Latching control of a power take off oscillator carried by a wave activated body.

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Abstract

This paper investigates the motion amplification brought by latching control to a mechanical oscillator installed on a wave activated body. This generic problem potentially applies to a familly of wave energy devices already existing or under development today. Assuming the excitation force being monochromatic, one can determine analytically the optimal latching duration. The analytical time domain approach allows for highlighting specific nonlinear features of such systems of oscillators as output period doubling, tripling,... It is shown that latching control applied to coupled power take off oscillators can be as efficient as it was demonstrated to be for simple oscillator in these wave energy recovering systems.

1 Introduction.

Conceptually, a lot of different methods has been proposed to extract power from the waves. One of them consists in using the relative motion between floating bodies, and one can say that it is one of the most promising today. In the Pelamis device [12], the rotational relative motions between floating cylinders is used to produce high pressure hydraulics, then electricity. In the sloped IPS buoy [11], it is a coupled submerged body which provides the reaction for floating body oscillation, whereas in the PS Frog MK5 [3] the coupled reaction providing body is located inside the floating unit. As others researchers, we believe that control is the key which could make one day wave energy conversion economically viable, whatever the device considered. When the device is composed of a single oscillator using the sea bottom as a reference – a point-absorber –, the law for optimum control is known from the early studies of Evans [6] and Falnes [2], but can not be applied in random waves, being then anti-causal. In the early eighties, Budal and Falnes proposed a sub-optimal control method known as *latching control* [2], which was further investigated in [7], [8], [1], for a single oscillator. On his side, Korde [9] applied this method to devices using an in-board active reference, but exerting the latching control on the power take off rather than on the motion of the system. In [10], he considered continuous reactive control to decide if the reaction providing body should be located inside or outside the floating unit. Here, we will expose a method that can be used to assess the benefit which can be brought by applying latching control on the relative motion of such a device.

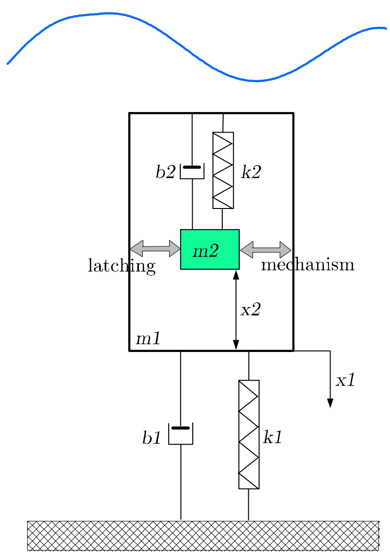

Figure 1: principle of the double oscilllator point absorber

2 General formulation.

Let's consider a simplified generic wave energy converter as described on figure (1). It is made of a positively buoyant cylindrical hull of mass m_1, moored to the sea bottom and restrained to move only in heave motion (measured by x_1). We assume all the other motions to be ideally restricted. Inside this cylinder lies an additionnal mass m_2 which can slide without friction along the vertical x axis. Let x_2 be the displacement of m_2 appart from its equilibrium position. When the system is excited by the sea, both the cylinder and the internal mass are set into motion, in such a way that the relative motion between the two parts can be converted into energy by means of a *power-take-off* (PTO). We will assume that the PTO can be modelized by linear spring k_2 and damper b_2. Time-domain equations of the motions are :

$$
\begin{aligned}
m_1\ddot{x}_1(t) + m_2\left(\ddot{x}_1(t)+\ddot{x}_2(t)\right) &= f_{ex}(t) - \mu_\infty \ddot{x}_1(t) \\
&- \int_0^t \dot{x}_1(\tau)\, K(t-\tau)\, d\tau - k_1 x_1(t) \\
m_2\left(\ddot{x}_1(t)+\ddot{x}_2(t)\right) &= -b_2\dot{x}_2(t) - k_2 x_2(t)
\end{aligned}
$$

where μ_∞ is the added mass for the heave motion, K is the impulse response function in heave mode, f_{ex} is the excitation force due to incident and diffracted waves, k_1 is the stiffness of the cylinder. Using Prony's method [4], one can approximate the kernel function K by a sum of N pair of conjuguate complex variables I_i. The value of each I_i is then given by a ordinary differential equation $\dot{I}_i = \beta_i I_i + \alpha_i \dot{x}_1$where (α_i, β_i) can be calculated using the method described in [5]. The equation of the motion then becomes a standard state equation:

