IWWWFB20, Spitzbergen, May 2005

Can lateral asymmetry of the hulls reduce catamaran wave resistance?

by E.O. Tuck

The University of Adelaide, SA 5005, AUSTRALIA

Abstract

The answer is a qualified yes. The qualification is that such a reduction is only available if the hull separation is sub-optimal, and then choosing a better hull separation is likely to be more beneficial than cambering the hulls. This matter is investigated via a generalised thin-ship theory, where the hulls are represented by both sources (as in the Michell theory) and vertical-axis vortices. Such vortices are formally needed on each thin hull, whether or not it is laterally symmetric, to take account of the side-wash induced by the thickness of the other hull. This effect is generally considered small for conventional uncambered catamarans of small draft, but here we allow for it, together with vortices directly created by camber (or yaw; in general by any lack of lateral symmetry of the individual hull). The wave resistance is then a quadratic expression in the source and vortex distributions, and the source-vortex interaction term can be negative, so allowing in principle a net reduction in wave resistance. However, we find that the source-vortex interaction term depends on the hull separation in such a way that (under some approximate but reasonable assumptions) it vanishes whenever the Michell wave resistance is minimised as a function of hull separation.

Formulation

Our interest is in a catamaran of width $2w$ moving steadily forward at speed U in the $-x$ direction, in a calm sea of infinite depth. The overall vessel is laterally symmetric, but the two thin hulls themselves could be unsymmetric about their centreplanes $y = \pm w$, such that the hull near $y = +w$ has right side $y = w + f^+(x,z)$ and left side $y = w + f^-(x,z)$. The disturbance caused by that hull to the uniform stream U is then modelled by distributions of sources of strength $\beta(x,z)$ (modelling thickness) and vortices of strength $\gamma(x,z)$ (modelling camber, both geometric and induced by sidewash from the other hull).

Thus we write the fluid velocity as $\nabla(Ux + \Phi)$, where

$$\begin{aligned}\Phi(x,y,z) &= \iint d\xi d\zeta\ \beta(\xi,\zeta)\left[G(x-\xi,y-w,z;\zeta) + G(x-\xi,y+w,z;\zeta)\right] \\ &\quad + \gamma(\xi,\zeta)\left[H(x-\xi,y-w,z;\zeta) - H(x-\xi,y+w,z;\zeta)\right].\end{aligned}$$

The double integral is over the centreplane of the right hull, the distributions over the left hull being taken account of by means of images, so that the whole flow is symmetric about the vessel centreplane $y = 0$. The kernel $G(x,y,z;\zeta)$ is the usual double-integral Havelock source at $z = \zeta < 0$, and H is a corresponding vertical-axis vortex, such that

$$H(x,y,z;\zeta) = -\int_{-\infty}^{x} G_y(\xi,y,z;\zeta)\, d\xi\ .$$

Now for thin hulls, the linearised boundary condition is

$$\Phi_y(x, w \pm 0, z) = U f_x^{\pm}(x,z)\ ,$$

so we must determine β and γ from the equations

$$U f_x^{\pm}(x,z) = \pm\frac{1}{2}\beta(x,z) + v(x,z)\ ,$$

where

$$\begin{aligned}v(x,z) &= \iint d\xi d\zeta\ \beta(\xi,\zeta) G_y(x-\xi, 2w, z;\zeta) \\ &\quad + \gamma(\xi,\zeta)\left[H_y(x-\xi,0,z;\zeta) - H_y(x-\xi,2w,z;\zeta)\right].\end{aligned}$$

Thus (by subtraction) the source distribution β is as usual determined explicitly from the slope of the local hull thickness $f^T = f^+ - f^-$, namely

$$\beta(x,z) = U f_x^T(x,z)\ .$$

On the other hand, by addition,

$$v(x,z) = U f_x^C(x,z)\ ,$$

where $f^C = (f^+ + f^-)/2$ is the local camber. However, this is not an explicit formula for the vortex strength γ, given f^C; rather it is an integral equation of lifting-surface character. We make no attempt to solve this very difficult integral equation here, instead proceeding in an inverse or "design" manner, by prescribing $\gamma(x,z)$.

