IMPACT OF A CYLINDRICAL SHELL ON A THIN LAYER OF WATER

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A 2D unsteady analysis of an elastic cylindrical circular shell that enters a thin layer of an ideal incompressible liquid is considered. At an initial instant of time, the cylinder touches the liquid free surface at a single point and then begins to penetrate the liquid layer at a constant vertical velocity. The flow field is divided into four distinct regions that exhibit different properties: the region beneath the entering surface, the jet root, the spray jet, and an outer region. A complete solution is obtained via matching of the solutions within the four subdomains. The problem is coupled because the liquid flow, the shape of the elastic shell and the geometry of the contact region between the body and the liquid must be determined simultaneously. The analysis is based on the normal-mode method with focus on strain-time histories of the inner surface of the cylinder.

1. Problem formulation and general assumptions

The flow is sketched in Figure 1. Initially, the liquid is at rest and occupies a region ($-h' < y' < 0$), and the elastic cylinder touches the free surface ($y' = 0$) at a single point, which is chosen as the origin of the Cartesian coordinate system $x'Oy'$ (dimensional variables are denoted by a prime). At the initial instant of time ($t' = 0$), the cylinder of radius R' begins to penetrate the liquid with initial vertical velocity V. The shape and velocity of the cylinder vary due to the interaction between the elastic cylinder and the liquid. The contact points between the free surface and the elastic body is the main feature of the problem. The positions of these points are not known and must be determined together with the liquid flow and the shell deformation.

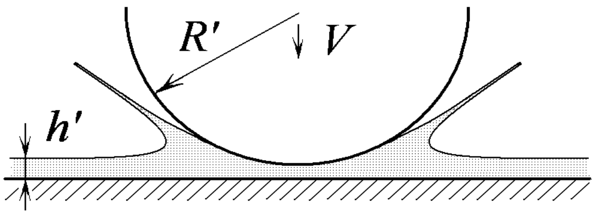

Figure 1.

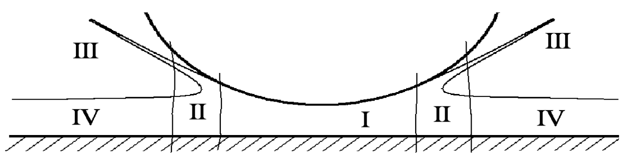

Figure 2.

The deformations of the elastic cylinder, the distribution of bending stresses, and the positions of the contact points are determined under the following assumptions: (i) the liquid is ideal and incompressible; (ii) the flow of the liquid layer is 2D and symmetrical with respect to the y'-axis; (iii) the liquid flow in the region beneath the entering body is represented using the shallow water model; (iv) the shell thickness is constant and small; (v) external mass forces and surface tension are negligible; (vi) the wetted area of the cylinder is a monotonically increasing function of time.

The problem is formulated in terms of nondimensional variables, with $L = h'$ and $T = h'/V$ as length and time scales, V as velocity scale for the liquid flow, and ρV^2 as the hydrodynamic pressure scale (ρ is the liquid density).

The flow is analyzed using the method of matched asymptotic expansions [1]. Specifically, the flow field is divided into the four regions depicted in Figure 2: I – the region beneath the entering body; II – the jet root; III – the spray jet; IV – the outer region.

The velocity of the fluid particles in the jet region III is tangential to the body surface. The flow inside this region is analyzed in [3], where the jet motion is shown to have negligible influence on the flow in the jet root. Here, the flows in regions I and II are analyzed, and matched to each other and to the state of rest in region IV.

