \varepsilon_n F_{mn}^{(2D)} \, \mathrm{I}_m (n\pi r/h_1) / \mathrm{I}_m (n\pi b_1/h_1) \cos(n\pi z/h_1) \right\} e^{\mathrm{i}m\theta} \tag{2}$$

where ε_n=1 for n=0 and ε_n=2 for n≥1 and $R_{mn}^{(2B)}(r)$ and $\widetilde{R}_{mn}^{(2B)}(r)$ are properly constructed functions that depend on the radial solutions of the potentials, e.g. the modified Bessel functions I_m and K_m.

Following Huang and Eatock Taylor (1996) and Mavrakos and Chatjigeorgiou (2006), the total second-order diffraction potential in the outer fluid domain **A** will be composed by the incident wave φ_2^I, a 'locked' wave and a 'free' wave component. The latter two, which are denoted by the superscripts *DD* and *ID* respectively, are expressed as:

$$\varphi_{2A}^{DD} = -\mathrm{i}\omega (H/2)^2 \sum_{m=-\infty}^{\infty} \mathrm{i}^m \left\{ \sum_{j=0}^{\infty} Z_j^{(2)}(h) Z_j^{(2)}(z) \int_1^{\infty} \frac{\xi}{b} Q_m^{(A)}\left(\frac{\xi}{b}\right) G_{mj}^{(A)}\left(\frac{r}{b};\frac{\xi}{b}\right) d\left(\frac{\xi}{b}\right) \right\} e^{\mathrm{i}m\theta} \tag{3}$$

$$\varphi_{2A}^{ID} = -\mathrm{i}\omega (H/2)^2 \sum_{m=-\infty}^{\infty} \mathrm{i}^m \left\{ \sum_{j=0}^{\infty} F_{mj}^{(2A)} \, \mathrm{K}_m(\kappa_j r) / \mathrm{K}_m(\kappa_j b) Z_j^{(2)}(z) \right\} e^{\mathrm{i}m\theta} \tag{4}$$

where κ_{j} are the solutions of the second-order transcendental equation satisfied at the free surface of the outer field **A** and Z_j are the corresponding vertical eigenfunctions. Furthermore, $Q_m^{(A)}$ is the nondimensional form of the inhomogeneous term of the second-order boundary condition on the free surface and $G_{mj}^{(A)}$ is the appropriate Green function which is obtained by solving the relative Sturm-Liouville problem.

Recently, Mavrakos and Chatjigeorgiou (2006) developed a solution for the upper inner region defined by the geometry of a vertical compound cylinder. The specific formulation can be easily extended for describing the second-order diffraction potential in the moonpool **C**, provided that the height of the step on the compound cylinder will be set to zero. As a result, the total second-order velocity potential in the moonpool will consists of the incident wave φ_2^I, a free wave component and a trapped wave component, denoted by *ID* and *DD* respectively. These will be given by the following:

$$\varphi_{2C}^{ID} = -\mathrm{i}\omega (H/2)^2 \sum_{m=-\infty}^{\infty} \mathrm{i}^m \sum_{j=0}^{\infty} \left[R_{mj}^{(2C)}(r) F_{mj}^{(2C)} + \widetilde{R}_{mj}^{(2C)}(r) \widetilde{F}_{mj}^{(2C)} \right] Z_j^{(2)}(z) e^{\mathrm{i}m\theta} \tag{5}$$

$$\varphi_{2C}^{DD} = -\mathrm{i}\omega (H/2)^2 \sum_{m=-\infty}^{\infty} \mathrm{i}^m \left\{ \sum_{j=0}^{\infty} Z_j^{(2)}(h) Z_j^{(2)}(z) \int_1^{b_2/b_1} \frac{\xi}{b_1} Q_m^{(C)}\left(\frac{\xi}{b_1}\right) G_{mj}^{(C)}\left(\frac{r}{b_1};\frac{\xi}{b_1}\right) d\left(\frac{\xi}{b_1}\right) \right\} e^{\mathrm{i}m\theta} \tag{6}$$

Again, $R_{mj}^{(2C)}(r)$ and $\widetilde{R}_{mj}^{(2C)}(r)$ are properly constructed radial functions that depend on the modified Bessel functions I_m and K_m, $Q_m^{(C)}$ is the nondimensional form of the radial dependent effective pressure distribution on the free surface of the moonpool and $G_{mj}^{(C)}$ is the one-dimensional Green's function obtained by the solution of the corresponding Strum-Liouville problem for region **C**.

The problem will be considered completely resolved after obtaining the Fourier coefficients in all subdomains defined in Fig. 1. This is achieved by enforcing the continuity of velocities and potentials along the boundaries of the individual fluid regions **A**, **B**, **C** and **D**.

