Development of a fully nonlinear water wave simulator based on Higher Order Spectral theory

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Introduction

In this paper we report on the development of a time accurate fully non linear potential flow model for simulating of the generation and propagation of gravity waves in finite depth, in 2D or 3D geometries. This Numerical Wave Tank (NWT) relies on a new non-periodic HOS technique based on the original High-Order Spectral (HOS) model of West *et al.* [1] and Dommermuth and Yue [2], and was presented in the previous workshop of this series [3]. The generation process is either based on a model of the physical wavemaker, or on submerged dipoles (see e.g.[4]). In the first case, the accurate geometry of the flaps, and advanced control strategies of the wave maker motion can be taken into account. Perfectly reflective sidewalls and an absorbing beach at the wall facing the wave maker are modelled. Besides its use as a tool for preparing and optimising experiments in ECN's ocean engineering basin, this model is also intended to provide an accurate description of non linear waves, with numerous potential applications such as the study of the statistical properties of irregular sea states in severe conditions (one of the benchmark studies of 24th ITTC), or to represent the incoming wave model for irregular wave diffraction calculations using the SWENSE approach [5]. For such applications, further developments related to global numerical efficiency, and to the determination of velocity and pressure fields in the fluid volume were considered necessary. Such improvements are reported in this paper, together with recent validation results.

Description of the model

Potential theory is used and free surface boundary conditions are rewritten as evolution equations for the surface elevation and free surface potential. These two unknowns expressed on collocation points are time-marched once the vertical velocity has been obtained through the solution of a Dirichlet problem for the potential. The latter is solved by the HOS expansion of the potential in orders of the wave elevation in parallel with the order consistent formulation of West *et al.* [1]. In this technique, spatial derivatives are calculated in the Fourier domain and the potential at each order is originally expanded on the natural modes of the basin. This enables the use of Fast Fourier Transforms and results in fast computations. Nonlinear products in physical space are carefully de-aliased to keep a good accuracy. The numerical absorbing zone is modelled through a local modification of the free surface dynamic boundary condition. The physical wavemaker motion may be included, presently only in a linear way, through an inlet flux condition. To provide easy comparisons with measurements, the same procedure is used both in numeric and experiments to calculate the wavemaker motion. For multi-directional wave fields, the Dalrymple method [6] is used rather than the snake principle to avoid spurious diffraction on the sides of the wavemaker and reflection on the sidewalls. In 2D calculations, the submerged dipole method may be used [3], which allows the generation of steeper waves without local breaking.

Typical validations results

In the previous workshop, a significant validation results were presented, in which the present model was used to generate regular nonlinear waves which very accurately recovered results of stream function theory [7]. More practical validation results are presented here, in which the HOS NWT is used to model the new ECN wave basin (50x30x5 m). We present here three different wave fields. The first comparison to laboratory measurements deals with 2D irregular waves. A Bretschneider spectrum with two parameters (Hs=16 cm and Tp=2 s) has been generated in with the same wavemaker motion in both the experimental and the numerical basins. Long time simulations are carried out. Figure 1 shows an example of a high amplitude wave packet reaching the probe at time t=760 s. Nonlinear numerical results (long dashed line) fits well the experimental probe signal (solid line) and further comparison with a previous second order model (dashed line) clearly illustrates the gain of the fully nonlinear model.

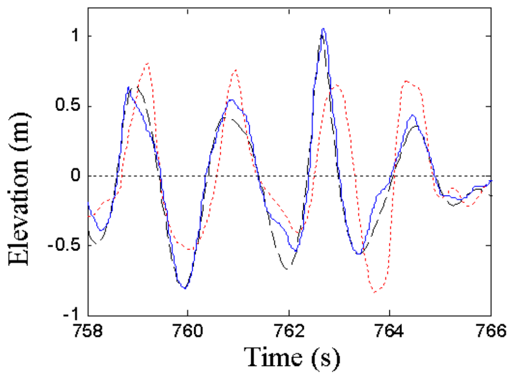

Figure 1: 2D calculations compared to experiments: irregular wave field

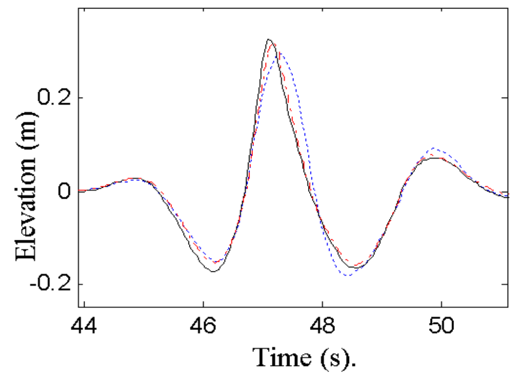

Figure 2: 2D calculations compared to experiments: focused wave packet

The second application is related to the generation of focused wave packets. Figure 2 shows the wave elevation at the focusing point (x=25 m from the wavemaker). The dotted line is the target wave packet we want to reproduce. It is used to predict the wavemaker motion, following linear wave theory. The generated wave field involves large amplitudes and incipient breaking. Measured wave elevation (dashed line) is different from the target: the main differences being a different amplitude explained by second order effects and an advance in time due to phase velocity modifications by third order effects. Numerical elevation (solid line) correctly reproduces these two effects.