2. Elastic shell vibrations

[2] shows that the elastic-shell problem is defined as

$$\ddot{w} + \alpha\,(w - v_\theta) + \beta\,(v_{\theta\theta\theta} + w_{\theta\theta\theta\theta}) = \gamma p_0(\theta, t) \quad (-\pi < \theta < \pi), \tag{1}$$

$$\ddot{v} + \alpha\,(w_\theta - v_{\theta\theta}) - \beta\,(v_{\theta\theta} + w_{\theta\theta\theta}) = 0 \quad (-\pi < \theta < \pi), \tag{2}$$

$$v(\theta, 0) = w(\theta, 0) = 0 \qquad (-\pi < \theta < \pi), \tag{3}$$

$$v_t(\theta, 0) = -sin\,\theta, \;\; w_t(\theta, 0) = -cos\,\theta \qquad (-\pi < \theta < \pi), \tag{4}$$

$$\alpha = \frac{E}{\rho_0 R^2 V^2 (1-\nu^2)}, \quad \beta = \frac{E h^2}{12 \rho_0 R^4 V^2 (1-\nu^2)} \quad \gamma = \frac{\rho}{\rho_0 h}, \quad R = \frac{R'}{h'}, \quad h = \frac{h_0}{h'}.$$

Here, w and v are the radial and angular components of the absolute displacements of the shell elements, respectively. r, θ are polar coordinates, $\theta = 0$ corresponds to the lowest point of the body. h_0 is the thickness of the shell, ρ_0 is the density of the shell material, E is the elasticity modulus, ν is Poisson's ratio, $p(x, y, t)$ is the hydrodynamic pressure, $p_0(\theta, t)$ is the external (hydrodynamic) load acting on the elastic shell. Within the contact region, $|x| < c(t)$ and $|\theta| < \theta_c(t)$, one has $p_0(\theta, t) = p(x(\theta, t), y(\theta, t), t)$, where $x(\theta, t)$ and $y(\theta, t)$ are the horizontal and vertical coordinates of the entering elastic cylinder with $x(\pm\theta_c(t), t) = \pm c(t)$. During the initial stage of the impact, i.e. for $\theta_c \ll 1$, one may use the approximate formulae $x \approx R\theta$, $\theta_c(t) \approx c(t)/R$. A dot stands for derivative with respect to time. The initial conditions (3) and (4) imply that the shell is undeformed before the impact and moves vertically.

Equations (1)-(4) correspond to the structural part of the problem. The coupled structural and hydrodynamic problem requires that the size of the contact region and the hydrodynamic pressure distribution along the wetted region be determined simultaneously.

3. Liquid flow beneath the entering shell

Estimates of the liquid flow parameters and physical reasoning indicate that within region I, the pressure p and the horizontal component u of the velocity are approximately y-independent. The dimensionless equations of the liquid motion are:

$$u_t + u u_x = p_x, \tag{5}$$

$$u_x + v_y = 0 \qquad (|x| < c(t), \quad -1 < y < f(x, t) - t), \tag{6}$$

$$v = f_x(x, t) u - f_t(x, t) - 1 \qquad (y = f(x, t) - t, \quad |x| < c(t)), \tag{7}$$

$$v = 0 \qquad (y = -1, \quad |x| < c(t)). \tag{8}$$

Here $u = u(x, t)$, $p = p(x, t)$, $v = v(x, y, t)$. Integration yields

$$u(x, t) = \frac{x - \int_0^x f_t(\xi, t) d\xi}{f(x, t) + 1 - t}, \tag{9}$$

for the symmetrical case.

The pressure distribution over the contact line can be determined by integrating (1) and using (5) and the boundary condition $p(c(t), t) = p_c(t)$, with p_c not known in advance. Integration yields

$$p(x, t) = p_c(t) + \frac{1}{2}[u^2(c, t) - u^2(x, t)] + \int_x^c u_t(\xi, t) d\xi. \tag{10}$$

It will be noted that the functions $f(x, t)$ and $c(t)$, which define the shape of the body and the size of the contact region, are unknown in equations (9-10).

4. Liquid flow in region II and matching conditions

Following [1], the flow in region II is assumed to be approximately quasi-stationary. Thus, the entering body velocity is neglected and the body surface is taken as a horizontal plate; see Figure 3.

