Fully Non-linear Diffraction Calculations of a Floating Sphere in Regular Waves

A. Ballast P.J. Zandbergen
IMPACT, Institute of Mechanics, Processes and Control of the University of Twente
Enschede, The Netherlands

1 Introduction

The assessment of the behaviour of Floating Production and Storage units Offshore (FPSO's) in survival wave conditions could benefit considerably from the application of fully non-linear potential theory calculations.

In an earlier study we calculated the diffraction of moderate to high non-linear water waves by vertical cylinders [1] to get a first idea of the computational difficulties involved. Further confidence in our method was obtained by comparing to more detailed experiments, see [2]. Now we include a freely floating body in our diffraction calculations. Berkvens [3] already included the algorithm for solving the equations of motions for a floating body in our method, but he did not do any diffraction calculations.

In this paper we show the results of a comparison with linear diffraction theory for the case of a sphere floating in regular waves. We compare with results from Pinkster [7]: the RAO's (Response Amplitude Operators) for surge and heave, and the horizontal drift force in regular waves. The latter is a second order quantity that only depends on first order quantities.

2 Set-up of the computations

For this comparison we need to do calculations for a range of frequencies. The interesting range of wave numbers goes from $ka = 0.4$ up till $ka = 1.5$, where a is the radius of the sphere and k is the wave number of the incoming wave. To start with, we take a domain with a radius of 64 metres and a sphere with a radius of 8 metres. For $ka = 0.4$ we then have a wavelength of 128 metres (cf. [1]). Since we have a limited resolution, 80 panels in the circumference of the domain, it is necessary to use a larger sphere with a radius of 12 metres for the higher ka values. For $ka = 1.0$ both spheres have been used for comparison. The wave height of the incoming waves is 0.50 metres. To get an idea of the possible nonlinearities, three extra wave heights have been calculated for $ka = 1.0$ and $ka = 1.1$: 0.25 m, 1.00 m, and 2.00 m. The smallest wave height has a steepness H/λ of 0.003, the largest wave height has a steepness of 0.04.

The calculations are done with our fully non-linear potential theory code HYPAN, which is based on a higher order panel method as described in e.g. Broeze [4] or de Haas et al. [5]. For the floating body algorithm see Berkvens [3], Ballast [2], or Tanizawa [9]. A circular domain is used. On the outer boundary we prescribe a Rienecker & Fenton [8] type of solution for a regular wave of finite amplitude. A Sommerfeld radiation condition is used on the difference of the total solution with the prescribed incoming wave:

$$\phi_t = \phi_{t,RF} - c((\phi_n - \phi_{n,RF}) + (\phi - \phi_{RF})/2r) \quad (1)$$

where the subscripts t and n denote partial differentiation and RF denotes the prescribed incoming wave. The phase velocity of the incoming wave is denoted by c and the horizontal radius by r. In front of this Sommerfeld radiation condition a pressure damping zone is implemented, also working on the difference with the incoming wave:

$$p_{damp} = b \cdot (r - r_0) \cdot (\phi_n - \phi_n^{RF}) \quad (2)$$

with r the horizontal radius and b a constant.

The calculations are started with the Rienecker & Fenton type of solution prescribed on the entire surface, including the sphere. In one or two periods the sphere then changes smoothly to an impermeable one, see Ferrant [6]. During this startup phase the position of the sphere is kept fixed, but when the normal velocity on the boundary of the sphere has become zero, the sphere is set free.

Different from the work of Ferrant cited above, we do solve the entire solution explicitly instead of only the diffraction field (using the known Rienecker & Fenton solution) and we do not use frozen coefficients in our RK4 time stepping. The grid points on the sphere move along meridians on the sphere, with the pole at the top. Far away from the sphere the grid points are free to move in the vertical direction only. In between they do a little bit of both. The drift forces on the sphere are compensated for by a soft spring mooring. For the entire computations no smoothing techniques were employed.

The calculations used 80 panels in the circular direction and 7000 panels in total. The calculation of one time step takes about 1500 seconds on one MIPS R14000 processor running at 500MHz; we used 16 or 32 of them on a SGI Origin 3800.

