Interference Resistance of Multi-Hulls per Thin-Ship Theory

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To a colleague and good friend, Nick Newman, in celebration of his 70th birthday. A leader and staunch supporter of marine hydrodynamics, Nick has expanded the reach and influence of this field through his insights and publications.

1 Introduction

Speed is an important consideration in all transportation systems. It is the solution to excursion-time reduction. However, this may be attained at the expense of increase in power consumption and in exhaust pollution. Ship designers are well aware of that conventional monohulls experience a rapid increase in *wave drag* at Froude number around 0.37, and the "first hump" of resistance at the Froude number of 0.5 is difficult to overcome. Higher speeds can be achieved by raising the hulls above water using foils or air cushions, or simpler still, by reconfiguring the original hull into a formation of smaller hulls. Other solutions involving a combination of air cushion, multi-hulls, and foils have also been proposed.

In the San Francisco Bay Area, local and state authorities have authorized a ten-year plan for a "quadruple expansion" of ferry routes, using new, high-speed ferries with a design involving a combination of catamaran and air-cushion. A fast tetra-hull has also been successfully developed by Lockheed Martin in Sunnyvale, CA, one that has excellent motion characteristics in moderate seas. This strong demand for fast multi-hulls has also led to a scrutiny on environmental concerns, especially in connection with operation in harbor and estuarine areas. Since large wave drag is almost synonymous to large wake wash, the design of "environmentally friendly" hulls needs an effective model for wave-resistance prediction.

2 Wave Resistance Theory

We will focus on the subject of wave resistance, which is the inhibiting component of drag at high Froude number. In a revisit of the theory of Michell (1898), which had provided the classical expression for the wave-making resistance of a monohull based on the assumption that the beam-to-length ratio is small, we were able to obtain a generalized expression useful for analyzing the effects of multi-hull interferences. An extensive collection of references on ship resistance in various contexts can be found in, for instance, Kostyukov (1958), Wehausen (1973), and more recently, Gotman (2002), and Tuck et al. (2002). Michell's theory was considered inadequate in the 1970's as few practical monohulls would meet the stringent "thin-ship" assumption and most design conditions were aimed at speed below the first hump. Figure 1, perhaps, summarizes the difficulty of this situation at lower Froude number, using a moderately thin Taylor Standard Series model as an example. Yet, at the resistance hump, despite the apparent importance of sinkage and trim (Yeung, 1972), Michell's theory as computed by Tuck et al. (1997) and our present procedure (CMML, 2004) yields predictions that are rather effective as a "first-cut" evaluation in ship design. Furthermore, for a given displacement, a multi-hull ship system will invariably consist of thinner hulls, thus making them more favorable to thin-ship modeling.

The presence of multi-hulls generates cross-flow effects. When this so-called lifting or camber contribution are neglected, based on either a slenderness assumption, or on an appropriate camber reshaping of the member hulls, the total resistance of the hull system can be represented simply by the interference of the wave systems in the far-field. Among many works on this subject, one should recall Eggers' (1955) excellent treatise on two hulls in stagger formation; this study includes the consideration of the effects from finite depth and channel walls. The cross-flow ef-

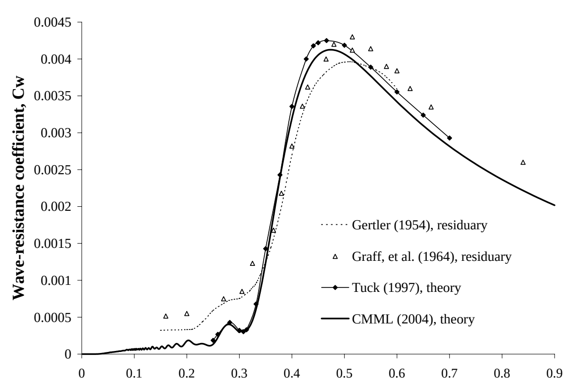

Figure 1: *Comparative plots of the wave-making resistance coefficient for a Taylor Standard Series hull ($C_p = 0.56,\ B/T = 3.0,\ C_\forall = 1.7 \times 10^{-3}$).*

fects can be modeled by a dipole distribution (Scragg et al., 1998), which requires the solution of an integral equation in a way similar to the Kelvin-Neumann problem (Yang et al., 2000). The computations would be quite demanding.