$$
\begin{aligned}
(m_1+m_2+\mu_\infty)\,\ddot{x}_1(t) + m_2\ddot{x}_2(t) &= f_{ex}(t) \\
&- \sum_{i=1}^{N} I_i - k_1 x_1(t) \\
(m_1+m_2)\,\ddot{x}_2(t) + b_2\dot{x}_2(t) + k_2 x_2(t) &= 0 \\
\dot{I}_i - \beta_i I_i &= \alpha_i \dot{x}_1
\end{aligned}
\quad (1)
$$

in which all the hydrodynamical radiation effects lie in the added states I_i. To help understanding, we will temporarily consider a simplified system consisting only in two coupled mechanical oscillators by skipping all the I_i terms in eq.(1) which degenerates into a damper. However, all the forthcoming calculations can be performed for the initial global problem by simply re-introducing the hydrodynamic I_i terms in eq.(1). Motion equations for the coupled oscillators problem are :

$$
\begin{array}{rcl}
m_1\ddot{x}_1 + m_2(\ddot{x}_1+\ddot{x}_2) &=& F_{ex}(t) - k_1x_1 - b_1\dot{x}_1 \\
m_2(\ddot{x}_1+\ddot{x}_2) &=& -k_2x_2 - b_2\dot{x}_2
\end{array}
\quad (2)
$$

From now on, we will consider monochromatic excitation force. Let us define the state vector $X = \begin{bmatrix} x_1 & x_2 & \dot{x}_1 & \dot{x}_2 \end{bmatrix}^T$.By inverting the mass matrix, we get the the first order differential equation of the motion:

$$\dot{\mathbf{X}} = \mathbf{A}.\mathbf{X} + \mathbf{B}\cos(\omega t + \varphi_0) \quad (3)$$

with :

$$
\mathbf{A} = \begin{pmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-\omega_1^2 & \eta\omega_2^2 & -\varepsilon_1 & \eta\varepsilon_2 \\
\omega_1^2 & -(1+\eta)\,\omega_2^2 & \varepsilon_1 & -(1+\eta)\,\varepsilon_2
\end{pmatrix}
$$

$$
\mathbf{B} = \begin{pmatrix} 0 \\ 0 \\ f_{ex} \\ -f_{ex} \end{pmatrix}
$$

and with $\eta = \frac{m_2}{m_1}$, $\omega_1^2 = \frac{k_1}{m_1}$, $\varepsilon_1 = \frac{b_1}{m_1}$, $\omega_2^2 = \frac{k_2}{m_2}$, $\varepsilon_2 = \frac{b_2}{m_2}$ and $f_{ex} = \frac{F_{ex}}{m_1}$.

It is well known that the general solution of this equation (3) can be expressed in term of matrix exponentials. Using the given initial condition $X(t = t_i) = X_i$., we get :

$$
\begin{aligned}
\mathbf{X}(t) &= \exp(\mathbf{A}(t-t_i)).\mathbf{X}_i + \\
&\Re\left(\begin{array}{c} \left(\mathbf{I}e^{i\omega(t-t_i)} - \exp(\mathbf{A}(t-t_i))\right) \\ \times(i\omega\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}e^{i(\omega t_i+\varphi_0)} \end{array}\right)
\end{aligned}
\quad (4)
$$

2.1 Equations of the latched system.

Latching control consists in locking the x_2 motion when the velocity vanishes (at say $t = 0$ the origin of time being free up to now), and releasing it (at $t = t_0$) after a certain delay to be determined. In a previous paper [1] where the present method was applied to a single oscillator (as if m_2 were discarded, and the PTO in b_1 instead of b_2 and the latching on the motion of mass m_1), this manoeuvre was shown to improve dramatically the response amplitude, and consequently the extracted power. Let's assume that, at time $t = 0$ (initial conditions), we lock the motion x_2 of the internal mass relative to the external mass as described above. During the latching period $(0 < t < t_0)$ the equation of motions then degenerates into :

$$
\begin{array}{rcl}
(m_1+m_2)\,\ddot{x}_1 &=& F_{ex}(t) - k_1x_1 - b_1\dot{x}_1 \\
\dot{x}_2 &=& 0
\end{array}
\quad (5)
$$

Under the same matrix form :

$$\dot{\mathbf{X}} = \mathbf{A}'.\mathbf{X} + \mathbf{B}'.\cos(\omega t + \varphi_0) \quad (6)$$

but now with :

$$
\mathbf{A}' = \begin{pmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
-\frac{\omega_1^2}{1+\eta} & 0 & -\frac{\varepsilon_1}{1+\eta} & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\quad
\mathbf{B}' = \begin{pmatrix} 0 \\ 0 \\ \frac{f_{ex}}{1+\eta} \\ 0 \end{pmatrix}
$$

Again the general solution is given by eq.(4), assuming the initial condition $X(t = 0) = X_0$.

