A simple theory of overturning ship bow waves

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Introduction

Common observation shows that an overturning thin sheet of water is typically generated at the bow of a ship that advances in calm water. Numerical computation of this highly nonlinear, two-phase, breaking flow is complex. A simple fully-nonlinear steady inviscid flow theory is summarized here. The theory involves three main ingredients and distinct steps, explained below.

The flow is observed from a system of coordinates (X,Y,Z) attached to the moving ship. The Z axis is vertical and points upward, and the mean free surface is taken as the plane $Z=0$. The X axis lies along the ship path and points toward the ship bow. Nondimensional coordinates $(x\,,y\,,z)=(X,Y,Z)\,g/V_s^2$ and related components $(u\,,v\,,w)=(U,V,W)/V_s$ of the flow velocity are defined in terms of the ship speed V_s and the acceleration of gravity g. Here, (U,V,W) is the velocity of the flow due to the ship. Thus, the nondimensional velocity of the total flow (uniform stream opposing the forward speed of the ship + flow due to the ship) is $(u-1,v,w)$.

A local system of coordinates is also used. Specifically, the orthogonal unit vectors $\mathbf{t}=(t^x,t^y,0)$ and $\mathbf{m}=(-t^y,t^x,0)$ are defined. These vectors lie in a horizontal plane. The vector $\mathbf{m}$ is colinear with the projection, onto the mean free-surface plane $z=0$, of the unit vector $\mathbf{n}=(n^x,n^y,n^z)$ normal to the ship hull. The vectors $\mathbf{n}$ and $\mathbf{m}$ point outside the ship. The vector $\mathbf{t}$ is tangent to the ship hull surface and, on the positive side $0\le y$ of the ship hull considered here, points toward the ship bow. One has

$$\mathbf{t}=(\cos\alpha\,,-\sin\alpha\,,0)\qquad \mathbf{m}=(\sin\alpha\,,\cos\alpha\,,0)\qquad \mathbf{n}=(\sin\alpha\,\cos\gamma,\cos\alpha\,\cos\gamma,-\sin\gamma)\tag{1a}$$

with $-\pi/2\le\alpha\le\pi/2$ and $-\pi/2\le\gamma\le\pi/2$. In the bow region, the angle α between the unit vector $\mathbf{t}$ and the x axis is positive. The (flare) angle γ between the normal vector $\mathbf{n}$ to the ship hull and the mean free-surface plane $z=0$ is positive for a typical hull form, and negative for a tumble hull. The unit vector

$$\mathbf{s}=\mathbf{t}\times\mathbf{n}=(\sin\alpha\sin\gamma,\cos\alpha\sin\gamma,\cos\gamma)=\mathbf{m}\,\sin\gamma+\mathbf{k}\,\cos\gamma\tag{1b}$$

is tangent to the ship hull and points upward. Here, $\mathbf{k}=(0\,,0\,,1)$ is the unit vector along the vertical z axis. The components $(u-1\,,v\,,w)$ of the total flow velocity $\mathbf{v_{total}}$ along the unit vectors $(\mathbf{i}\,,\mathbf{j}\,,\mathbf{k})$ attached to the $(x\,,y\,,z)$ axes and the corresponding velocity components (u',v',w') along the unit vectors $(\mathbf{t}\,,\mathbf{n}\,,\mathbf{s})$ are related via the identity $\mathbf{v_{total}}=(u-1)\,\mathbf{i}+v\,\mathbf{j}+w\,\mathbf{k}=u'\,\mathbf{t}+v'\,\mathbf{n}+w'\,\mathbf{s}$. The free surface near the ship is defined by $z=\zeta(x\,,y)$ or $z=\zeta(t,m)$, and the slope ζ_m of the free surface in the direction of the unit normal vector $\mathbf{m}$ is expressed as

$$\zeta_m=\tan\mu\quad\text{with}\quad -\pi/2-\gamma\le\mu\le\pi/2-\gamma\tag{2}$$

where μ stands for the angle between the free surface and the mean free-surface plane $z=0$.