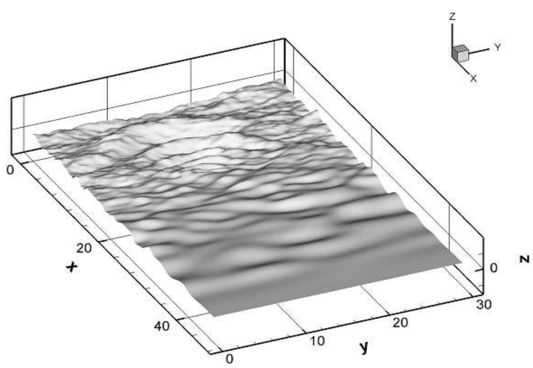

Figure 3: : 3D view of the focusing wave field

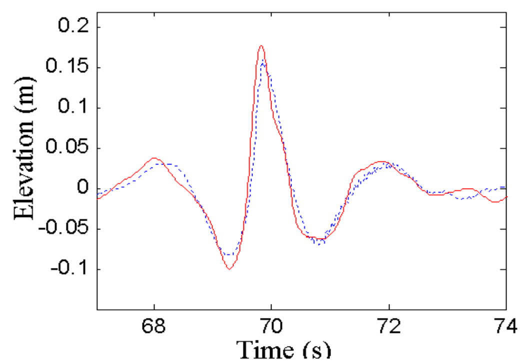

Figure 4: : time evolution of probe elevation

Figure 3 shows a view of a 3D wave packet while it focuses towards the middle of the basin. This wave packet is embedded in a directional irregular sea that we see at the front or behind the wave packet. Dalrymple's method [6] has been used to generate the oblique waves. The picture on the right represents the elevation of a probe located at the focusing point in the middle of the basin. The simulated elevation (solid line) shows a good agreement with the experimental one (dotted line).

Efficient calculation of wave kinematics

In a Higher Order Spectral Methos (HOS), kinematics within the fluid domain have to be calculated from values at the free surface. We have developed such a technique in our model, based on a previous work by Bateman [8], who developed the so-called H and H_2 operators in a Dirichlet to Neumann Operator method (DNO). It has to be mentioned that the DNO method in its accelerated version is strictly equivalent to the HOS scheme, a fact that motivated us to adapt the H and H_2 operators to our HOS scheme.

The starting point of the method is to consider a variable known along the free surface (ϕ for example) as a spectral expansion:

$$\phi(x,y,\eta,t) = \sum_{n_x=0}^{N_x}\sum_{n_y=0}^{N_y} A_{n_x n_y}(t)\psi^s_{n_x n_y}(x,y) \quad \text{with} \quad \psi^s_{n_x n_y}(x,y) = \cos(k_{n_x}x)\cos(k_{n_y}y)\cosh\Big[k_{n_x n_y}(\eta(x,y)+1)\Big]$$

From this expression, Fast Fourier Transforms could not be used directly to derive $A_{n_x n_y}$. The underlying idea in the development of H and H_2 operators is to transfer the variable of interest from the free surface $z = \eta(x,y,t)$ to its value on $z = 0$. Then, FFT's may be applied for an efficient calculation of $A_{n_x n_y}$. The reconstruction of the variable of interest along the z axis is straightforward. H_2 operator differs from the H one in the computation of the transfer described previously: the H_2 operator split it into several steps. Bateman [8] noticed that significant errors could appear in specific cases when using only the H operator. Then, we chose to use the H_2 operator.

Two different options are available for the determination of the fluid kinematics using this approach. In the first one, the velocity potential is first reconstructed from values at the free surface, and then derived analytically to access velocity components. In the second one, velocity components are directly reconstructed from their values at the free surface. Both methods have been implemented, and compared to results of Rienecker & Fenton's stream function theory [7] in the case of regular non linear waves.