2.2 Calculating the optimal latching duration.

Let's assume that the excitation and the control has started from $t = -\infty$ and that a periodic regime has been reached. We assume that at $t = 0$, the velocity $\dot{x}_2$ of the mass m_2 vanishes. From this we get the initial condition :

$$\mathbf{X}_0 = \mathbf{X}(t=0) = \begin{bmatrix} x_{1,0} & x_{2,0} & \dot{x}_{1,0} & 0 \end{bmatrix}^T$$

The distance $x_{2,0}$ between the two masses will be kept constant during a duration t_0 to be determined. We assume that the periodic motion is established, in such a way that the system reach a final state X_1 at $t = t_1$ (with $\dot{x}_2(t_1) = 0$) opposite of the initial condition. This implies two conditions at $t = t_1$. First one is that $\mathbf{X}_1 = \mathbf{X}(t = t_1) = -\mathbf{X}_0 = \begin{bmatrix} -x_{1,0} & -x_{2,0} & -\dot{x}_{1,0} & 0 \end{bmatrix}^T$. The second one requires that necessarily $F_{ex}(t = t_1) = -F_{ex}(t = 0)$ resulting in $\omega t_1 = \pi + 2k\pi$,with $k \in \mathbb{N}$. From this we get $t_1 = (2k+1)\frac{\pi}{\omega}$ and we see that the ratio between the period of the system response (say T_{out}) and the excitation period (say T_{ex}) is necessarily odd.

2.3 Equation of latching

By writing the state vector at $t = t_0$, when the second mass m_2 is released, and at $t = t_1$, we find that $\mathbf{X}_0$ must be solution of

$$\mathbf{X}_0 = -\left(\mathbf{I} + \exp(\mathbf{A}\Delta).\exp(\mathbf{A}'t_0)\right)^{-1} \times \Re\left(\begin{pmatrix} \exp(\mathbf{A}\Delta)\left(\mathbf{I} - \exp(\mathbf{A}'t_0)e^{-i\omega t_0}\right) \\ \times\left(i\omega\mathbf{I} - \mathbf{A}'\right)^{-1}\mathbf{B}' \\ +\left(\mathbf{I}e^{i\omega\Delta} - \exp(\mathbf{A}\Delta)\right) \\ \times\left(i\omega\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{B} \end{pmatrix} e^{i(\omega t_0 + \varphi_0)}\right) \quad (7)$$

with $\Delta = t_1 - t_0$. Moreover we must have : $\Delta = (2k+1)\frac{\pi}{\omega} - t_0$, $t_1 = (2k+1)\frac{\pi}{\omega}$. The last field of vector X_0 represents the velocity of mass m_2. From the previous conditions it should vanish, and this gives the equation of latching delay.

So now let's choose an integer value for k. We can calculate for each value of φ_0 all the possibilies for $t_0 \in \left[0, (2k+1)\frac{\pi}{\omega}\right]$ satisfying the equation of latching. For periodicity arguments, it is not necessary to seek for solutions for values of $\varphi_0$outside the range $[0, \pi]$. Finally, among all the solution couples (φ_0, t_0), we retain the only one which maximizes the amplitude of the motion of the second mass during the free motion period. Note that the form of the equation (7), and therefore the solution method, would have been absolutely identical if we had made the calculation with the hydrodynamic terms I_i included.

3 Results

On figure (2,a), we have plotted the amplitude of the response of the system, controlled or free. The parameters are set to $\omega_1 = 0.75$ rad/s, $\omega_2 = 1.0$ rad/s, $\eta = 0.1$, $f_{ex} = 1.0$, $\varepsilon_1 = \varepsilon_2 = 0.1$. In this first calculation, we have set $k = 0$, which means that the period of the response T_{out} is equal to the response of the excitation force T_{ex}. Results are globally the same as what we observed in the case of a single mechanical oscillator : non trivial solutions of the above latching problem can be found when the excitation frequency is lower than the natural frequency (i.e. ω_2) of the working mass (here m_2) on which latching is applied. The first (from top) two curves fig.3 show the

