Transients in Wave Diffraction by Cylinders and Cylinder Arrays

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Introduction

At the 2005 Workshop, results were given comparing a fully non-linear boundary element analysis of propagating waves with calculations based on the classical linear analytical solution for a transient wavemaker (Eatock Taylor, 2005). Here the work is extended to consider diffraction by one or more circular cylinders, again in long crested propagating waves. The classical transient wavemaker solution is first adapted to facilitate the use of the efficient FFT algorithm to approximate the necessary integral.

The classical solution

The solution for long crested waves, generated by a piston wavemaker driven with a unit amplitude sinusoidal velocity at frequency ω, was given by Joo *et al* (1990) in the form

$$\eta(x,t) = \frac{2\omega}{\pi}\int_0^\infty \frac{\cos\beta t - \cos\omega t}{\omega^2 - \beta^2}\,\frac{\tanh k}{k}\cos kx\,dk\,. \qquad (1)$$

Here $\eta(x,t)$ is the wave elevation due to the imposed wavemaker velocity $u(x,t) = \sin\omega t$, and $\beta^2 = k\tanh k$. All results are non-dimensionalised with respect to the depth of the tank and the acceleration due to gravity. The origin of the coordinates (x,z) is at the intersection of the wavemaker and the mean free surface.

When the "steady state" of propagating sinusoidal waves has been reached at some point in the tank, their amplitude is $A_0 = \dfrac{\tanh k_0}{k_0 c_g}$, where $k_0 \tanh k_0 = \omega^2$, and the group velocity is

$$c_g = \frac{1}{2}\frac{\omega}{g}\left[1 + \frac{2k_0}{\sinh 2k_0}\right].$$

We now consider a position x sufficiently far from the wavemaker that evanescent waves are negligible, and we introduce shifted coordinates (x',t') such that $kx = kx' - \beta t'$. Equation (1) is then written in the form:

$$\eta(x,x',t,t') = \mathrm{Re}\left[\frac{\omega}{\pi}\int_{-\infty}^{\infty} \frac{\cos\beta t - \cos\omega t}{\omega^2 - \beta^2}\,\frac{2\beta\tanh k}{k(\tanh k + k\,\mathrm{sec}\,h^2 k)}e^{i(\beta t' - kx')}d\beta\right], \qquad (2)$$

after a change of integration variable from k to β. Equation (2) may be evaluated using the FFT algorithm, to perform the inverse transform from β (and k) to t'. At each equispaced discrete value β_n, the corresponding wavenumber k_n is obtained from the dispersion relation, prior to doing the discrete transform. The result is a time history in terms of t', for each specified position x'. The time t may be set at the length of the required simulation, and x in Eq. (1) may be set to zero.

Figure 1 shows the spatial profiles at $t = 60$ and $t = 80$, divided by A_0 to provide a unit amplitude in the steady state. Two curves are plotted in each case: one corresponds to direct evaluation of Eq. (1) by the FFT in the x domain; the other is based on use of $x' = 0$ and $t' = 80$ in Eq. (2). The FFT was performed with 2^{12} points. The case corresponds to $\omega = 1$, which was considered by Joo et al (1990). They gave a corresponding figure for $t = 60$, which (if similarly scaled) appears to be the same as Fig. 1a.