Nonlinear analysis of flow along contact curve

A main element of the theory is a fully-nonlinear analysis of the steady inviscid flow along the contact curve between the ship hull and the free surface. Thus, surface-tension and viscosity effects are ignored here. However, no other approximations are made, and the analysis is exact for steady inviscid flows.

The ship-hull boundary condition $n^x(u-1)+n^yv+n^zw=0$ and the kinematic and dynamic boundary conditions $w=(u-1)\,\zeta_x+v\,\zeta_y$ and $(u-1)^2+v^2+w^2=1-2\,\zeta$ at the free surface $z=\zeta(x\,,y)$ are presumed to hold along the contact curve between the ship hull and the free surface. These three boundary conditions yield three algebraic equations (two linear equations and a quadratic equation) that can be used to determine the three velocity components u, v and w. The resulting expressions for u, v and w are given in Noblesse et al. (1991), where experimental validation may also be found. Further experimental validation of the analysis is given in Waniewski et al. (2002).

The analysis of the ship-hull/free-surface contact flow given in Noblesse et al. (1991) is reconsidered here using the local system of coordinates $(\mathbf{t}\,,\mathbf{n}\,,\mathbf{s})$. The ship-hull boundary condition becomes $v'=0$, as expected. Thus, the velocity component v' along the unit vector $\mathbf{n}$ normal to the ship hull is null, and the flow velocity is given by

$$\mathbf{v_{total}} = u'\,\mathbf{t} + w'\,\mathbf{s} = u'\,\mathbf{t} + w'\;\mathbf{t}\times\mathbf{n} \tag{3}$$

The velocity components u' and w' along the unit vectors $\mathbf{t}$ and $\mathbf{s}$ tangent to the hull are then determined by the kinematic and dynamic free-surface boundary conditions, which yield

$$w'\,(\cos\gamma - \zeta_m \sin\gamma) = u'\,\zeta_t \qquad\qquad (u')^2 + (w')^2 = 1 - 2\,\zeta \tag{4}$$

Here, the ship-hull boundary condition $v' = 0$ was used, and the free surface is defined by $z = \zeta(t, m)$.

The kinematic free-surface boundary condition (4) and expression (2) yield

$$w' = \frac{u'\,\zeta_t \cos\mu}{\cos(\gamma+\mu)} = \frac{u'\,\zeta_t / \cos\gamma}{1 - \tan\gamma\,\tan\mu} \tag{5}$$

This expression and the dynamic free-surface boundary condition (4) then yield

$$u' = -\sqrt{\frac{1-2\,\zeta}{\cos^2(\gamma+\mu) + \zeta_t^2\,\cos^2\mu}}\;\cos(\gamma+\mu)$$

Here, the condition $0 \le \cos(\gamma+\mu)$, which follows from (2), and the condition $u' \le 0$ (no flow reversal) were used. Expressions (3) and (5) then show that the total flow velocity at the contact curve is

$$\mathbf{v_{total}} = u'\,\mathbf{t} + w'\;\mathbf{t}\times\mathbf{n} = -\sqrt{\frac{1-2\,\zeta}{\cos^2(\gamma+\mu) + \zeta_t^2\,\cos^2\mu}}\;[\,\cos(\gamma+\mu)\;\mathbf{t} + \zeta_t\cos\mu\;\;\mathbf{t}\times\mathbf{n}\,] \tag{6}$$

Thus, the total flow velocity $\mathbf{v_{total}}$ at the contact curve between a ship hull and the free surface is defined by the simple analytical expression (6) in terms of the ship speed V_s, the flare angle γ, the elevation ζ of the contact curve, and the angle μ between the free surface and the horizontal plane. Expression (6) is based on exact boundary conditions (for steady inviscid flows), at the actual locations of the ship hull and the free surface, and thus is exact. This expression is equivalent to, but simpler than, the expressions for the velocity components u, v and w given in Noblesse et al. (1991).