When a multiple number of hulls are present, Michell's resistance is not simply the sum of the individual resistances of each individual hull alone. The quadratic form of the expression yields an extra term that accounts for the interaction between each pair of combination of the hulls. Analysis showed that this interference resistance can be expressed in a strikingly simple integral, mirroring somewhat Michell's original expression for a single hull. The new expression (Yeung et al., 2004) contains the explicit effects of stagger and separation and requires only the knowledge of the Kochin functions of each of the interacting hulls. The expression together with Michell's integral can be computed concurrently using specialzed quadrature methods. On a desk-top PC, thousands of combinations of geometric configurations and speeds would take only tens of seconds, thus enabling a rapid evaluation in the parameter space and a quick search for an optimum in the early stage of configuration design of multi-hulls. After providing a brief exposition of the analytical development, we present one case of the our validations against experimental measurements, and two sample applications.

3 Interference Resistance of a pair of hulls

With reference to Fig. 2, we assume two hulls to be moving at constant speed U in the $x-$direction, each defined by the hull function $y_j = f_j(x,z)$ in its own body coordinates. If these hulls were individually *alone*, Michell's well-known result gives the following expression:

$$R_{wj} = \pi\,\rho U^2 \int_1^\infty \frac{d\lambda}{\lambda^4\sqrt{\lambda^2-1}}\,|\,A_j(\lambda)\,|^2,\ (j=1,2) \quad (1)$$

where $A_j(\lambda)$, the wavemaking-amplitude (or Kochin) function of the j-th hull, and is given by:

$$A_j(\lambda) = \frac{2\,i}{\pi}\,k_0^2\,\lambda^4 \iint_{S_j} dxdz\, f_j(x,z)\, e^{k_0\lambda^2 z}\, e^{ik_0\lambda x}. \quad (2)$$

with $k_0 = g/U^2$. Here, λ may be written as $\sec\theta$, where θ denotes the orientation of the crest line of the Kelvin wave system relative to the x-axis (see Newman, 1977). The classical result was given in terms of hull slope f_{jx}. An integration by part in x was conducted to obtain the above after assuming the hulls to have closed ends.

Of interest is Eq. (1) states that the wave resistance is proportional to the beam squared, with all other variables kept constant. Thus, if we start with a baseline hull of beam B, and split it into two hulls identical and affine to the initial one, the two separate hulls will have beam $B'(=B/2)$, and $R_{w1} + R_{w2}$ would only be one-half of the baseline-hull resistance, yet having the same displacement. This argument is definitely true if the two hulls are infinitely far apart (not a practical situation). In reality, the resistance of the two hulls with finite separation and stagger is given by:

$$\begin{aligned} R_{wT} &= R_{w1} + R_{w2} + R_{w1\rightleftharpoons 2} \\ &\equiv R_{w1} + R_{w2} + R_{w1\to 2} + R_{w1\leftarrow 2} \end{aligned}$$

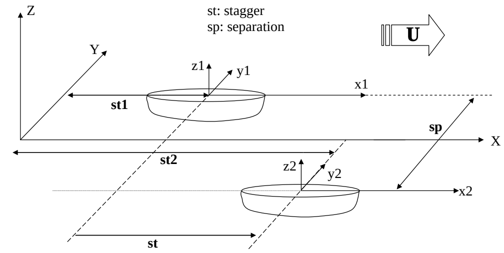

Figure 2: *Frames of references for two hulls with separation and stagger.*

The interference resistance $R_{w1\rightleftharpoons 2}$ sums the effect of hull 2 on hull 1, $(R_{w1\to 2})$ and the the effect of hull 1 on hull 2 $(R_{w1\leftarrow 2})$. Clearly, these effects can be expressed as:

