The Electromagnetic Field Induced by a Submerged Body Moving in Stratified Sea

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Introduction

Wave motion at sea induces variation of the electromagnetic field of the Earth [1]. Surface waves generated by a moving ship also affect the ambient electromagnetic field. Algorithms for detecting such variations have been discussed in [2,3]. For a body moving below the free surface, the variations in the magnetic field may be too weak to be observed. However, if the body moves in a stratified fluid medium and consequently generates internal waves, the magnetic field disturbances may be significant. The purpose of this study is to develop a mathematical model allowing us to evaluate the order of magnitude of the magnetic field induced by a submerged body moving in a stratified sea that is modeled as two layers of different density. It is also shown that such changes can be successfully detected by existing magnetometers of a SQUID (Superconducting Quantum Interference Devices) type [4,5].

Mathematical model

Consider a submerged vehicle moving in a fluid with two separate layers of different depth $h_{1,2}$ and of different density $\rho_{1,2}$ with a rectilinear speed V in the negative direction of the x-axis (Fig. 1). Within the framework of linear water wave theory, such a totally submerged slender body may be represented by a line of sources distributed along its axis. To simplify the analysis, the free surface is replaced by a rigid lid. The corresponding Green function [6] is used, and the resulting velocity field is calculated. Once the velocity field is known, the variations in the ambient magnetic field in the sea are described by the following set of Maxwell equations:

(1) $\nabla \times \mathbf{H} = \varepsilon_o \, \partial \mathbf{E} / \partial t$ (for air and soil),

(2) $\nabla \times \mathbf{H} = \sigma(\mathbf{E} + \mathbf{q} \times \mathbf{B_E})$ (for water),

(3) $\nabla \cdot \mathbf{H} = 0$,

(4) $\nabla \times \mathbf{E} = -\mu_o \, \partial \mathbf{H} / \partial t$,

where $\mathbf{E}$ and $\mathbf{H}$ represent the total electric and magnetic fields respectively, t is the time, $\mathbf{q}$ is the induced velocity field in the water, $\mathbf{B_E}$ is the associated earth magnetic vector, σ is the electric conductivity, ε_o is the electric permittivity of vacuum and μ_o is a physical constant. These equations are supplemented by the condition that the variations of the fields decay at large distance from the body. It is also assumed that the magnetic field varies continuously across the interface between different media. To solve the linearized problem (1)-(4), a Fourier transform technique is used, yielding a set of ODE's. In a laboratory coordinate system (fixed to Earth), the expression for the magnetic field variation can be represented in a form similar to that obtained in [2,3,7], i.e.;

(5) $$\mathbf{H_o} = Re \int_{-\pi/2+\theta_{xy}}^{\pi/2} \mathbf{A}(k,\theta,z) e^{i\omega t + ik(x\cos\theta + y\sin\theta)} d\theta,$$

where $k = k(\omega)$ denotes the dispersion relation, $\theta_{xy} \equiv arctan(|y|/x)$ and $\mathbf{A}$ is an amplitude function to be determined.

Spectral analysis of the magnetic field induced by a moving body

It is assumed next that an airborne magnetometer can provide quantitative information about the variation of the magnetic field at a given point or along a straight line. A practical question may arise then whether such a method can be applied for a moving ship. Let us consider for instance, a cut of the wave wake along a straight line (the flying direction of the magnetometer) constituting an angle β with the body track (Fig. 2). Expressed in a coordinate system attached to the magnetometer, equation (5) can be written as:

(6) $$\mathbf{H_o} = Re \int_{-\pi/2+\theta_{xy}}^{\pi/2} \mathbf{A_o}(k,\theta,x_o,y_o,z_o) e^{i\Omega t} d\theta,$$

where (x_o, y_o, z_o) represent the location of the magnetometer at $t=0$ and

(7) $\Omega(\theta) \equiv k(\theta)[c\,cos(\theta-\beta)+V\,cos\,\theta]$.

Here c denotes the speed of the airborne magnetometer and V is the speed of the ocean going body. Determining the variations of the magnetic field is reduced to analyzing its spectral density. The corresponding spectral density function of the magnetic field variations:

(8) $S(\omega_e) = lim_{T\to\infty} \left|\hat{\mathbf{H}}_{\mathbf{0}}(\omega_e, T)\cdot \hat{\mathbf{H}}^*_{\mathbf{0}}(\omega_e, T)\right| / T$,

is defined by its Fourier transform:

(9) $\hat{\mathbf{H}}_{\mathbf{0}}(\omega_e, T) = \dfrac{1}{2\pi}\displaystyle\int_0^T \hat{\mathbf{H}}_{\mathbf{0}}(t) e^{i\omega_e t}\,dt$,

where $\hat{\mathbf{H}}^*_{\mathbf{0}}$ denotes its complex conjugate. For large values of T and by using the stationary phase method, it can be shown that the maximum value of the Fourier transform $\hat{\mathbf{H}}_{\mathbf{0}}(\omega_e)$ occurs at the frequency $\omega_e^{max} = \Omega(\theta_o)$ such that,

(10) $\hat{\mathbf{H}}_{\mathbf{0}}(\omega_e^{max}) \propto \sqrt{\dfrac{T}{|\Omega''(\theta_o)|}}$,

where θ_o is the root of $\Omega'(\theta)=0$. It should be noted that θ_o depends on the body speed and on the cutting angle β. Here Ω' and Ω'' denote the first and second derivatives of Ω with respect to the argument.