Expression (5) and the inequalities $u' \le 0$ and $0 \le \cos(\gamma+\mu)$ yield $\text{sign}(w') = -\text{sign}(\zeta_t \cos\mu)$. This relation yields $\text{sign}(w') = -\text{sign}(\zeta_t)$ for $-\pi/2 \le \mu \le \pi/2$, and one then has $0 < w'$ in the region between a ship stem and a ship bow-wave crest where $\zeta_t < 0$. Expression (5) yields

$$w'\cos\gamma \approx u'\,\zeta_t \quad \text{if} \;\; \tan\gamma\,\tan\mu \ll 1 \tag{7}$$

This approximation may be expected to hold except near a ship stem (or stern) where μ or γ can be large. The approximation (7) and the dynamic free-surface boundary condition (4) yield

$$\mathbf{v_{total}} = u'\,\mathbf{t} + w'\;\mathbf{t}\times\mathbf{n} \approx -\sqrt{\frac{1-2\,\zeta}{\cos^2\gamma + \zeta_t^2}}\;(\,\cos\gamma\;\mathbf{t} + \zeta_t\;\,\mathbf{t}\times\mathbf{n}\,) \tag{8}$$

This approximation defines the flow velocity at the free-surface and ship-hull contact curve in terms of the ship speed V_s and flare angle γ, and the elevation ζ of the contact curve.

Lagrangian analysis of detached flow sheet

Another main element of the theory is the determination of the shape of the detached sheet of water that leaves the ship hull along the ship-hull/free-surface contact curve. This step of the theory consists in an elementary Lagrangian analysis of the motions of fluid particles that leave the ship hull at the contact (flow-detachment) curve with velocity

$$u'\,\mathbf{t} + w'\,\mathbf{s} = u'\,\mathbf{t} + w'\,(\,\mathbf{m}\,\sin\gamma + \mathbf{k}\cos\gamma\,)$$

with u' and w' given by (6) or the related approximation (8).

The nondimensional time $\theta = \Theta\,g/V_s$ and the nondimensional coordinates $t = Tg/V_s^2$, $m = Mg/V_s^2$, $z = Zg/V_s^2$ show that the path of a water particle is determined by Newton's equations $d^2t/d\theta^2 = 0$,

$d^2m/d\theta^2 = 0$, $d^2z/d\theta^2 = -1$. Here, t and m are the nondimensional horizontal distances along the unit vectors $\mathbf{t}$ and $\mathbf{m}$ tangent and normal to the ship hull in a horizontal plane. Thus, a water particle that leaves a point $(t_0\,, m_0\,, \zeta)$ of the flow-detachment curve, with velocity given by (6) or (8), at the time $\theta = 0$ follows the path $t = t_0 + \theta\, u'_0$, $m = m_0 + \theta\, w'_0 \sin\gamma$, $z = \zeta + \theta\, w'_0 \cos\gamma - \theta^2/2$. These parametric equations yield

$$m - m_0 = \frac{t-t_0}{u'_0}\, w'_0 \sin\gamma \qquad\qquad z - \zeta = \frac{t-t_0}{u'_0}\left(w'_0 \cos\gamma - \frac{t-t_0}{2\, u'_0} \right) \tag{9a}$$

$$z - \zeta = \frac{m-m_0}{\tan\gamma}\left(1 - \frac{m-m_0}{(w'_0)^2 \sin(2\gamma)} \right) \tag{9b}$$

Thus, the projections of the paths of water particles on the horizontal plane $(\mathbf{m}\,, \mathbf{t})$ and the vertical planes $(\mathbf{k}\,, \mathbf{t})$ and $(\mathbf{k}\,, \mathbf{m})$ are a straight line and parabolas, respectively, as expected.