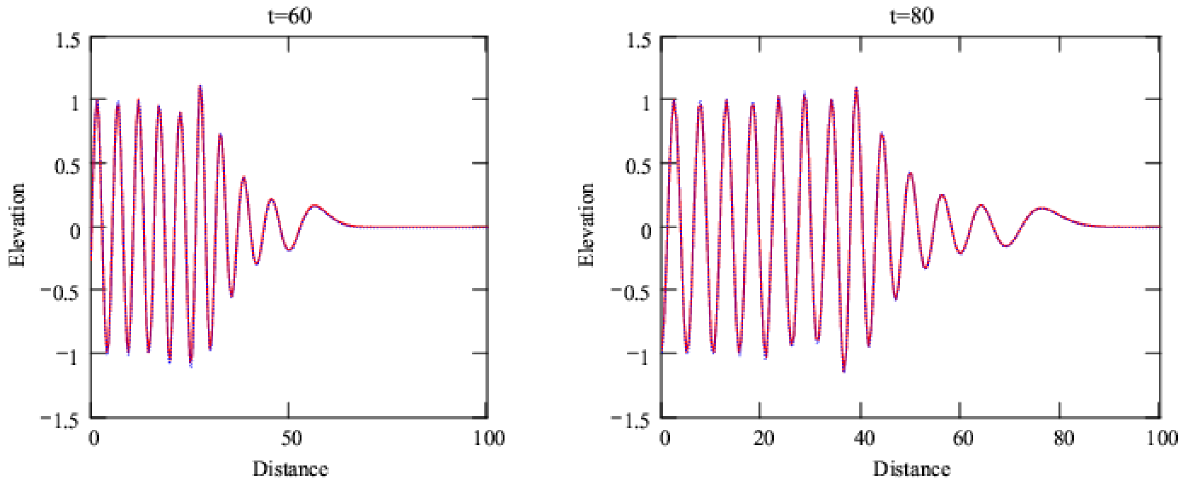

Figure 1 Profiles of transient wave, by direct evaluation of Eq. (1) and by Eq. (2)

Diffraction by a cylinder

The well known frequency domain solution for linear diffraction of a wave of amplitude A and frequency β, by a cylinder of radius a (McCamy & Fuchs,1954) is written in polar coordinates with x=r cosθ:

$$\eta(r,\theta,t) = \mathrm{Re}\left[\left\{e^{-ikr\cos\theta} - \sum_{m=-\infty}^{\infty} B_m H_m{}^{(2)}(kr)\cos m\theta\right\} e^{i\beta t}\right], \tag{3}$$

where $B_m = (-i)^{m+1}\dfrac{J'_m(ka)}{H_m^{(2)'}(ka)}$.

If we now replace the exponential term in Eq. (2) by the term in the square brackets in Eq. (3), we can obtain the solution for diffraction of the transient wave by the cylinder. Again it is convenient to evaluate this by the FFT algorithm.

Figure 2 shows results at $t = 60$ and $t = 80$ for the diffraction of the above transient wave by a vertical circular cylinder, of radius $a = 1$ in unit water depth $(k_0 a = 1.20)$. The incident and total diffracted wave elevation are plotted, as well as the radiated wave (shifted down by one