3 Results

First we have a look at the RAO's. For the surge motion the deviation with respect to the linear theory is around 1% and at most 2% for the amplitude, and around 1% for the phase. These results are not shown. For the heave motion the results are shown in figures 1 and 2. The deviations from the linear theory range from 2 till 9% for the amplitude. At heave resonance one can see from the difference between the small and the large sphere, that a large part of these deviations can be attributed to the limited size of the computational domain and thus to the limited distance from the sphere to the numerical beach. The calculation with the small sphere has a deviation from the linear results of around 2%, whereas the calculations with the larger sphere have a deviation from the linear results of 9% for the small wave heights. For the phase of the heave motion, the deviations from the linear theory are around 1% for the small wave heights. The largest wave heights show a clearly non-linear behaviour with their deviation from linear theory of 3 or 4%. For both heave and surge motions the small differences between the small wave heights at $ka = 1.0$ and $ka = 1.1$ suggest that for these wave steepnesses we are still in the linear regime.

The results for the horizontal drift force are shown in figure 3. Here there are deviations from the linear theory of up to 20%. In the linear diffraction theory the total drift force is the sum of several components, each depending on first order quantities only. To get a better idea of where the deviations come from, we also tried to calculate some of the components of the horizontal drift force separately. Of course, in a fully non-linear diffraction program not all of these components are readily available. The second component, see equation (4), can be obtained quite easily. See figure 4(b). The agreement with the linear diffraction theory is comparable to the agreement seen for the first order motions. This is a good indication of the accuracy of the code. The first component, see equation (3), can be calculated indirectly by using the relative wave heights on the sphere. Indirectly, because this is not the way the pressure is calculated in the code. See figure 4(a). Again the agreement is comparable to that what is seen for the first order motions. This is a good indication of the quality of the wave field obtained by the code. Combining the results for these two components, we can conclude that the deviations of the first order quantities from the linear diffraction theory can not explain the underestimation of the horizontal drift force.

4 Conclusion

We can do long stable computations in mild regular waves. The first order motions of the floating sphere show good agreement with linear diffraction theory, although for a more detailed comparison a larger computational domain with a larger numerical beach would be necessary. The horizontal drift force is underestimated by almost 20%. Looking at the relative wave height term of that drift force shows that this difference cannot be attributed to deviations in the calculated wave heights around the sphere. Also the velocity squared term of the horizontal drift force shows a much better agreement then the total drift force does. This needs some further investigation.

5 Acknowledgements

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Appendix: Components of the mean horizontal drift force

From Pinkster [7] we have the following expressions for the contributions to the mean horizontal drift force for a sphere submerged down to its middle. These expressions depend only on the mean first order quantities and are accurate up to second order. The superscript $^{(1)}$ denotes first order quantities, ζ_r is the relative wave height with respect to the centre of mass of the sphere, S_0 is the mean wetted surface, and $\vec{X}$ is the displacement vector from the equilibrium position.

$$F_I^{\text{mean}} = \oint_{\text{mean WL}} \frac{1}{2}\rho g (\zeta_r^{(1)})^2 \, (\hat{n} \cdot \hat{k}) \; dl_{\text{mean}} \tag{3}$$

$$F_{II}^{\text{mean}} = \iint_{S_0} -\frac{1}{2}\rho (\nabla\phi^{(1)})^2 \, (\hat{n} \cdot \hat{k}) \; dS_{\text{mean}} \tag{4}$$

$$F_{III}^{\text{mean}} = \iint_{S_0} -\rho (\vec{X}^{(1)} \cdot \vec{\nabla}\phi_t^{(1)}) \, (\hat{n} \cdot \hat{k}) \; dS_{\text{mean}} \tag{5}$$

$\vec{k}$ is the wave vector of the incoming wave, thus $(\hat{n} \cdot \hat{k})$ has the opposite sign as the longitudinal direction cosine used by Pinkster, resulting in the same overall sign convention.

