Water wave kinematics of steep irregular waves — systematic perturbation approach, empirical law, PIV measurements and engineering practice

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The kinematics of steep irregular waves is an important topic of continuing interest for the offshore industry. One of the challenges is how to deal with continuous design spectra with decay ω^{-4} or ω^{-5}, for which second- or higher-order terms in traditional engineering approaches may diverge near the equilibrium water surface unless an (arbitrary) high-frequency cutoff is employed. In these cases, common practice is often to employ the method of Wheeler stretching, which is not based on rigorous hydrodynamic theory.

A systematic perturbation approach, called the nonlinear Schrödinger method, has been derived using the strategy that the peak of the spectrum is accounted for as mainly free waves, and that all bound waves arising from the free waves near the peak are accounted for as well. The method was reported by Trulsen (1999) for second-order nonlinear waves on deep water, and by Trulsen, Gudmestad & Velarde (2001) for second-order nonlinear waves on finite depth. The method is now being expanded to third-order nonlinear waves. Second-order nonlinear Schrödinger kinematics corresponds to the cubic nonlinear Schrödinger equation for spatiotemporal evolution, while third-order Schödinger kinematics corresponds to the modified nonlinear Schrödinger equation of Dysthe (1979). For the first time, this method is now validated with the precise measurements of Grue *et al.* (2003) obtained by Particle Image Velocimetry (PIV) near the peaks of steep irregular waves. Third-order kinematics gives excellent comparisons with the experiments, and it turns out that non-local contributions at the third order are important for the successful computation of the fluid velocity field.

The kinematics of extreme waves in deep water was recently analyzed by Grue *et al.* (2003). They discovered that after proper normalization, the kinematics profiles under a variety of extreme wave crests fitted surprisingly well with a universal exponential profile e^{kz}, where z is the vertical coordinate and k is a local wavenumber. The key to the successful collapse of data demonstrated by Grue *et al.* (2003) was to use the local trough-to-trough "wave period" as basis for normalization at the crest to be considered. Furthermore, the local wavenumber and steepness had to be determined by careful use of third-order nonlinear formulas for Stokes waves.

We show that the method of Grue *et al.* (2003) may be considered as a limiting case of the nonlinear Schrödinger kinematics method for deep water (Trulsen 1999) in the limit that the length of the computational domain is exactly one local trough-to-trough wave period, and provided the perturbation expansion of Trulsen (1999) is extended to the next order of nonlinearity with respect to local nonlinear effects only, excluding non-local effects and finite bandwidth effects.

Thereby, the nonlinear Schrödinger perturbation method explains analytically why the measured velocity profiles fit so well with the universal exponential profile e^{kz} in deep water.

The main contribution of the nonlinear Schrödinger method beyond the universal kinematics profile of Grue *et al.* (2003) is to include the non-local effect of the induced current under modulated wave groups. Under an extreme wave, this current is typically a return flow.

A typical comparison between the method of Grue *et al.* (2003), the nonlinear Schrödinger method at the first three orders, common engineering practice using the Wheeler stretching method, and PIV measurements for the horizontal velocity field under an extreme crest, is shown in figure 1.

In figure 1 the Wheeler stretching kinematics and the nonlinear Schrödinger kinematics are obtained using an extract of the measured time series comprising approximately 88 peak periods centered around the extreme crest. The overall steepness for the time series is estimated to be $k_c\bar{a} \approx 0.13$, significantly less than the local steepness estimated for the extreme wave event. The normalization is determined considering only the local trough-to-trough period, which is also the basis for the universal exponential profile. The three Wheeler stretching profiles are obtained using cutoff frequencies of, respectively, three, four and five peak frequencies.

The Schrödinger kinematics discussed here employs the standard scaling that bandwidth is comparable to steepness, corresponding to the work of Dysthe (1979). The Schrödinger profiles at the first two orders are slightly different due to the fact that the time of the true extreme crest was estimated to be between two experimental measurement times, thus the time derivative of the wave envelope at the experimental measurement time chosen for computation is non-zero. Computation at the time of the true extreme crest would have yielded identical Schrödinger profiles at the first two orders. The third-order Schrödinger profile also includes the induced current under the modulated group, which is seen to be important to achieve the correct tilt of the velocity profile in the crest region.