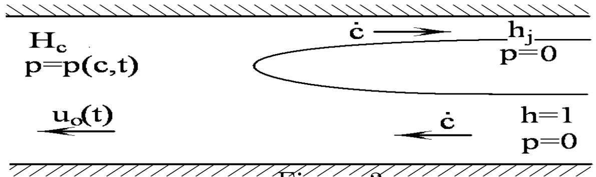

Figure 3.

Within this approach, the jet of thickness 1 moves left with velocity dc/dt. A part of the jet mass continues to move left between the two rigid horizontal plates, $H_c = f(c,t) - t + 1$. At left-hand-side infinity, the pressure is $p_c(t)$ and the horizontal velocity is $u_0 = dc/dt - u(c,t)$. Another part of the jet is deflected and forms a spray jet of thickness h_j. The dynamic condition at the free surface requires that the magnitude of the flow velocity at the free surface be constant. The jet horizontal velocity at infinity is then dc/dt [4]. Here, matching of the flow parameters in region I and II was used. A detailed analysis of the flow can be obtained using conservation laws:
mass-conservation law

$$c' = h_j c' + H_c u_0, \tag{11}$$

Bernoulli's equation (equivalent to energy-conservation law)

$$(c')^2 = 2p_c + u_0^2, \tag{12}$$

momentum-conservation law

$$(p_c + u_0^2)H_c = (c')^2(1 + h_j). \tag{13}$$

The three equations (11)-(13) determine the three unknown functions h_j, c_t, $p_c(t)$, as follows $h_j = (\sqrt{H_c} - 1)^2$,

$$c' = \frac{c - \int_0^c f_t(\xi,t)d\xi}{2(H_c - \sqrt{H_c})}, \tag{14}$$

$$p_c(t) = -\frac{(c(t) - \int_0^c f_t(\xi,t)d\xi)^2}{2H_c^2(\sqrt{H_c} - 1)}. \tag{15}$$

Equations (14)-(15) define the flow in region I, the size of the contact region, and the pressure distribution along the contact line by virtue of (9)-(10).

5. Normal modes method

The coupled hydroelasticity problem is solved here using the normal mode method. Within this approach, the solution of the boundary-value problem (1)-(4) is sought in the form

$$p(R\theta, 0, t) = \sum_{n=0}^{\infty} p_n(t)\cos n\theta, \quad w(\theta,t) = \sum_{n=0}^{\infty} a_n(t)\cos n\theta, \quad v(\theta,t) = \sum_{n=1}^{\infty} b_n(t)\sin n\theta, \tag{16}$$

where $-\pi < \theta < \pi$, and the principal coordinates $a_n(t)$ and $b_n(t)$ define the elastic deformation of the shell. Within the parabolic approximation, the shape of the shell is given by

$$f(x,t) = \frac{x^2}{2R} + \sum_{n=0}^{\infty} a_n(t)\cos\frac{nx}{R}. \tag{17}$$

Formulae (9) gives in this case

$$u(x,t) = \frac{x - \dot{a}_0 x - \sum_{n=1}^{\infty} \frac{a_n(t)R}{n}\sin\frac{nx}{R}}{\frac{x^2}{2R} + \sum_{n=0}^{\infty} a_n(t)\cos\frac{nx}{R} + 1 - t} = \frac{U(x,\dot{\vec{a}}(t))}{H(x,t,\vec{a}(t))}. \tag{18}$$

