PFFT-NASTRAN Coupling for Hydroelastic Problem of VLMOS in Waves

by Ken Takagi and Jun Noguchi

Department of Naval Architecture and Ocean Engineering, Osaka University, Japan

Email takagi@naoe.eng.osaka-u.ac.jp , Jun_Noguchi@naoe.eng.osaka-u.ac.jp

INTRODUCTION

We proposed a concept of floating wind power plant, which is composed of slender beams and lower-hulls so that the structure is constructed in lightweight possible for its propulsive performance. The structure is advancing with sails and propellers, which are equipped under the lower-hull, and it is navigated so that wind turbines are in service at beam wind. Many vertical struts are equipped on lower-hulls, which induce a lateral lift force to counter the wind drag force. We called this structure VLMOS (Very Large Mobile Offshore Structure). Since the plant is in service at an ocean with wind and waves, the hydroelastic behavior is one of major concerns. We propose pFFT-NASTRAN coupling technique for this problem. Since the structure is very large, a great number of panels are required to get an accurate solution. The pFFT (precorrected Fast Fourier Transformation) technique is effective to accelerate the numerical computation. The NASTRAN by MSC-software is one of popular commercial FE codes in the shipbuilding industry, and it gives trusty numerical results for the variety of structural problems.

PRECORRECTED FFT

The pFFT technique has been developed by Phillips and (1997) to accelerate the electrostatic analysis of complicated 3-D structures. Korsmeyer et al. (1999) has extended this method to the periodic free surface flow. We apply this technique to the hydro-elastic problem of the very large mobile offshore structure.

We solve the problem the linearized radiation/ diffraction problem in the frequency domain. It is assumed in this problem that the fluid is ideal, the flow is irrotational and the wave slope is small. Under the above assumptions, the fluid velocity is a scalar potential $\Phi(\vec{x},\omega)$, for $\vec{x}\in\Re^3$ and ω is the angular frequency of the incident waves. The linearized free surface condition is represented by

$$\Phi(\vec{x},\omega)+K\frac{\partial\Phi}{\partial z}=0 \quad \text{on} \quad z=0\,, \tag{1}$$

where K is the wave number of the incident waves. In case of the deep water, $K=\omega^2/g$, where g is the gravitational acceleration. It is noted that the advance speed of the structure is neglected for the sake of simplicity, although the VLMOS is slowly moving to induce the lift force.

It is well known that the integral equation to be solved for the velocity potential appears in the form

$$\begin{aligned}&-2\pi\Phi(\vec{x},\omega)+\int_{S_b}\Phi(\vec{x}',\omega)\frac{\partial G(\vec{x},\vec{x}',\omega)}{\partial\nu(\vec{x}')}dS(\vec{x}')\\&=\int_{S_b}\frac{\partial\Phi(\vec{x}',\omega)}{\partial\nu(\vec{x}')}G(\vec{x},\vec{x}',\omega)dS(\vec{x}') \quad \text{for} \quad \vec{x}\in S_b\end{aligned} \tag{2}$$

where S_b is the portion of the surface of the structure under analysis for which $z<0$ when at rest, $\nu(\vec{x})$ is its unit surface normal directed into the fluid domain. The Green function $G(\vec{x},\vec{x}',\omega)$ is defined by

$$G(\vec{x},\vec{x}',\omega)=\frac{1}{r}+\frac{1}{r_1}+\frac{2K}{\pi}\int_0^\infty\frac{e^{K(z+z')}}{k-K}J_o(kR)dk \tag{3}$$

where
$$r=\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}\,,$$
$$r_1=\sqrt{(x-x')^2+(y-y')^2+(z+z')^2}\,,$$
$$R=\sqrt{(x-x')^2+(y-y')^2}\,.$$

$J_0(x)$ is the zeroth-order Bessel function. The Green function can be calculated quickly and accurately by a subroutine, which is found in a book by Kashiwagi et al. (2003).

NUMERICAL ALGORITHMS

The numerical algorithms of the pFFT is available in Korsmeyer et al. (1999) and in Phillips and White (1997). Following their algorithms, we show the procedure briefly for the convenience.