$$\begin{aligned} R_{w1\rightleftharpoons 2} &= R_{w1\to 2} + R_{w1\leftarrow 2} = \\ &\frac{\rho U^2}{\pi} \iint_{S_1} dx_1 dz_1\,(f_1)_{x_1} \iint_{S_2} d\xi_2 d\zeta_2\,(f_2)_{\xi_2} \\ &\qquad \times G_{x_2}(x_1-\xi_2;\ sp;\ z_1,\ \zeta_2) \\ &+\frac{\rho U^2}{\pi} \iint_{S_2} dx_2 dz_2\,(f_2)_{x_2} \iint_{S_1} d\xi_1 d\zeta_1\,(f_1)_{\xi_1} \\ &\qquad \times G_{x_1}(x_2-\xi_1;\ -sp;\ z_2,\ \zeta_1). \end{aligned} \quad (3)$$

where G is the Havelock source function given in Wehausen & Laitone (1962). As expected, the inner double integral represents the linearized dynamic pressure driven by a source distribution of the neighboring hull, while the outer double integral integrates this pressure using the longitudinal component of the hull in question. After substituting the expressions of the Green functions in each of the two terms, making appropriate trigonometric expansions, and changing variables to relate the coordinate systems, one finds that the odd terms of G_{x_1} and G_{x_2} cancel out in the sum of $R_{w1\to 2}$ and $R_{w1\leftarrow 2}$ (Yeung et al., 2004). Similar "internal force" cancellation was observed by Eggers (1955). In the end, we arrive at a rather simple expression involving the Kochin functions of the interacting hulls:

$$\begin{aligned} R_{w1\rightleftharpoons 2} &= 2\,\pi\,\rho U^2 \int_1^\infty \frac{d\lambda}{\lambda^4\sqrt{\lambda^2-1}}\,\cos\left[k_0 sp\lambda\sqrt{\lambda^2-1}\right] \\ &\times \left(\Re(A_1\overline{A_2})\,\cos\left[k_0\,st\,\lambda\right] - \Im(A_1\overline{A_2})\,\sin\left[k_0\,st\,\lambda\right]\right), \end{aligned} \quad (4)$$

with $\Re$ and $\Im$ denoting real and imaginary parts, respectively. Eq. (4) shows explicitly how the stagger $st \equiv (st_2 - st_1)$ and separation sp of the two hulls can influence the total wave resistance. The more negative the interference is, the less wave drag the pair of hulls has. $R_{w1\rightleftharpoons 2}$ is independent of the sign of st if the two hulls are identical. If $A_1 \neq A_2$, $R_{w1\rightleftharpoons 2}$ does depend on the sign of st.

The above analysis can be quickly generalized to a family of n hulls. The total wave resistance exerted on the *whole system* is given by:

$$R_{wT} = \sum_{i=1}^{n} R_{wi} + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} R_{wi\rightleftharpoons j}. \quad (5)$$