3 Results and Discussion

In this section we present the results for the first- and second-order vertical exciting forces on the inner 'piston' and for the free surface elevation in the moonpool. In particular the wave run-up depicted herein corresponds to the wave frequency of the resonant water motion in the interior basin. First- and second-order vertical exciting forces have been normalized by $1/2\rho g b^2(H/2)$ and $1/2\rho g b(H/2)^2$ respectively, while the depicted second-order force component was obtained by the second-order diffraction potential only. The second-order wave elevation $|\eta_2| = |\eta_2^{(1)} + \eta_2^{(2)}|$ was normalized by $(H/2)^2/b$. The relative dimensions of the two bodies are determined by b_1/b=0.3846, b_2/b=0.5385, h_2/h=0.68 and h/b=1.923 while three different draughts for the inner body were investigated: h_1/h=0.5, 0.68 and 0.80. The resonant water motions occur approximately at $kb\approx1.25$ for h_1/h=0.5 and 0.68 and at $kb\approx1.4$ for h_1/h=0.8. (Figs 6-8). As can be seen the second-order force peaks occur exactly at the same wave frequencies. Although the magnitudes of the first-order vertical exciting wave forces at the resonant water motion frequencies appear to increase for smaller piston draughts, no similar trend is observed for their second-order counterparts. On the other hand the values of the second-order forces are extremely high compared to the first-order ones. The latter remark could be proven very useful in practical applications as the total hydrodynamic force will be definitely underestimated by the linear potential theory. As can be easily seen, the abrupt jump of the second-order diffraction force is closely related to the resonant water motions in the moonpool. The free surface elevation curves depicted in Figs 2-4 represent the total wave run-up that was calculated by the first- and the second-order diffraction potentials at the frequency of the resonant water motion. In all cases the wave run-up on the exterior of the outer cylinder is rather smaller than the water elevation in the moonpool. Furthermore, no worth mentioning variation of the wave run-up is observed with respect to the azimuthal angle in the moonpool for h_1/h=0.5 and 0.68 while for h_1/h=0.8 the wave elevation appears to be extremely high at the lee side of the piston.

It should be also highlighted that the level of the water on the endmost limits of the moonpool, i.e., the outer surface of the inner body and the inner surface of the outer body, is approximately the same. Additionally, the variation of the water level on the circumferences of the endmost limits follows more or less the same trend. Nevertheless, it should be mentioned that the free surface in the moonpool at the location, where the resonant water motions occur, exhibits strong disturbances. Although this is not noticeable in the surface plot of Fig. 5 due to small width of the moonpool, the validity of the latter remark is supported by the variety of the levels of the contours.

4 References

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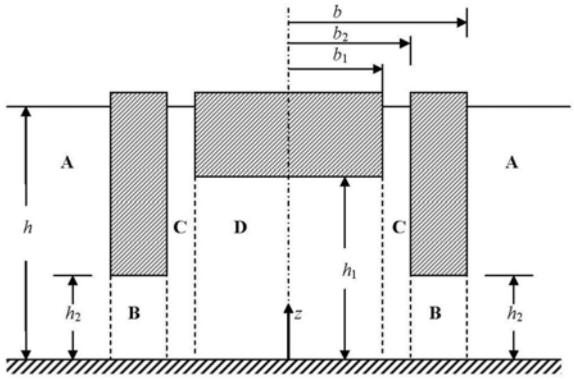