A straight forward method for solving on the offshore problem is to discretiz the surface S_b with N planar elements upon which the potential and its normal derivative are taken to be constant and to enforce the discrete equation at N collocation points on S_b, usually taken to be the element centroids. We proceed in this manner and arrive at the linear equation system

$$[D]\{\Phi\}=[P]\left\{\partial\Phi/\partial\nu\right\} \tag{4}$$

where the $N\times N$ influence matrices are

$$D_{ij}=\begin{cases}\int_{S_j}\vec{\nabla}G(\vec{x}_i,\vec{x}')\bullet\vec{\nu}_j dS(\vec{x}') & i\neq j\\ 2\pi & i=j\end{cases} \tag{5}$$

and

$$P_{ij}=\int_{S_j}G(\vec{x}_i,\vec{x}',\omega)dS(\vec{x}') \tag{6}$$

in which S_j is an element of the surface S_b, with constant surface normal vector $\vec{\nu}_j$ .If equation (4) is sufficiently well conditioned, it may be solved by an iterative method, such as GMRES), with order-N^2 cost.

The acceleration of the solution of (4) is accomplished by filling only an order-N subset of $[D]$ and $[P]$, and computing the application of $[D]$ or $[P]$ to a vector in two parts, that is:

$$\{d^k\} = \{d^k_{far}\} + [D_{sparse}]\{\Phi^k_{near}\} \tag{7}$$

and

$$\{p^k\} = \{p^k_{far}\} + [P_{sparse}]\left\{ \partial\Phi^k_{near} / \partial\nu \right\} \tag{8}$$

with $\{d^k_{far}\}$ and $\{p^k_{far}\}$ computed by FFT convolution integral technique, which costs only order-$N \ln N$. The computation is proceeded as the following steps.

Grid set-up: Overlay the problem geometry with a uniform right-parallelepiped grid. Find the nearby elements of a given element, which are those elements in the 27 cells that share a vertex with the given element's cell. Set point-singularities on the grid at the cell vertices.

Projection operators: Numerically evaluate the operators, which can replace the set of element singularity distributions in the cell with an equivalent set of point singularities on the grid. This is done with the singular value decomposition.

Interpolation operators: Numerically evaluate the interpolation operators. These are essentially the transpose of the projection operators.

Direct interaction and precorrection: Directly compute the small number of nearby influences for each element. Use the projection operators, the Green function, and the interpolation operators to pre-compute and subtract these nearby influences from the grid-based influences for these same nearby elements..

Projection: Project the element singularity distributions to point singularities on the grid by applying the projection operators.

Convolution: Compute the potentials at the grid points due to the singularities at the grid points according to the Green function (2) by FFT-accelerated convolution.

Interpolation: Interpolate the grid point potentials onto the elements by applying the interpolation operators and add these to the precorrected direct influences.

EQUATION OF MOTION

A popular method to analyze the elastic behavior of a thin structure is to use the bar element of a commercial FE code. In the NASTRAN program, the displacement of the structure is represented by a vector $\{\varsigma\}$ of the nodal displacement, which has six degrees of freedom, at both ends of the bar elements. The vector is represented by

$$\{\varsigma\} = \{\varsigma_{11}, \varsigma_{12}, \varsigma_{13}, \varsigma_{14}, \varsigma_{15}, \varsigma_{16}, \varsigma_{21}, \varsigma_{22}, \cdots\cdots \varsigma_{N_e5}, \varsigma_{N_e6}\}^T, \tag{9}$$

where the subscript n of ς_{nm} is the nodal number, m indicates the displacement and the rotation at the node and N_e is the total number of the nodes.

Using the function of the eigen mode analysis of the NASTRAN, we get an eigen mode of the structure $\{\varsigma^{(n)}\}$. Since an eigen mode is orthogonal to the other eigen modes, the nodal displacements are represented by a summation of the eigen modes as

$$\{\varsigma\} = \sum_n q_n \{\varsigma^{(n)}\}. \tag{10}$$

After discretizing with the FE, the equation of motion for the structure moving with a circular frequency of ω is represented by

$$\left(-\omega^2[M] + [K]\right)\{\varsigma\} = \{f\}, \tag{11}$$

where, $[M]$ is the mass matrix, $[K]$ is the stiffness matrix, $\{f\}$ is the external force vector acting on the nodes. Since the eigen modes by the NASTRAN are normalized, we get

$$\{\varsigma^{(n)}\}^T [M] \{\varsigma^{(m)}\} = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases}. \tag{12}$$

On the contrary, the definition of the eigen modes suggests

$$\left(-\omega_n^2[M] + [K]\right)\{\varsigma^{(n)}\} = 0. \tag{13}$$

Thus the mass matrix has the following relation with the stiffness matrix

$$\begin{aligned} &\{\varsigma^{(n)}\}^T [K] \{\varsigma^{(m)}\} = \\ &\quad \omega_n^2 \{\varsigma^{(n)}\}^T [M] \{\varsigma^{(m)}\} = \begin{cases} \omega_n^2 & m = n \\ 0 & m \neq n \end{cases} \end{aligned} \tag{14}$$

Substituting (10) into (11) and utilizing (14), we get

$$\left(\omega_n^2 - \omega^2\right) q_n = \{\varsigma^{(n)}\}^T \{f\} \tag{15}$$

(15) suggests that the mass matrix and the stiffness matrix are not necessary for the equation of motion with the eigen modes by the NASTRAN, even if it contains the interaction of the fluid motion.