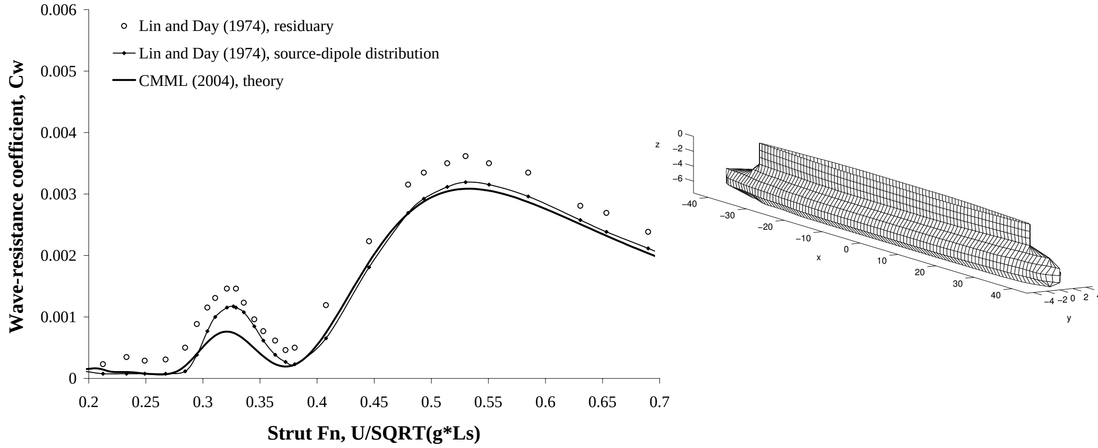

Figure 3: *Comparison of the wave-resistance coefficients of the MODCAT-IV catamaran (Lin & Day, 1974) and perspective view of the demi-hull.*

This involves the interference resistance $R_{wi\rightleftharpoons j}$ of *any pair* of hulls i and j. The indices $(1, 2)$ in (4) need only be replaced by (i, j), with $st = (st_j - st_i)$ and sp is the lateral separation between hull i and j.

4 Validations and Applications

Computations of the integrals given by Eqs. (1), (2), and (4) were made by first developing a spline-surface of the hulls. Then for a given λ, the Kochin functions were computed using Gaussian quadrature and Filon quadrature. The final integration in λ or θ uses either regular or adaptive numerical integration as appropriate. The fast but accurate computations allow a full definition of the resistance function in the parametric space quickly.

For a di-hull example, we compare the computations from our "CATRES" code for the Lin & Day (1974) twin-hull SWATH hull. Figure 3 shows the frame lines of the model and the present results, compared with both experiments and those of Lin (1974), the latter using a dipole distribution to account for cross-flow effects on the strut.

The humps and hollows occur at the same locations, but there are some discrepancies regarding the values of the resistance around Froude number of 0.325.

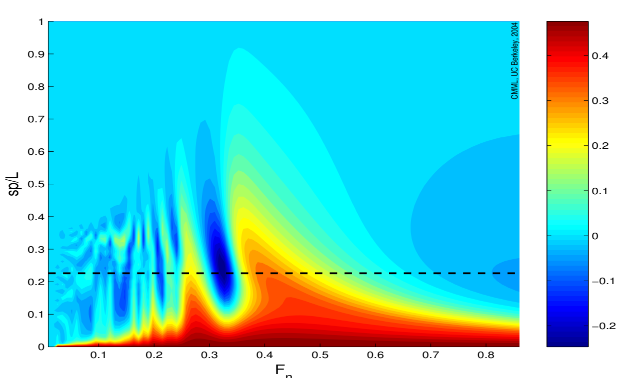

Figure 4: *Contour plot of R_{interf}/R_0 for two Series-60 hulls in parallel configuration as function of Fn and sp/L.*

Next, we show a combination of two Series 60 (demi-)hulls (Model 4210W) in a catamaran formation, with $st = 0$, but sp and F_n taken as variables. Fig. 4 is a contour projection of the 3-D surface function $R_{1\rightleftharpoons 2}/R_0$, labeled as R_{interf}/R_0. Here R_0 is the resistance of a mono series 60 hull of the same displacement. The complexity of the interference is evident. However, one can pick out the existence of an optimal sp of $0.226L$ and Fn=0.33, which is an achievable value in practice.

A tri-hull resistance code, "TRIRES", was also developed to design/configure a combination of three hulls, with stagger, separation, speed, and volumetric distribution, as possible variables. To illustrate the application of one of its several options for a tri-hull problem, we consider the problem of three hulls with the outriggers being identical, but geometrically scaled from the center hull. A Wigley hull form is chosen and assumed to operate at a design speed of $U_d = 12m/s$. The separation (sp) between the center hull (Hull 1) and each of the outriggers (Hull 2 and 3) is assumed fixed at $9m$.