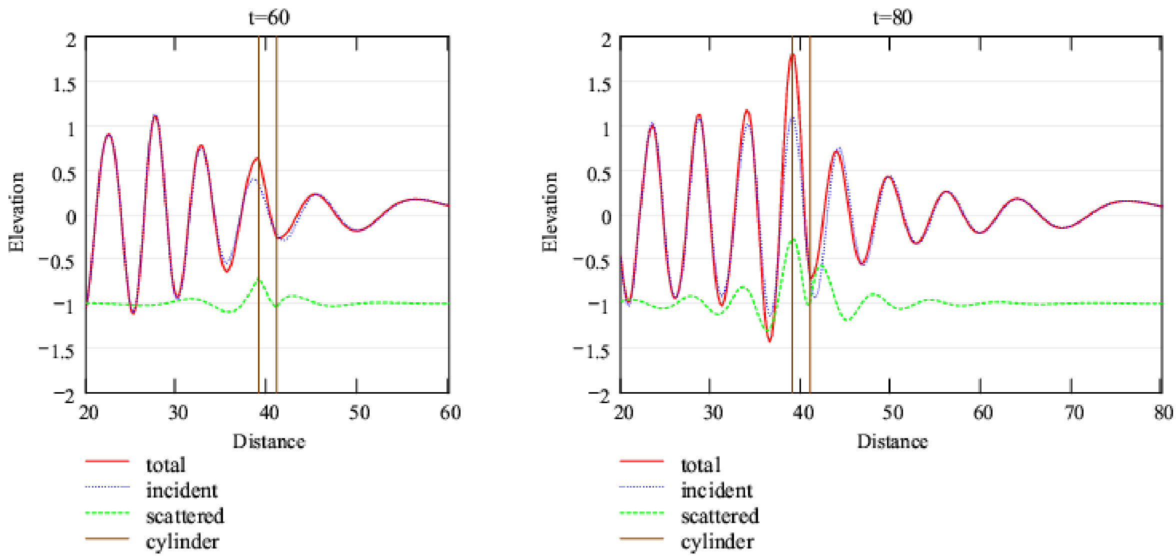

Figure 2 Diffraction of a transient wave by a cylinder

unit). Note the different axes as compared with Fig. 1. Figure 3 shows a comparison with time histories of runup on the upwave face of a cylinder, obtained from a fully non-linear boundary element analysis (Eatock Taylor, Wu, Bai and Hu, 2005). This is for a cylinder of radius 0.1416 in a tank of depth 1, and the waves are driven by a piston wavemaker at frequency $\omega = 2$. A cosine ramp function is imposed on the piston displacement time history over the first two periods, in both linear and BEM analyses. The elevations are divided by the piston amplitude, and time is expressed in periods. Results are superimposed for two amplitudes, and the effect of non-linearity may be observed.

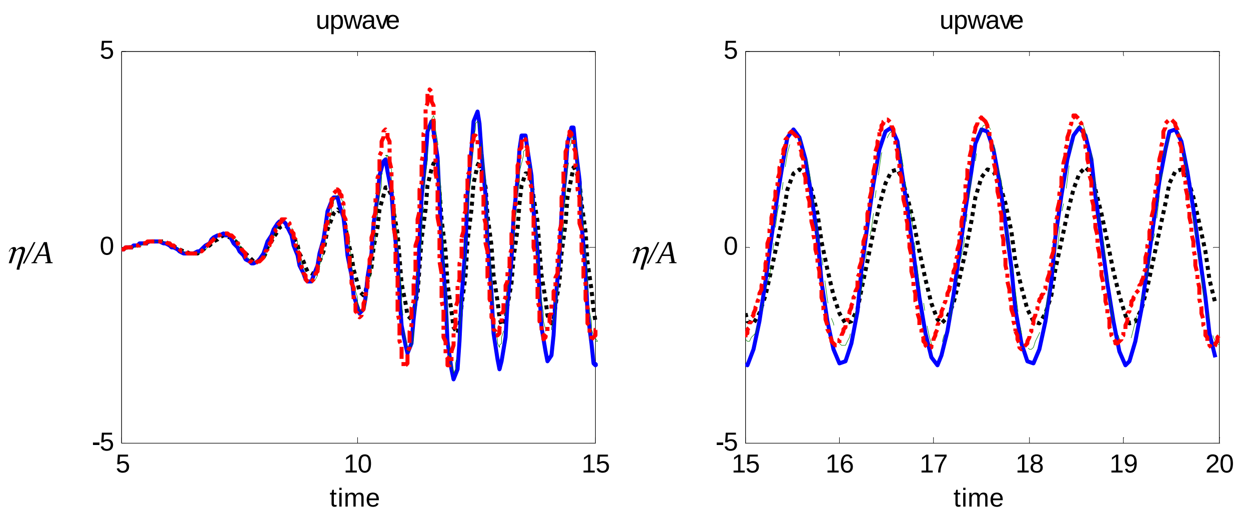

Figure 3 Linear and BEM analyses. ··········· incident; ——— linear diffracted; ------ BEM (A = 0.01); —·—·— BEM (A = 0.02)

Diffraction by an array

Maniar & Newman (1997) showed the strong magnification of wave forces due to diffraction by a long array of cylinders, linked to near trapping in regular waves. Some limited results for a linear array in a transient (focussed) wave group were given by Walker *et al* (2005), including forces and free surface elevations. A typical elevation along the centreline of a linear array of 10 cylinders is shown in Figure 4, obtained by the linear transient analysis summarised above. The half spacing d between the axes of the cylinders is twice the radius, and the regular wavemaker frequency corresponds to the near trapping wavenumber given by $k_0 d = 1.346352$. The figure includes the non-dimensional local wave amplitude at steady state (the envelope, based on a direct linear frequency domain analysis); the total diffracted wave at an instant; and the corresponding incident wave. The wave front has passed the rightmost cylinder, and the trapping phenomenon is building up. Further results illustrating the build-up will be presented at the workshop.