In order to compute the interaction with the fluid motion, the external force is divided into two components, which are the radiation force and the diffraction force as follows:

$$\{f\} = \sum_n q_n \{f^{(n)}\} + \varsigma_a \{f_D\} \tag{16}$$

where ς_a is the amplitude of incident waves.

Substituting (16) into (15), we get

$$\begin{aligned} &\left(\omega_n^2 - \omega^2\right) q_n - \sum_m q_m \{\varsigma^{(n)}\}^T \{f^{(m)}\} \\ &\qquad = \varsigma_a \{\varsigma^{(n)}\}^T \{f_D\} \end{aligned}. \tag{17}$$

(17) is the equation of motion for the amplitude q_n of the n-th eigen mode.

HYDRODYNAMIC FORCE

Since the NASTRAN out-put data gives only a discrete information for the displacement of the structure, it is necessary to reconstruct the continuous data of the displacement for the computation of the hydrodynamic forces. The present process is similar to Yoshida's idea (Yoshida and Ozaki : 1984), however it is modified to suit the data structure of the NASTRAN and to gain the accuracy. In order to reconstruct it from the NASTRAN out-put data, the following assumptions are used.

(a)The displacement between two adjacent nodes is linear.

(b)The displacement due to the revolution at the node is negligible. However, the rigid revolution, i.e. the revolution around the element axis, is taken into account.

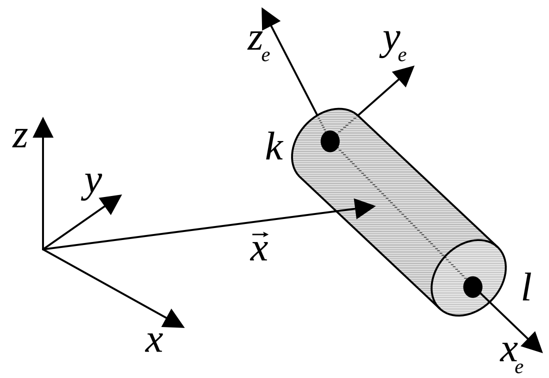

Fig.1 Definition of the element coordinate and the earth fixed coordinate.

When the j-th element is defined by the origin-node l and the end-node k, the velocity $\vec{\mathrm{v}}^{(n)}$ at a point $\vec{x}$ on the element is given by

$$\vec{\mathrm{v}}^{(n)} = i\omega\left\{\vec{\xi}_k^{(n)} + \vec{\theta}_j^{(n)} \times \left(\vec{x} - \vec{x}_k\right)\right\}, \tag{18}$$

where

$$\begin{aligned} \left\{\xi_k^{(n)}\right\} &= \left\{\xi_{k1}^{(n)}, \xi_{k2}^{(n)}, \xi_{k3}^{(n)}\right\}^T, \\ \left\{\hat{\xi}_k^{(n)}\right\} &= \left\{\xi_{k4}^{(n)}, \xi_{k5}^{(n)}, \xi_{k6}^{(n)}\right\}^T, \end{aligned} \tag{19}$$

$$\begin{aligned} \vec{\theta}_j^{(n)} &= \frac{1}{L_j^{\,2}}\left(\vec{x}_l - \vec{x}_k\right)\times\left(\vec{\xi}_l^{(n)} - \vec{\xi}_k^{(n)}\right) \\ &\quad + \frac{1}{2L_j^{\,2}}\left(\vec{x}_l - \vec{x}_k\right)\bullet\left(\vec{\hat{\xi}}_l^{(n)} + \vec{\hat{\xi}}_k^{(n)}\right)\left(\vec{x}_l - \vec{x}_k\right) \end{aligned} \tag{20}$$

and

$$L_j = \left|\vec{x}_l - \vec{x}_k\right|. \tag{21}$$

If we define the radiation velocity potential for the n-th mode as

$$\Phi^{(n)} = i\omega\phi^{(n)}, \tag{22}$$

the boundary condition for it may be

$$\frac{\partial\Phi^{(n)}}{\partial\nu} = \vec{\nu}\bullet\vec{\mathrm{v}}^{(n)} \quad \text{on the body surface}. \tag{23}$$

Therefore, the boundary condition for the radiation problem is given by

$$\frac{\partial\phi^{(n)}}{\partial\nu} = \vec{\nu}\bullet\left\{\vec{\xi}_k^{(n)} + \vec{\theta}_j^{(n)} \times \left(\vec{x} - \vec{x}_k\right)\right\}, \tag{24}$$

where $\vec{\nu}$ is the normal vector inward to the fluid.