$p_n(t)$ can be determined by multiply the first expansion in (16) by $\cos(nx/R)$ and integrating it with respect to x with $-c \leq x \leq c$. After integration, $p_n(t)$ is obtained in the form

$$p_m(t) = K_m(t, c(t), \vec{a}(t), \dot{\vec{a}}(t)) - \sum_{n=0}^{\infty} S_{nm}(t, c(t), \vec{a}(t)) \ddot{a}_n(t), \tag{19}$$

where

$$K(t, c(t), x, \vec{a}(t), \dot{\vec{a}}(t)) = \frac{(U(c(t), \dot{\vec{a}}(t)))^2}{2H_c^2(\sqrt{H_c} - 1)} + \frac{1}{2}\left[\frac{(U(c(t), \dot{\vec{a}}(t)))^2}{(H(c(t), t, \vec{a}(t)))^2} - \frac{(U(x, \dot{\vec{a}}(t)))^2}{(H(x, t, \vec{a}(t)))^2}\right] -$$

$$- \int_x^c \frac{\xi U(\xi, \dot{\vec{a}}(t))[\sum_{n=0}^{\infty} \dot{a}_n(t) \cos \frac{n\xi}{R} - 1]\, d\xi}{(H(\xi, t, \vec{a}(t)))^2}, \tag{20}$$

$$K_n(t, c(t), \vec{a}(t), \dot{\vec{a}}(t)) = \pi R \int_{-c}^{c} K(t, c(t), x, \vec{a}(t), \dot{\vec{a}}(t))\ \cos \frac{nx}{R}\ dx, \tag{21}$$

$$S_{0m}(t, c(t), \vec{a}(t)) = 2\pi R \int_0^c \frac{\xi^2 \cos \frac{m\xi}{R}}{H(\xi, t, \vec{a}(t))}\, d\xi, \quad S_{nm}(t, c(t), \vec{a}(t)) = \frac{\pi R^2}{n} \int_0^c \frac{\xi \sin \frac{n\xi}{R} \cos \frac{m\xi}{R}}{H(\xi, t, \vec{a}(t))}\, d\xi, \tag{22}$$

The system (1)-(2) can be expanded to obtain expressions for the principal coordinates $a_n(t)$ and $b_n(t)$.

$$\ddot{a}_n + a_n(\alpha + \beta n^4) - b_n(\alpha n + \beta n^3) - \gamma p_n = 0, \tag{23}$$

$$\ddot{b}_n - a_n(\alpha n + \beta n^3) + b_n(\beta n^2 + \alpha n^2) = 0 \tag{24}$$

$$\ddot{a}_0 + \alpha a_0 - \gamma p_0 = 0. n = 0 \tag{25}$$

(19) can be used and combined with second time derivatives $\ddot{a}_n(t)$. Following [2] and [5], an infinite system of ordinary differential equations is obtained for the principal coordinates $\vec{a} = (a_0, a_1, a_2, ...)^T$, $\vec{b} = (b_0, b_1, b_2, ...)^T$ and auxiliary vector-functions $\vec{q} = (q_0, q_1, q_2, \ldots)^T$, $\vec{r} = (r_0, r_1, r_2, \ldots)^T$,

$$\begin{aligned} \dot{q}_n &= (I - \gamma S)^{-1}(\gamma K_n(t, c(t), x, \vec{a}(t), \dot{\vec{a}}(t)) - D_1 a_n - D_2 b_n) \\ \dot{r}_n &= -(D_3 a_n + D_4 b_n) \\ \dot{a}_n &= q_n \\ \dot{b}_n &= r_n \end{aligned} \tag{26}$$

$$D_1 = \mathrm{diag}(\alpha + \beta n^4), \quad D_2 = \mathrm{diag}(\alpha n + \beta n^3), \quad D_3 = \mathrm{diag}(\alpha + \beta n^3), \quad D_4 = \mathrm{diag}(\alpha n^2 + \beta n^2).$$

System (26) was solved numerically under zero initial conditions at time $t = 0$ by Runge-Kutta method. Integrals (20)-(22) was calculated numerically.

As in previous studies, it is convenient to replace the independent variable t by c. However, the connection $c(t)$ does not follow here from the Wagner condition, but from equation (14).

Results and discussion of this calculation will be presented at the workshop.

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