The water trajectory defined by (9) intersects the mean free-surface plane $z = 0$ for

$$\frac{t_{z=0} - t_0}{u'_0} = w'_0 \cos\gamma + \sqrt{(w'_0)^2 \cos^2\gamma + 2\,\zeta} = \frac{m_{z=0} - m_0}{w'_0 \sin\gamma} \tag{10}$$

If $0 < w'_0$, the water trajectory reaches a top height for $(t_{\,top} - t_0)/u'_0 = w'_0 \cos\gamma = (m_{\,top} - m_0)/(w'_0 \sin\gamma)$ and the top height is given by

$$z_{\,top} - \zeta = (w'_0 \cos\gamma)^2/2 \approx (u'_0\, \zeta_{\,t})^2/2 \tag{11}$$

where the approximation follows from (7). Thus, the maximum height $z_{\,top}$ reached by water particles that leave the ship-hull/free-surface contact curve at a height $z = \zeta$ is significantly larger that ζ only if $|\,\zeta_{\,t}\,|$ is large, e.g. near a ship stem where the contact curve is tangent to the ship stem (Noblesse et al. 1991).

Two options : semi-analytical and analytical theories

Expressions (9)–(11) define the detached sheet of water that leaves the ship hull along the ship-hull and free-surface contact curve in terms of the location of the contact curve and the related velocity components u' and w'. These velocity components are defined by (8) in terms of the ship speed, the hull geometry, and the location of the contact curve. Thus, the detached sheet of water generated at a ship bow is explicitly determined in terms of the ship speed, the hull geometry, and the location of the contact curve.

The location of the contact curve can be determined using a number of well-established calculation methods, including approximate methods (Michell thin-ship theory, slender-ship approximation, 2D+T approximation), panel methods based on boundary distributions of Rankine sources or free-surface Green functions, and CFD methods that solve discretized Euler or RANS equations. This mixed approach — in which the analytical expressions for the flow at the ship-hull/free-surface contact curve and the related detached bow sheet given here are coupled with a numerical determination of the location of the contact curve (taken as the computed bow wave) — provides a semi-analytical theory of overturning ship bow waves.

An alternative, fully-analytical, theory is obtained if the simple analytical expressions for the height and location of a ship bow wave given in Noblesse et al. (2005) are used (instead of a computational tool) to determine the location of the ship-hull and free-surface contact curve. Specifically, the height $z_b = Z_b\, g/V_s^2$ and the location $x_b = X_b\, g/V_s^2$ of the bow wave generated by a ship that advances at constant speed V_s in calm water is explicitly determined in terms of the ship speed V_s , the draft D, and the waterline entrance angle $2\,\alpha_E$ by

$$z_b \approx \frac{C^Z}{1+F_D}\,\frac{\tan\alpha_E}{\cos\alpha_E} \qquad\qquad x_b \approx \frac{C^X}{1+F_D} \qquad\qquad \text{with } F_D = \frac{V_s}{\sqrt{gD}} \tag{12}$$

These simple analytical expressions, with $C^Z \approx 2.2$ and $C^X \approx 1.1$, are shown in Noblesse et al. (2005) to be in excellent agreement with experimental measurements for wedge-like ship bows, and for other ship-bow forms if a simple procedure is used to define an effective draft D and an effective waterline entrance angle $2\,\alpha_E$.