The diffraction problem for the hydro-elastic motion is as same as that for the rigid structure. The velocity potential of the incident wave is defined as

$$\phi_0 = e^{Kz - iK\left(x\cos\chi + y\sin\chi\right)}. \tag{25}$$

The boundary condition is given by

$$\frac{\partial}{\partial\nu}\left(\phi_0 + \phi_d\right) = 0, \tag{26}$$

where ϕ_d is the velocity potential for the diffraction problem.

Once we get the velocity potential with the pFFT technique, the pressure of the fluid is easily obtained, and integrating it the hydrodynamic forces are computed. If the overall velocity potential is Φ, it is represented as

$$\Phi = -i\varsigma_a\frac{g}{\omega}\left(\phi_0 + \phi_d\right) + i\omega\sum_n q_n\phi^{(n)}. \tag{27}$$

The linearized pressure due to this velocity potential is

$$\frac{p}{\rho} = -g\varsigma_a\phi_D + \sum q_n\left(\omega^2\phi^{(n)} - gZ^{(n)}\right) - gz, \tag{28}$$

where $\phi_D = \phi_0 + \phi_d$ and $Z^{(n)}$ is the vertical component of the n-th eigen mode. In order to compute the nodal force in (17), the hydrodynamic forces acting on an element is decomposed into the nodal forces. Thus, the following assumptions are used.

(a)Moments acting on a node are negligible.

(b)As an exception of the assumption (a), the twisting moment is taken into account. However, since it is impossible to distribute the twisting moment onto two nodes consistently, it is evenly distributed.

(c)Similarly, the axial force is equally distributed onto two nodes.

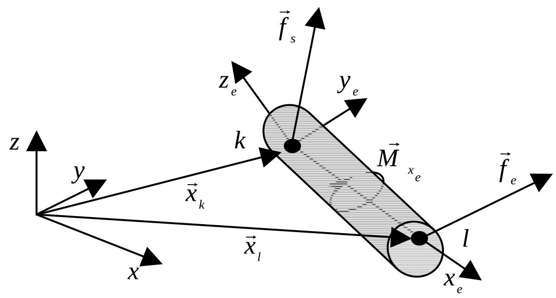

Fig.2 Definition of forces acting on a element.

According to the definitions shown in Fig.2, the equation of equilibrium for each element is given by

$$-\int_{S_j} p\,\vec{\nu}\,dS = \vec{f}_s + \vec{f}_e, \tag{29}$$

$$-\int_{S_j} p\left(\vec{x} - \vec{x}_k\right)\times\vec{\nu}\,dS - \vec{M}_{x_e} = \left(\vec{x}_l - \vec{x}_k\right)\times\vec{f}_e \tag{30}$$

and

$$\vec{M}_{x_e} = \left(-\int_{S_j} p\left(\vec{x} - \vec{x}_k\right)\times\vec{\nu}\,dS\bullet\vec{e}_x\right)\vec{e}_x, \tag{31}$$

where $\vec{e}_x = \left(\vec{x}_l - \vec{x}_k\right)/\left|\vec{x}_l - \vec{x}_k\right|$ is an unit vector representing the axial direction of the element, S_j denotes the surface of the j-th element, $\vec{f}_s$ and $\vec{f}_e$

are nodal force acting on the origin node and end node respectively and $\vec{M}_{x_e}$ is a moment in axial direction acting on the element. Combining (29) and (30), and solving it , we can get the nodal forces. In the actual procedure, according to (28) and (17), the radiation forces and the diffraction forces are separately computed

VORTEX INDUCED FORCE

Since the VLMOS is composed of thin structural component, the vortex induced force plays an important role at the resonant frequency for the elastic modes. It is assumed that the vortex induced force is represented as a drag force

$$f_D = \frac{1}{2}\rho C_D U|U| \qquad (32)$$

where f_D is the drag force per unit length, C_D is the drag coefficient and U is the relative flow velocity taken at the section center of the lower-hull. Since (32) is not sinusoidal in time, the Fourier averaged value is used for the drag force in the frequency domain. In addition, an iterative method is used to solve the equation of motion for taking the nonlinearity of the drag force into account accurately. Further assumption is that the vortex induced force is parallel to the direction of relative velocity. The vortex induced force is integrated on the element and decomposed into the nodal forces with the same manner as presented in the previous section.