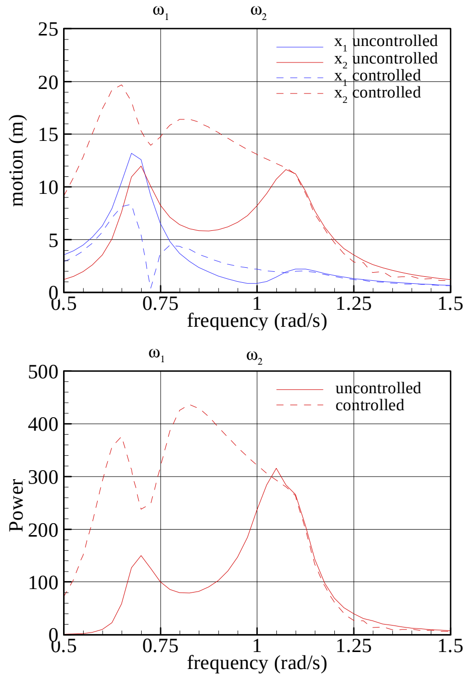

Figure 2: Homochromatic ($T_{out} = T_{ex}$) response amplitude and extracted power with and without control. $\omega_1 < \omega_2$

amplification obtained in a time-domain simulation. On the bottom plot, figure (2, b), where the absorbed power is plotted, one can appreciate all the benefit brought by latching control, and the large broadening of the bandwidth. We made another calculation for the same choice $k = 0$, then $T_{out} = T_{ex}$, but now with $\omega_2 < \omega_1$. Results were quite disappointing: latching control still works for $\omega < \omega_2$, as we could anticipate, but it cover only a narrow band in low frequency, and is globally unefficient in terms of absorbed power. On figure (4), we have plotted results corresponding to the case $\omega_1 < \omega_2$ but now with $k = 1$, which means that the period of the response is three times the excitation period. ($T_{out} = 3T_{ex}$). A example of such a behavior is plotted in fig.3,c. As in the case of a single mechanical oscillator, one can see that the amplitude of the motion is amplified whatever is the frequency, even for $\omega > \omega_2$, and the absorbed power is significantly improved, except around the natural frequency ω_2. Period tripling allows to improve the functionment of the system, even for frequencies higher than the natural frequency of the working oscillator. However, the gain in terms of power is less than in the case $k = 0$ when the frequency is lower than ω_2. Applications of the present method including all the hydrodynamic terms will be presented at the Workshop, and explained in detail in a forthcoming paper.

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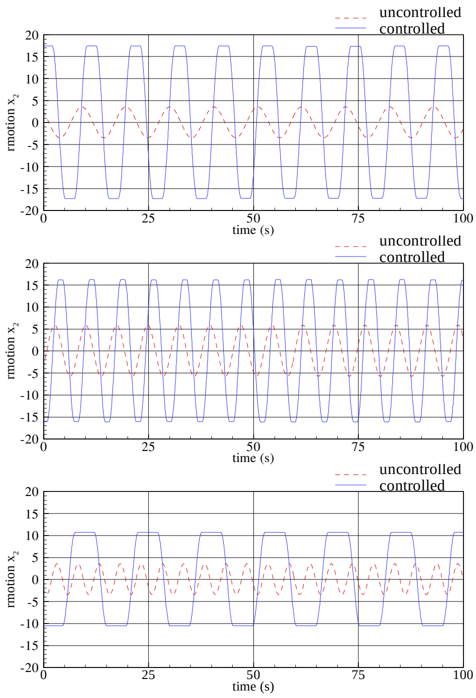

Figure 3: Time domain simulations of the relative motion between the two masses for $\omega = 0.6$, $\omega = 0.85$ and $\omega = 1.25$.(top to bottom)

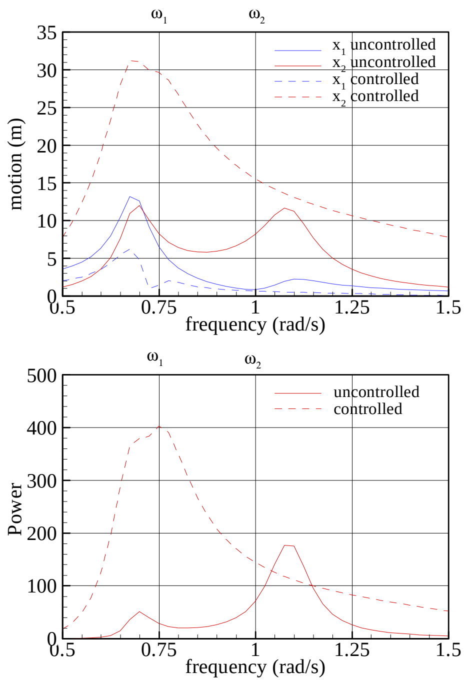

Figure 4: Triperiodic ($T_{out} = 3T_{ex}$) response amplitude and extracted power with and without control when $\omega_1 < \omega_2$

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