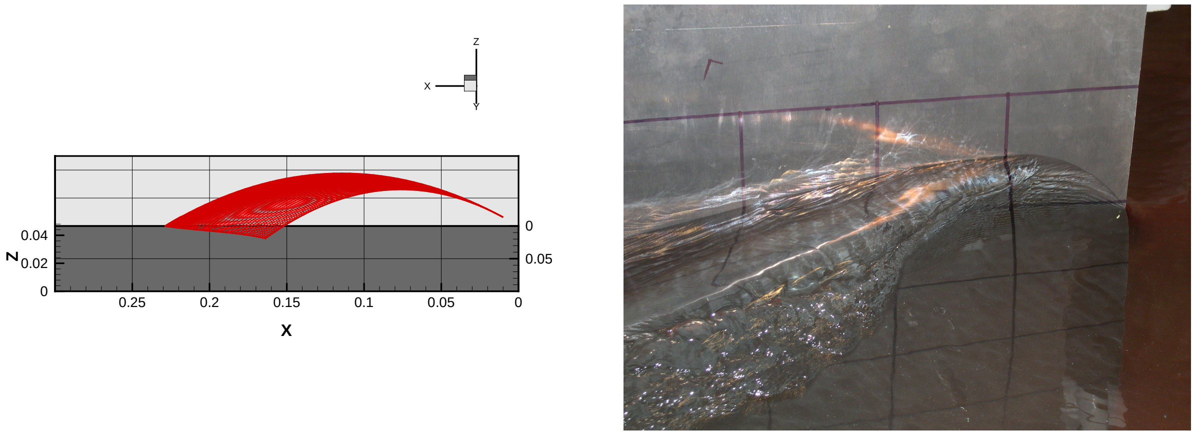

Figure 1: Overturning bow sheet for $\alpha_E = 10°$, $\gamma = 10°$ and $F_D = 0.67$

A ship bow wave can then be reasonably approximated by the expression

$$z = \frac{C^Z}{1+F_D}\,\frac{\tan\alpha_E}{\cos\alpha_E}\left(1 - \frac{(1+F_D)^2\,t^2}{(C^X)^2}\right) \quad \text{with} \quad |\,t\,| \le \frac{1}{1+F_D} \tag{13}$$

Here, the crest of the bow wave is chosen as the origin $t = 0$ of the axis associated with the previously-defined unit vector $\mathbf{t}$ tangent to the ship hull. The foregoing expression shows that the normalized bow-wave elevation $(1+F_D)\,(\cos\alpha_E/\tan\alpha_E)\,Z\,g/V_s^2$ only depends on the normalized horizontal distance $(1+F_D)\,X\,g/V_s^2$ from the bow-wave crest. This similarity rule implies that all bow-wave profiles (for every ship speed V_s, draft D and waterline entrance angle $2\,\alpha_E$) approximately coalesce into a single curve if represented in terms of the foregoing normalized variables.

Expression (13) for a ship bow-wave profile can be improved if one enforces the property — theoretically established in Noblesse et al. (1991) and experimentally verified in Waniewski et al. (2002) — that, at a ship stem, the bow-wave profile is tangent to the stem line. This property and expressions (12) for a bow-wave height and location can indeed be used to obtain a useful analytical approximation to a ship bow wave.

This analytical approximation for the ship-hull/free-surface contact curve, or expression (13), can be used with expressions (8) and expressions (9)–(11) to explicitly determine the overturning sheet of water that is typically generated at a ship bow in terms of the ship speed V_s and the hull geometry.

Comparison with experimental observations

Calculations based on expressions (13), (8) and (9)–(11) in fact are reported in Delhommeau et al. (2005) and in Fig.1 for an idealized ship bow, taken as an inclined flat plate with flare angle $\gamma = 10°$ and half waterline-entrance angle $\alpha_E = 10°$, at a Froude number $F_D = 0.67$. These analytical predictions are in reasonable agreement with the experimental observations. Additional, more detailed experimental measurements are in progress and will be reported at the Workshop, with comparisons with theoretical predictions.

Conclusion

In summary, a remarkably simple, fully-analytical, theory of the overturning thin sheet of water that is typically generated at the bow of a ship advancing in calm water has been summarized.

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Discusser - H.B. Bingham:

Would you comment on the significance of the fact that your experiments are unsymmetric with respect to the ship's centerline. This would seem to produce a rather different flow from both the real ship bow and your theory.

Reply:

Our experiments are made with a flat plate in heel and drift which leads to an unsymmetric flow. We check by 3-D perfect fluid computations that this effect is not too large and that the non vertical leading edge has little influence on the shape of the wave at moderate heel angles. In practice this model is more convenient to study rapidly the influence of flare and entrance angles.