RESULTS

We have carried out a model experiment for measuring the hydroelastic behavior of the VLMOS in waves with 1/100 scale backbone type elastic model.

The dimensions of the model are 4m length and 4.5m width. A picture of the model is shown in Fig. 3. Fig. 4 shows a comparison of strains at the lower hull between the numerical results and the experimental ones in head waves. Fig. 5 shows the effect of the vortex induced force. It is apparent that the vortex induced force plays an important roll at the resonant frequency of the two-nodes vibration mode of the transverse beam, where the drag coefficient is 1.0.

Further results will be presented at the workshop.

REFERENCES

Kashiwagi, M. et al. : Hydrodynamics of the floating body, Seizando, Tokyo, Japan, 2003. (in Japanese)

Korsemeyer, T., Klemas T., White, J.K. and Phillips, J.R. : Fast Hydrodynamic Analysis of Large Offshore Structures, Proc of the 19th ISOPE , Brest, France, pp.27-34, 1999.

Phillips, J.R. and White, J.K. : A Precorrected-FFT method for Electrostatic Analysis of Complicated 3-D Structures, IEEE Transactions on Computer-Aided Design of Integrated Circuits and systems, Vol. 16, No.10, pp1059-1072, 1997.

Yoshida, K. and Ozaki, M.: A Dynamic Response Analysis Method of Tension Leg Platforms Subjected to Waves, J. of the Faculty of Eng., the University of Tokyo, Vol. XXXVII, No.4, 1984.

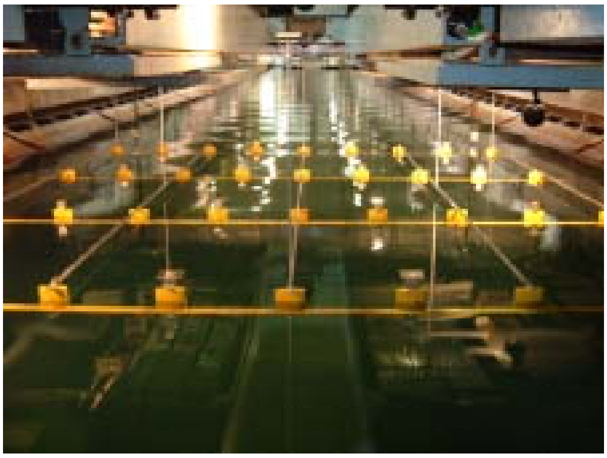

Fig.3 Picture of the experimental model at the towing tank of Osaka University

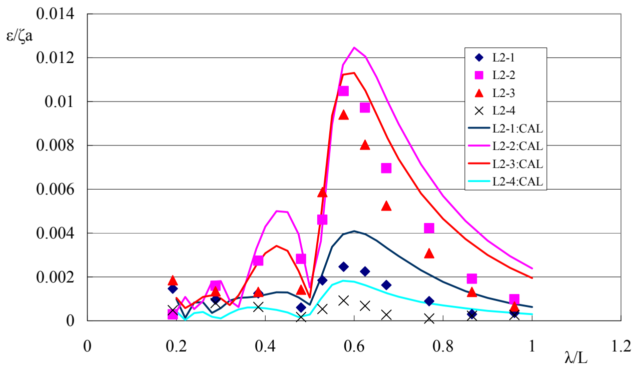

Fig.4 Comparison of the strain of the lower hull at χ=180deg

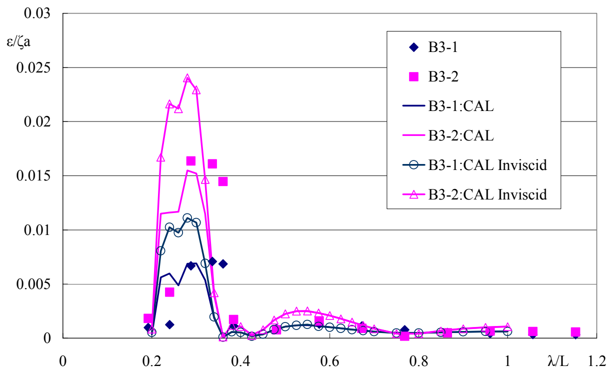

Fig.5 Effect of the vortex induced force for the strain of the transverse beam at χ=180deg