Wind waves magnetic spectrum

In real seas the variations in the magnetic field, which are induced by the presence of a submerged body, are fused with the corresponding variations of the magnetic field affected by the ambient wind waves. The first quantity is considered as a signal to be determined, whereas the second is treated as noise. The ratio of signal-to-noise serves as an important characteristic parameter of the present problem. It can be shown that in many practical cases the standard deviation of the magnetic field variation induced by the moving body, is smaller than the standard deviation of the noise. In order to detect the moving object it is essential to compare the spectral densities of signal and noise. If the wave induced magnetic spectrum is wide enough in comparison with the body induced magnetic spectrum, it is then possible that the latter can be accurately detected. In the coordinate system attached to the moving magnetometer, the relation between the wave amplitude spectrum of the wind waves $S_\zeta(\omega_e)$, and the magnetic spectrum $S_H(\omega_e)$, induced by the same waves, is defined by the following transfer function $T_f(\omega_p)$:

(11) $S_H(\omega_e) = T_f^2(\omega_p) S_\zeta(\omega_e)$,

where ω_p is the frequency defined in the coordinate system attached to the Earth (laboratory). The following expression for the transfer function (12) has been derived by Weaver [1];

(12) $T_f(\omega_p) = \pi\sigma B_E g \omega_p^{-1}\left(sin^2 I + cos^2 I\,cos^2\alpha\right) exp\left(-s\omega_p^2/g\right)$,

where s is the altitude above sea surface, g is the gravitaional acceleration, α is the angle between the direction of waves propagation and the magnetic north, and I is the dip angle, i.e., the angle between the geomagnetic vector and a plane tangent to Earth.

Numerical computations

The component parallel to the geomagnetic vector of the magnetic field induced by the moving body is presented in Fig. 2. The order of magnitude of the extremal values of the magnetic field in the range of tenths body lengths is 10^{-4} nano Tesla (nT), whereas the sensitivity of existing magnetometers based on the SQUID technology, is about 10^{-6} nT [4,5]. Therefore, the current SQUID technology is capable of detecting conventional ship wakes. However, this is possible only if the trajectory of the airborne moving magnetometer cuts the far-reaching ship wake at different values of β. The painted area in Fig. 3 represents the conditions under which the magnetometer trajectory cuts the ship wake. It is seen from the figure that only for some combinations of body speed V and cutting angle β, the sensitivity of the prevailing magnetometers is sufficient to detect wakes induced by ocean going vehicles.

Once the accuracy of a specific magnetometer is found to be adequate to recognize the body wake, a question arises then whether it is possible from the available information to extract the speed of the body and its direction of motion. To answer this question, one has to analyze the magnetic field samples measured along a straight line, and evaluate the spectrum of the induced magnetic field. We assume that the measured spectrum has a local maximum at some particular frequency ω_e^{exp} pertaining to some particular speed of the body V^* and cutting angle β^*. On the other hand, the asymptotic estimate of the maximal value of the magnetic spectrum (10) is also attained at some particular frequency $\omega_e^{max}(V,\beta)$. The estimated body speed and direction of motion, can be then found under the assumption that $\omega_e^{exp} = \omega_e^{max}$, $V^* = V$ and $\beta^* = \beta$. However, it should be noted that these conditions cannot always be satisfied. In such cases, the detection procedure of the wake is likely to be problematic, at least within the framework of the discussed approach.

Another problem, which should be addressed, is that the magnetometer measurements also contain disturbances induced by the ambient wind waves. Theoretical estimated values of the maximal spectral density $S(\omega_e^{max})$ are presented in Fig. 4. The spectra of the magnetic field disturbances due to the presence of wind waves can be computed for a relatively rough sea by employing (11). For each value of V and β the value of the signal spectra function $S(\omega_e^{max})$ can be compared against the extremal value of the noise ambient spectral density $S_H(\omega_e^{max})$. Such a comparison is carried out for a certain number of discrete values of $0^o \le \lambda \le 180^o$, where λ is the angle between the prevailing wind speed (direction of the wind waves) relative to the flying direction of the magnetometer. For each numerical simulation, the inequality $S(\omega_e^{max}) > S_H(\omega_e^{max})$ is verified. The ratio between the number of simulations were such an inequality is satisfied to the total number of simulations can be defined as the probability to detect the ship wake on a noisy background. It is found that (excluding a limited range of small values of $|\beta| < 3^o$) the probability of detection is generally higher than 95%, which suggest that the proposed method can be effectively used for detecting submerged bodies.