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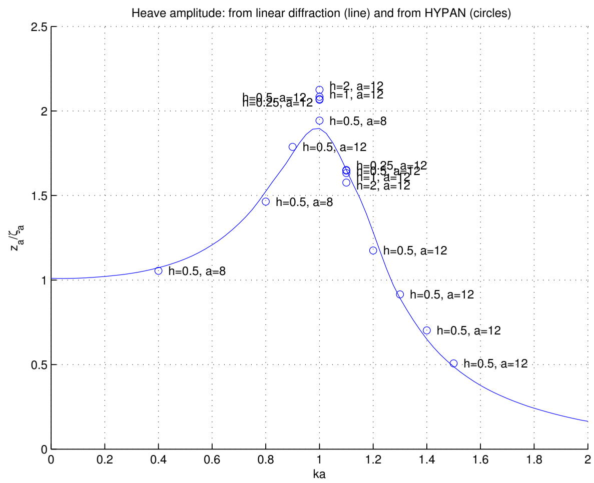


Figure 1: Heave amplitude: from linear diffraction (line) and from HYPAN (circles)

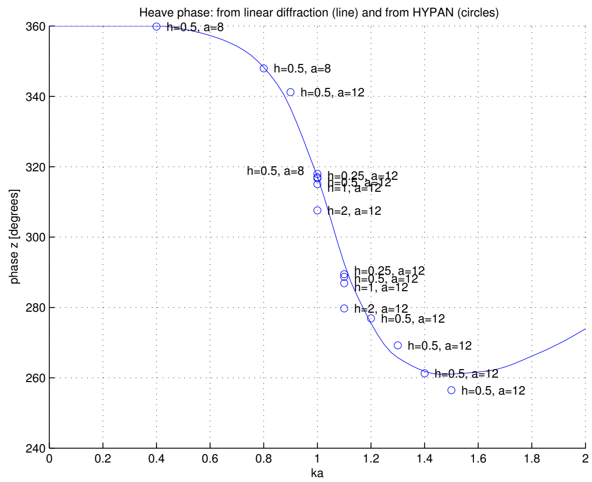


Figure 2: Heave phase: from linear diffraction (line) and from HYPAN (circles)