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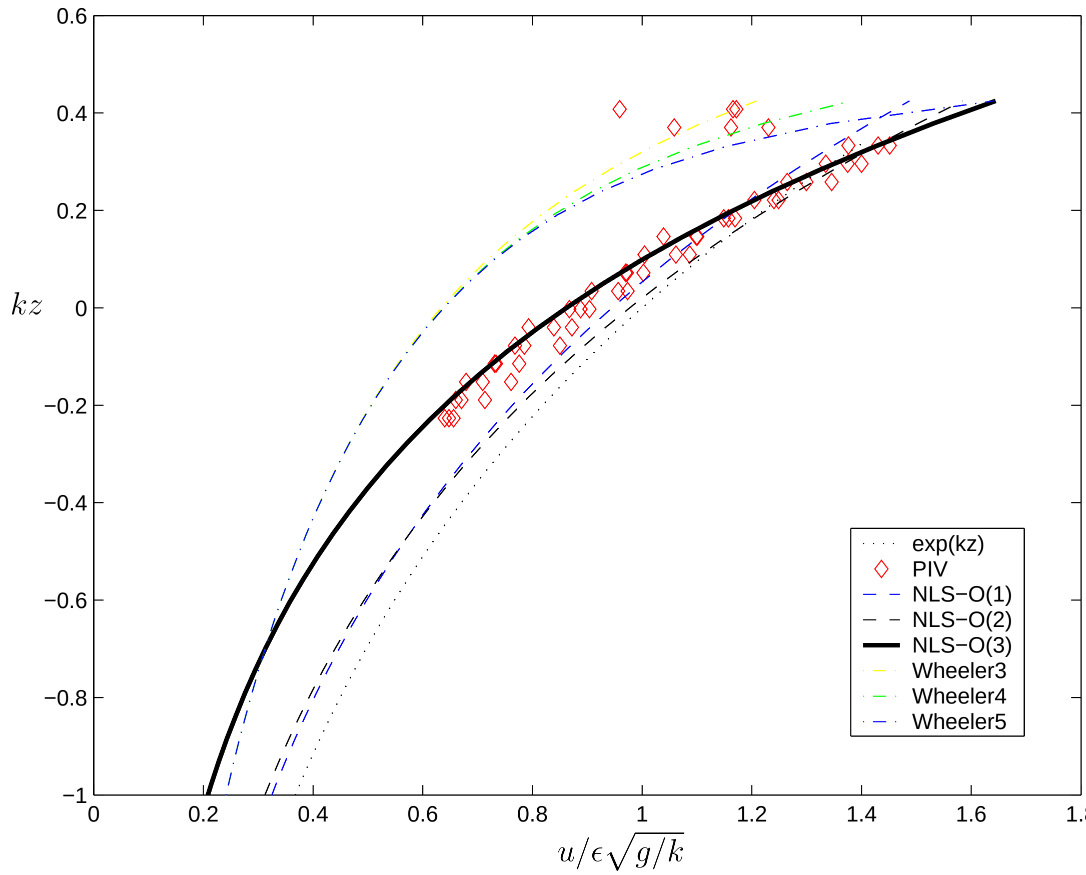

Figure 1: Horizontal velocity profiles below extreme wave crest in deep water. First axis horizontal velocity scaled with local phase speed and local steepness. Second axis vertical position scaled with local wavenumber. $- \cdot -$, Wheeler stretching profiles with cutoff at three, four and five peak frequencies; $\cdots$, exponential profile of Grue *et al.* (2003); $- -$, first- and second-order Schrödinger profiles; —, third-order Schrödinger profile; $\diamond$, PIV measurements of Grue *et al.* (2003). The scaling of all data is derived from using the length of the local trough-to-trough period containing the extreme crest.

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