Wave resistance and its minimisation

The wave resistance R_W as defined by the energy in the wave pattern far downstream is a quadratic expression in β and γ, namely

$$\begin{aligned} R_W = \frac{2}{\pi}\rho k_0^2 \int_{-\pi/2}^{\pi/2} d\theta \quad & \Big[C^2 |\Omega_\beta|^2 \sec^3\theta \\ & + \quad S^2 |\Omega_\gamma|^2 \sec^5\theta \sin^2\theta \\ & + \quad iCS(\bar{\Omega}_\beta \Omega_\gamma - \Omega_\beta \bar{\Omega}_\gamma) \sec^4\theta \sin\theta \Big] \end{aligned}$$

where

$$\Omega_\beta = \iint dx dz \; \beta(x,z) e^{ik_0 x \sec\theta + k_0 z \sec^2\theta}$$

and

$$\Omega_\gamma = \iint dx dz \; \gamma(x,z) e^{ik_0 x \sec\theta + k_0 z \sec^2\theta}$$

with $C = \cos(k_0 w \sec^2\theta \sin\theta)$ and $S = \sin(k_0 w \sec^2\theta \sin\theta)$, $k_0 = g/U^2$.

Even in the inviscid case, this is not the total drag on the vessel, as there is also an induced drag due to vortex shedding, which is half of the total aerodynamic induced drag on a “biplane”, each wing of which is a double body obtained by reflecting the original hull in the free surface. However, we shall not concern ourselves with induced drag here, noting that it depends only on the net circulation $\int \gamma(x,z)dx$ and in a design context this quantity can be chosen to vanish, so eliminating induced drag. Similarly we shall ignore viscous drag, which is largely fixed by the hull centreplane areas.

Hence our present aim is to minimise the wave resistance R_W by choice of the vortex distribution γ, holding fixed the source distribution β, which fixes the thickness distribution and hence the net displacement of the vessel. A choice of γ is essentially a choice of camber for the hulls, although the connection between vortex strength and camber is not as direct as that between source strength and thickness. The constraints on γ are the Kutta condition that $\gamma = 0$ at the stern, and the zero-induced-drag condition $\int \gamma(x,z)dx = 0$.

First let us assume that the thickness of the hull is fore-aft symmetric, so that the source distribution $\beta(x,z)$ is an odd function of x. This seems reasonable, especially since in the absence of vortices (e.g. for Michell’s integral) there is a formal result that fore-aft symmetry is optimal for wave resistance minimisation. Hence Ω_β is imaginary, and it can then be shown that in order to minimise R_W, we must choose Ω_γ to be real, so that γ must be even in x. In that case, the third (source-vortex interaction) term in the above wave resistance is proportional to $\int CS|\Omega_\beta|\Omega_\gamma \sec^4\theta \sin\theta \, d\theta$, and we hope to make this term sufficiently negative to provide a reduction in net wave resistance.

Let us now make a further assumption, namely that γ is proportional to β_x, so Ω_γ is proportional to $\Omega_\beta \sec\theta$. Then the interaction term is proportional to $\int CS|\Omega_\beta|^2 \sec^5\theta \sin\theta \, d\theta$, which is in turn proportional to the w-derivative of the first (source-only or Michell) term in the wave resistance. The net effect of all of these assumptions is that the source-vortex interaction term vanishes when the Michell wave resistance is minimised as a function of hull separation w. Although these assumptions (especially the last) are rather restrictive, they are qualitatively reasonable, and it is likely that hull vortices have little effect on wave resistance if the hull separation is optimal.

These ideas were tested by direct computation of the wave resistance for a Wigley-hulled catamaran with total displacement 31.25t, length 19.1m, draft 1.25m, and hull separation $2w = 9$m, the two hulls each having 1.47m beam. The Michell wave resistance of this vessel was studied in a recent paper (Tuck and Lazauskas, Schiffstechnik 1998), and the given hull separation of 9m is a compromise, the optimum vessel being wider at low speeds and narrower at high speeds. The wave resistance in the absence of vortices is $R_W = 10.11$kN at $U = 9.6\text{ms}^{-1}$ (Froude number 0.7), and $R_W = 10.91$kN at $U = 13.7\text{ms}^{-1}$ (Froude number 1.0). A monohull of the same total displacement would have about double that wave resistance. Can we do even better with vortices, i.e. by cambering or yawing the hulls?

Hardly at all! Assuming as above that the vortex strength is proportional to the x-derivative of the source strength, we computed the full three-term wave resistance, adding the contributions due to sources alone (Michell), vortices alone, and interactions. At the lower speed, there is less than 1% reduction in total wave resistance possible due to vortices. This must be compared to an available 3% reduction in the Michell resistance if we are allowed to go wider (to about 13m). At the higher speed, there is almost no improvement available due to vorticity, whereas going a little narrower (to about 7m) gives a 1% reduction in the Michell resistance.

Similar ideas have been used to analyse trimarans, where again there is little opportunity for wave resistance reduction by cambering the outrigger hulls. In particular, it is always preferable to place a small fraction of the displacement in uncambered source-like outriggers at optimal separation, rather than deliberate vortex generation by cambered outrigger foils of negligible displacement. Both strategies can have quite substantial beneficial effects relative to a bare monohull, but source-like outriggers are better.