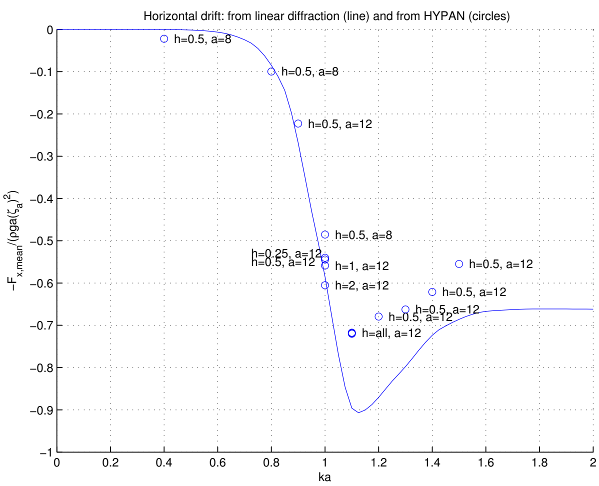
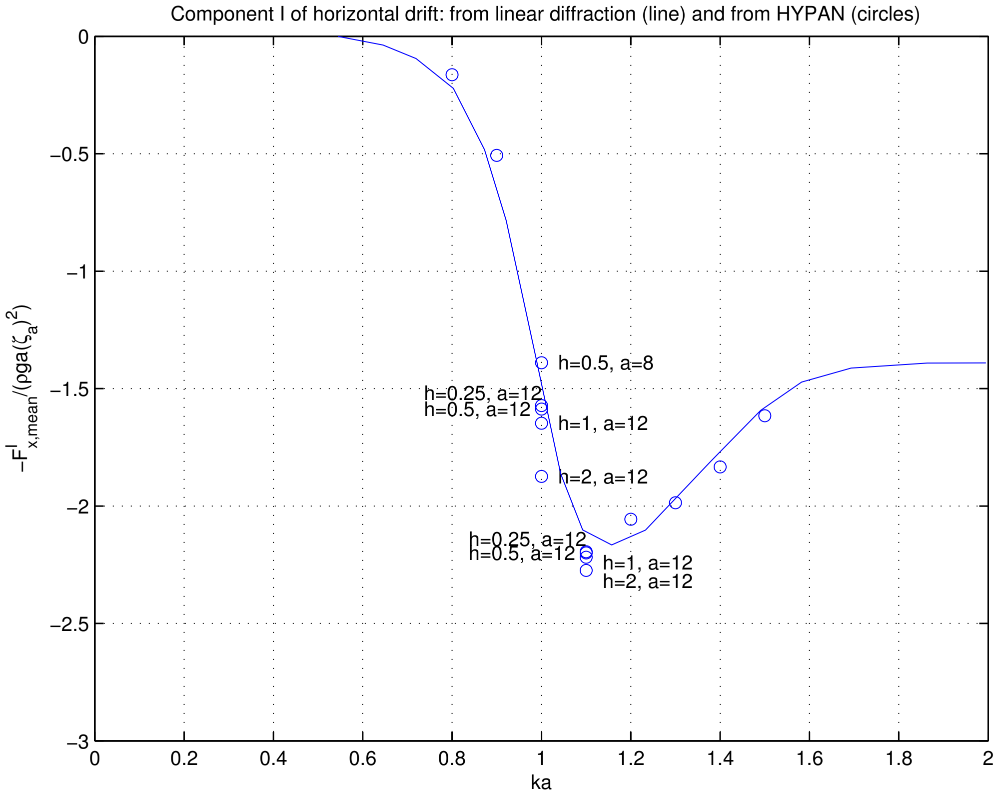
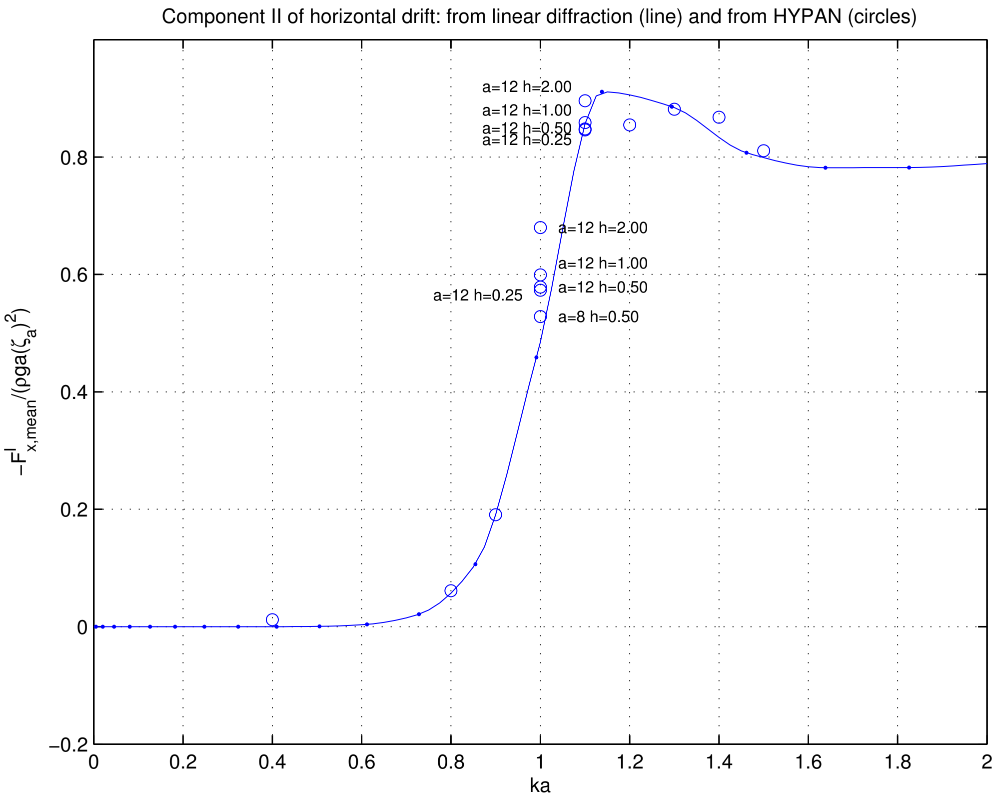


Figure 3: Horizontal drift: from linear diffraction (line) and from HYPAN (circles)



(a) Component I of horizontal drift: from linear diffraction (line) and from HYPAN (circles)



(b) Component II of horizontal drift: from linear diffraction (line) and from HYPAN (circles)

Figure 4: Comparing the results from linear diffraction theory and the results from HYPAN for two of the terms that make up the ‘linear’ horizontal drift force (eqns. (3) and (4))