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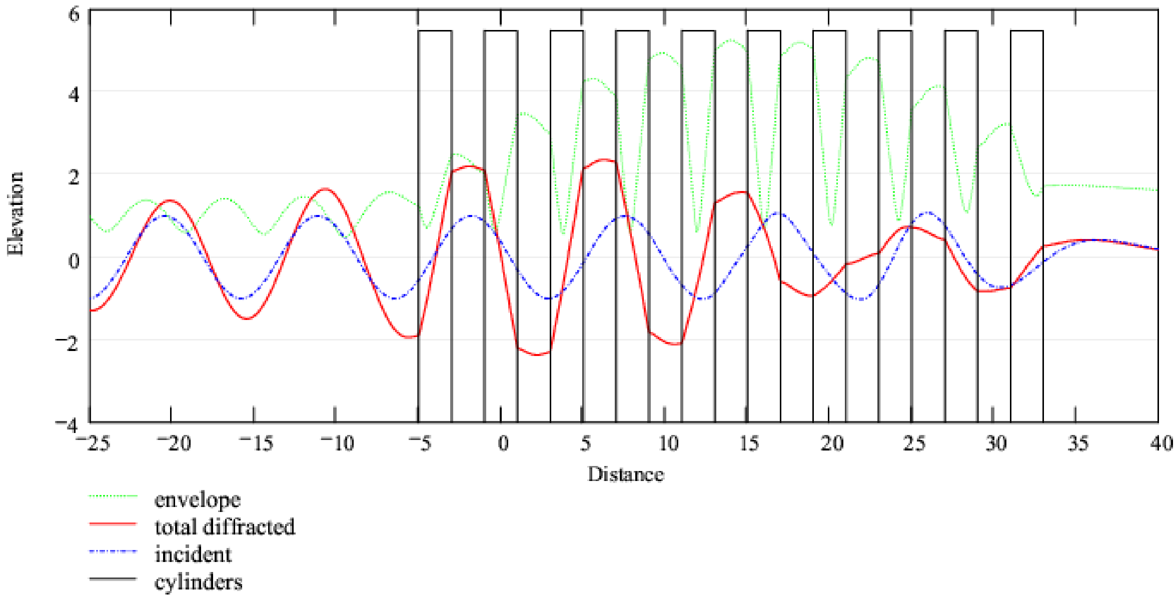

Figure 4 Wave elevation along 10 cylinders during start-up of wave at near trapping

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Discusser - M.J. Cooker:

What time scale T of *decay* in amplitude would you get if you turned off the incident waves? Is T the same or more or less than the time scale of increase towards steady (time-periodic) states which you have shown in your talk?

Reply:

Turning off the piston wave-maker would lead to another complex transient at the location of the array. One could use various types of ramp function, as I discussed at the Workshop last year in the context of turning on the wavemaker. In principle, for the same transient rise and transient decay in the incident wave I would expect the rise and decay of near-trapping response to have the same time scale in this linear model.

Discusser - M. Longuet Higgins:

The growth of the wave amplitude in some cases resembles the growth in the wave envelope at the front of a wave train, which is described by a Fresnel integral. Perhaps it would be possible to distinguish these two effects by arranging for the wavemaker to be at different distances from the array,

Reply:

This would certainly be worth investigating. Other possibilities are to use different control signals on the wavemaker to create different wave fronts. This is very easy to implement using the FFT analysis I have described, but I have not yet used these ideas to clarify the transient response of the near-trapping structure.

Discusser - M.H. Meylan:

Have you considered the complex resonance associated with the near trapping frequency. Maybe this can explain the rise time etc?

Reply:

No, but I plan to do this. The 1997 A.O.R. paper by Evans & Porter, and your paper at the 2003 Hydroelasticity conference, could provide a way forward to quantify the equivalent 'damping' in the near-trapping system. It is important to note that for the 4-column array at $ka = 1.66$, the cases $\beta = 0^o$ and $\beta = 45^o$ lead to different rise times in the corresponding maximum local wave amplitudes. The imaginary part of the complex, resonance frequency is not the only parameter governing the rise (or decay) of the near-trapping.