Here is presented a step to transform a variable of interest (ϕ for example) from the initial surface elevation $z = \eta(x,y,t)$ onto a new surface $z = \eta_2(x,y,t)$ (*i.e.* H_2 *operator*). We write the Taylor's series development for this new value of z :

$$\phi_2(x,y,\eta_2,t) = \phi(x,y,\eta_2,t) = \sum_{n=0}^{\infty}\frac{\eta_2^n}{n!}\frac{\partial^n\phi}{\partial z^n}(x,y,0,t)$$

To solve this system of equation we could write a triangular process:

$$\begin{aligned}
\phi^s &= \phi_0(x,y,0,t)\\
\phi_1(x,y,0,t) &= -\eta\frac{\partial\phi_0}{\partial z}(x,y,0,t)\\
\phi_2(x,y,0,t) &= -\eta\frac{\partial\phi_1}{\partial z}(x,y,0,t) - \frac{\eta^2}{2}\frac{\partial^2\phi_0}{\partial z^2}(x,y,0,t)\\
&\dots\\
\phi_p(x,y,0,t) &= -\sum_{n=0}^{p-1}\frac{\eta^{n+1}}{(n+1)!}\frac{\partial^{n+1}\phi_{p-1-n}}{\partial z^{n+1}}(x,y,0,t)
\end{aligned}$$

Then, we rebuild a second triangle to solve the quantity $\phi_2(x,y,\eta_2,t)$ we look for:

$$\begin{aligned}
\phi_{2_0}(x,y,\eta_2,t) &= \phi_0(x,y,0,t)\\
\phi_{2_1}(x,y,\eta_2,t) &= \eta_2\frac{\partial\phi_0}{\partial z}(x,y,0,t)\\
\phi_{2_2}(x,y,\eta_2,t) &= \eta_2\frac{\partial\phi_1}{\partial z}(x,y,0,t) + \frac{\eta_2^2}{2}\frac{\partial^2\phi_0}{\partial z^2}(x,y,0,t)\\
&\dots\\
\phi_{2_p}(x,y,\eta_2,t) &= \sum_{n=0}^{p-1}\frac{\eta_2^{n+1}}{(n+1)!}\frac{\partial^{n+1}\phi_{p-1-n}}{\partial z^{n+1}}(x,y,0,t)
\end{aligned}$$

And, finally:

$$\phi(x,y,\eta_2,t) = \phi_2(x,y,\eta_2,t) = \sum_{p=0}^{\infty}\phi_{2_p}(x,y,\eta_2,t)$$

We use here a fully nonlinear wave generation with submerged dipoles (see Le Touzé [4]). Computations for two steepnesses are shown there, $\epsilon = ka = 15.7\%$ ($H/\lambda = 0.05$) and $\epsilon = ka = 30\%$ ($H/\lambda = 0.09$) where we choose a wave length $\lambda = 0.5$ in a basin of length $L_x = 10$ and with a depth $h = 1$. The submerged dipole is located at $x_d = 2.5$; $z_d = -0.25$ and two numerical beaches are used between $x = 0$ and $x = 1$ and between $x = 8$ and $x = 10$.

Figure 5 & Figure 6 shows the perfect agreement between reference Rienecker & Fenton's solution and our computation of kinematics using the velocity on free surface (relative error does not exceed 1%). However, calculation using the potential on the free surface seems to be limited to small steepness where reconstruction is really good. The better choice is thus to reconstruct the wave kinematics directly from values of the fluid velocities at the free surface.

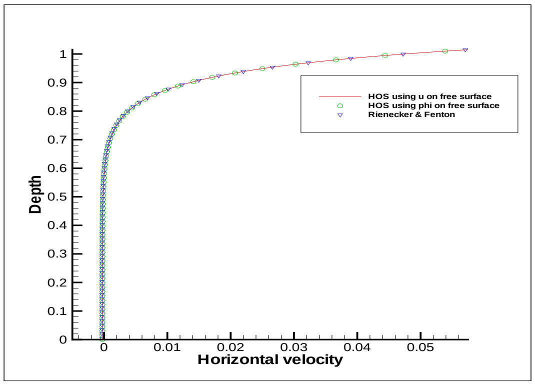

Figure 5: $\epsilon = ka = 15.7\%$

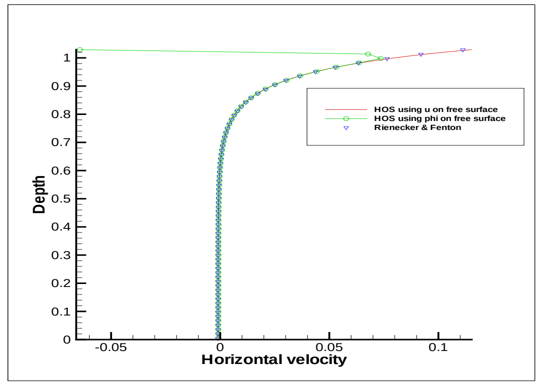

Figure 6: $\epsilon = ka = 30\%$

Calculation speed-up

In order to speed up our HOS calculations, in view of intensive simulations of irregular waves, we have considered the application of the scheme presented by Fructus et al [9]. First results seems to be promising, and more detailed conclusions will be presented at the workshop.

Conclusion

Recent developments of our HOS scheme for fully non linear water wave simulations have been presented. The resulting scheme combines high accuracy and numerical efficiency, essential for applications such as the study of the non linear behaviour of water waves, or the simulation of their interaction with ship or offshore structures, using the recently developed SWENSE approach for the non linear wave-body interactions in viscous fluid.

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