Fig. 1: Main dimensions and fluid regions

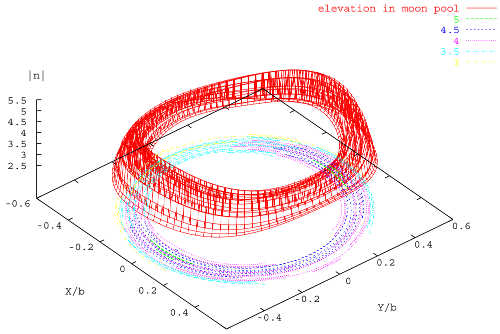

Fig. 5: Free surface elevation in the moonpool, h_1/h=0.68, kb=1.25

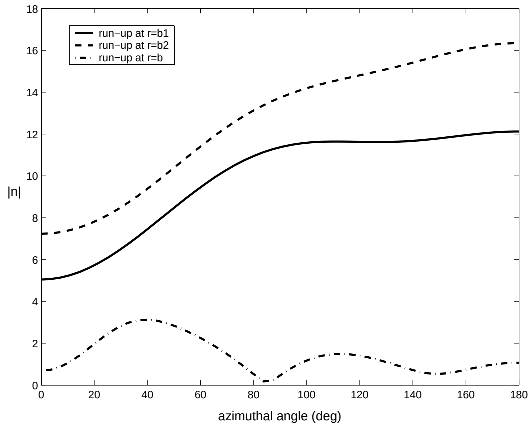

Fig.2: Wave run-up for h_1/h=0.50, kb=1.25

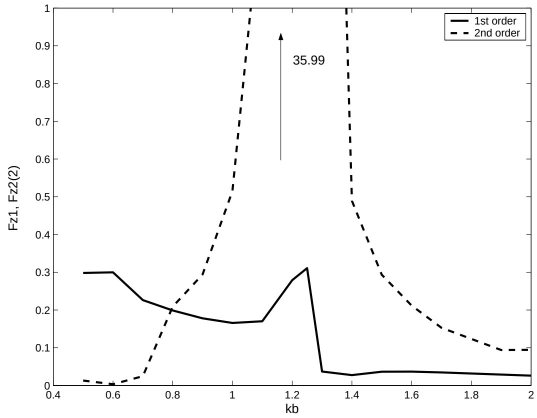

Fig. 6: Heave forces on the inner body, h_1/h=0.5

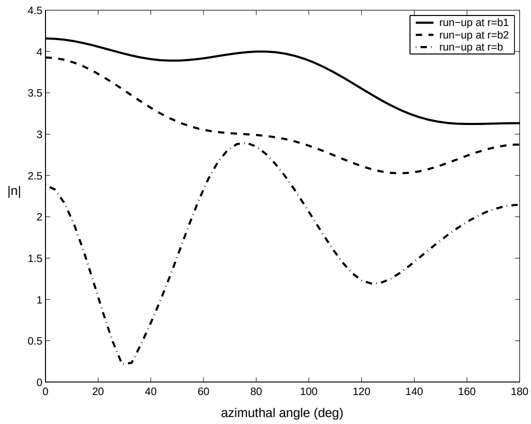

Fig.3: Wave run-up for h_1/h=0.68, kb=1.25

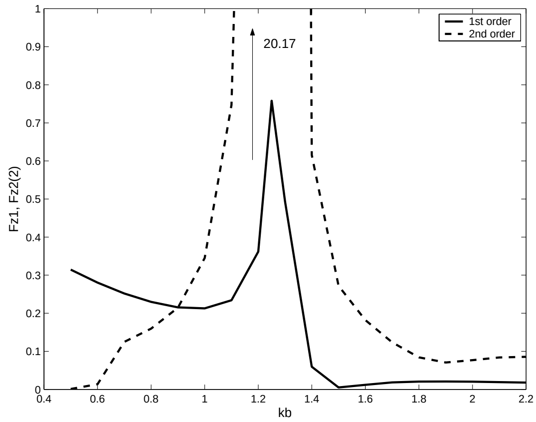

Fig. 7: Heave forces on the inner body, h_1/h=0.68

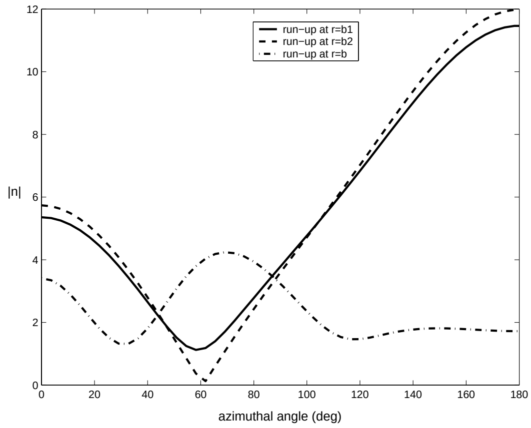

Fig.4: Wave run-up for h_1/h=0.80, kb=1.4

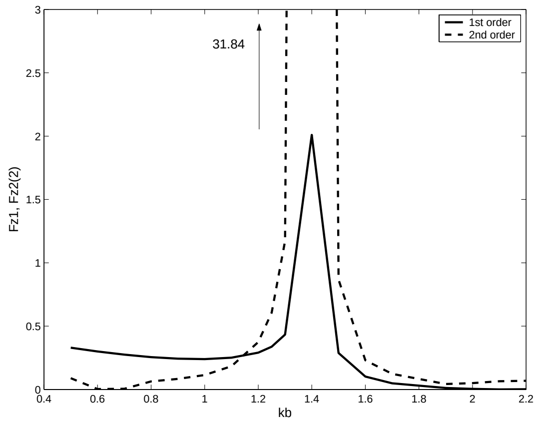

Fig. 8: Heave forces on the inner body, h_1/h=0.8

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Discusser - M. Longuet-Higgins:

(In reply to chairman's criticism that some second-order terms were larger than the first-order terms.)

The second-order pressure fluctuations in standing waves are not attenuated exponentially with depth like the first-order terms, and so can be much larger. This was first verified experimentally by R.I.B. Cooper and M.S. Longuet-Higgins in 'An experimental study of the pressure variations in standing water waves', Proc. R. Soc. Lond. A. **206** (1951), 424-435.

Reply:

Thank-you for your comments.