For comparison, a baseline Wigley hull ($L = 36m,\ B = 3.6m$ and $T = 2.25m$) is taken as reference. The effects of stagger st and the volumetric distribution among the three hulls on the total resistance of the system are investigated, with the constraint that the total volume is fixed at $\forall_o = 64.8m^3$. The distribution of the volume among Hull 1 and the Outriggers is governed by the relation: $\forall_1 = (1-2p)\forall_o$ and $\forall_{2,3} = p\forall_o$, with p in $[0., 0.5]$. This formulation allows one to recover a monohull when $p = 0$, a catamaran when $p = 0.5$. The stagger between Hull 1 and the Outriggers is negative (0 to -60 m). The surface plot of the wave resistance in Fig. 5 reveals a minimum point at ($st = -29.25m$ and $p = 0.2789$), with $R_{wT} = 20,200N$. A perspectivce veiw of the optimum configuration of the trimaran is shown in Fig. 6.

Figure 7 compares the perfomance of this optimum trimaran with that of the baseline monohull, in term of wave resistance alone and total resistance, the latter includes the frictional resistance based on the ITTC friction line. The wave resistance is reduced by 60.7% for the trimaran. Even though the increase in the friction contribution (because of larger wetted surfaces) diminishes this favorable redcution, the trimaran's total resistance is still 25% less than that of the monohull at the design speed $U_d = 12m/s$.

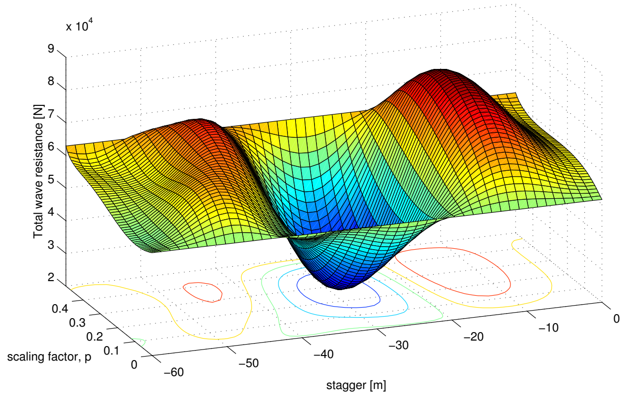

Figure 5: *Total wave resistance of three Wigley hulls with stagger st and scaling factor p as variables (at $U_d = 12\,m/s$, $sp = 9m$).*

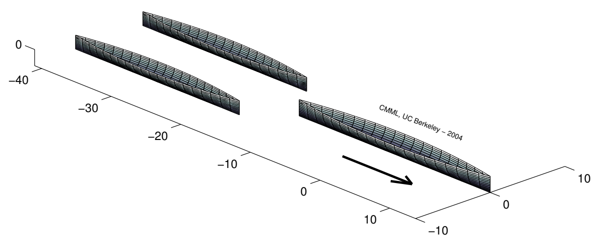

Figure 6: *Perspective view of the optimum trimaran at a design speed of $12\,m/s$.*

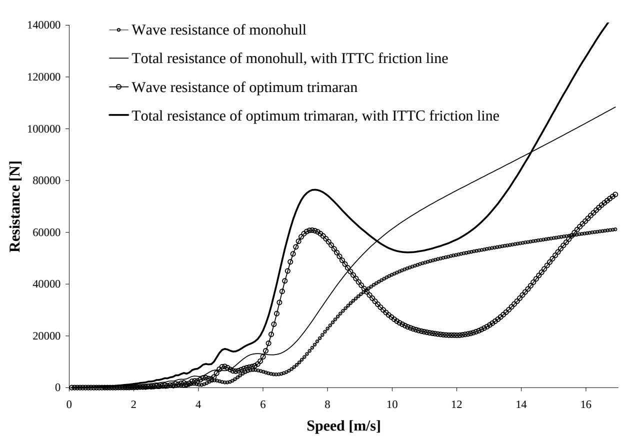

Figure 7: *Wave resistance and total resistance (using the ITTC (1957) friction line) of two configurations (monohull and trimaran) having the same total displaced volume* $\forall = 64.8\,m^3$.

The trimaran will have to overcome a higher resistance at a speed of about $7.8m/s$ in order to reach the design speed. More discussions and results of this web-based analysis tool will be further explained in the Workshop.

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