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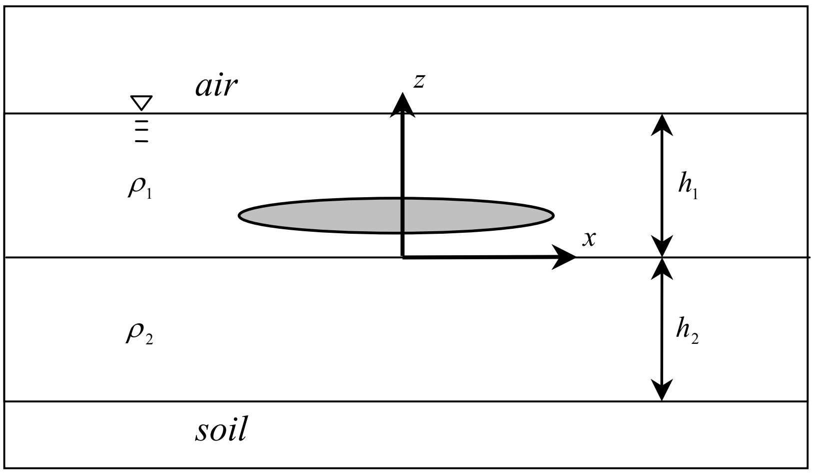

Fig. 1 - A submerged vehicle traveling in a stratified sea

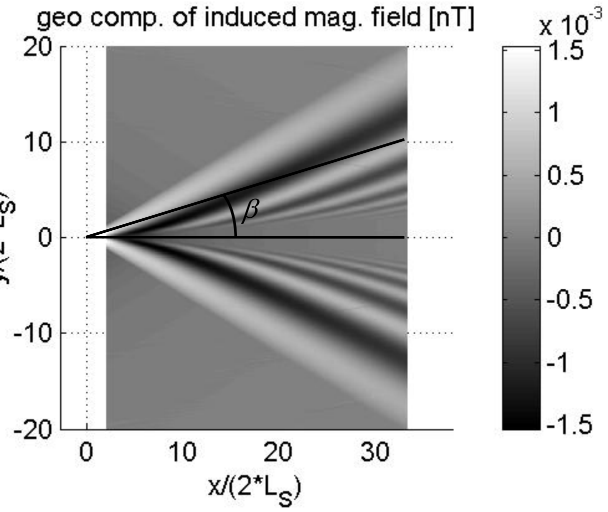

Fig. 2 - The magnetic field induced by a prolate spheroid 60 m length, 6 m beam, moving parallel to the magnetic north with speed 1 m/s 10 m above the interface of a two-layer fluid medium of different densities ($h_1 = 40\ \mathrm{m}$, $h_2 = 100\ \mathrm{m}$, $\rho_1/\rho_2 = 0.999$, $B_{\mathrm{E}} = 50{,}000\ \mathrm{nT}$, $I = 60^o$, $\sigma = 5\ \mathrm{mho/m}$).

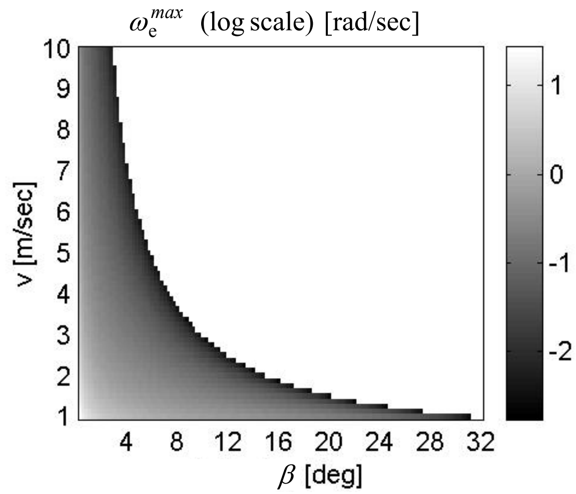

Fig. 3 - Frequency of the maximal value of spectra (conditions are the same as in Fig. 2)

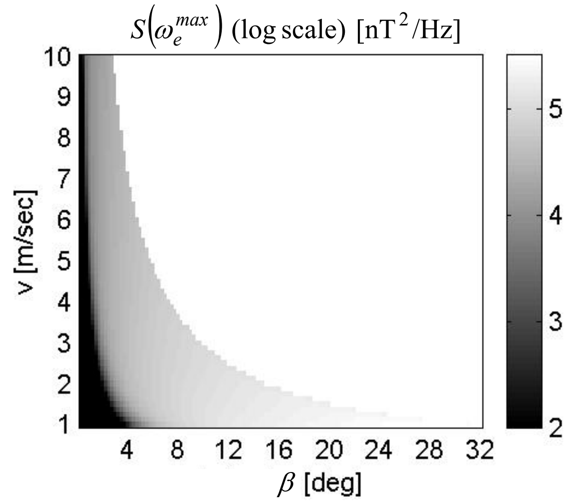

Fig. 4 - Maximal spectral density (conditions are the same as in Fig. 2)