SPECULATIONS ON THE ADEQUACY OF 2ND ORDER DIFFRACTION THEORY

J. Zang, P. H. Taylor, R. Eatock Taylor
Department of Engineering Science, University of Oxford

(jun.zang@eng.ox.ac.uk, paul.taylor@eng.ox.ac.uk, r.eatocktaylor@eng.ox.ac.uk)

SUMMARY

This paper contrasts the high accuracy of 2nd order diffraction theory when applied to the description of surface elevation around the bow of a fixed ship to the poor description of the flow structure around a small diameter circular column. For the ship all the main physics appears to occur at 1st and 2nd order whereas for the cylinder linear diffraction appears to be correct but 2nd order does not work well – possibly throwing into question the relevance of such diffraction calculations for a class of small structures.

INTRODUCTION

Second order diffraction theory is widely used for the design of offshore structures. Yet there is considerable evidence that for small diameter columns at the low end of the diffraction regime in terms of D/λ (but where viscous effects are relatively small), it does not provide an adequate description for steep waves – even ignoring high frequency effects such as ringing (which is usually taken to be predominantly 3rd order).

As part of the FP5 REBASDO Programme, we are examining the effects of directional wave spreading on the non-linear hydrodynamic loads and the wave run-up in the region of the bow of a floating vessel (FPSO) moored in random seas. The non-linear wave scattering problem is solved using quadratic boundary elements. An existing scheme (*DIFFRACT* developed in Oxford) has been extended to deal with bi-chromatic and bi-directional input wave systems, calculating second-order wave diffraction under regular waves and focussed wave groups. The second-order sum and difference (double and low frequency) terms have been investigated in detail to explore the various terms contributing to the nonlinear scattering field.

EXPERIMENTS AND 2ND ORDER MODELLING

A simplified model of an FPSO (a finite-draft fixed box with semi-circular end facing upstream, width 32.5*cm*, draft 12.5*cm*, length 1.1*m*) on deep water in regular waves and a narrow banded uni-directional focussed wave group has been analysed for comparison with experiments performed by Gibson and Swan in a wave flume at Imperial College.

Fig.1 shows the measured free surface time history at gauge 16 (10*mm* in front of the bow of the simplified FPSO of scaled length 1.1*m*) for a focussed wave group with crest and trough focussing (paddle signal ×-1) at the position of the bow but in the absence of the ship. With these original non-linear experimental data, it is easy to derive an experimental linearised incoming wave time series, shown in Fig.1, by subtracting the trough focus time history from the crest focus one. Based on this linear incoming wave time history, the corresponding amplitude spectrum can be obtained using an FFT. In order to fit the experimental measurements, 13 frequency components, which are marked by circles 'o' in Fig.2, are included in the numerical modelling of diffraction. Compared with the amplitude spectrum in Fig.2, the amplitude spectra derived from the experimental measurements for crest and trough focussing at the bow position when the ship is in place, shown in Fig.3, indicates the considerable enhancement of the spectrum due to diffraction. The linear wave is mostly within the range of angular frequency from 5.0 to 10.0 *rad/s* and the diffracted spectra for crest and trough focussed cases shown in Fig. 3 have almost the same shape within this range, but differ considerably for the frequency ranges 10.0 to 20.0 *rad/s* (2^{nd} order sum interactions) and 0 to 5.0 *rad/s* (2^{nd} order difference interactions). This clearly shows the significant effect of second order diffraction in the experiments.

The numerical mesh for the simplified FPSO model is shown in Fig.4. Two plane-symmetry is used to speed up the numerical modelling by ~75%. Quadratic panels permit the use of relatively large elements while achieving high accuracy for the demanding second order diffraction calculations.

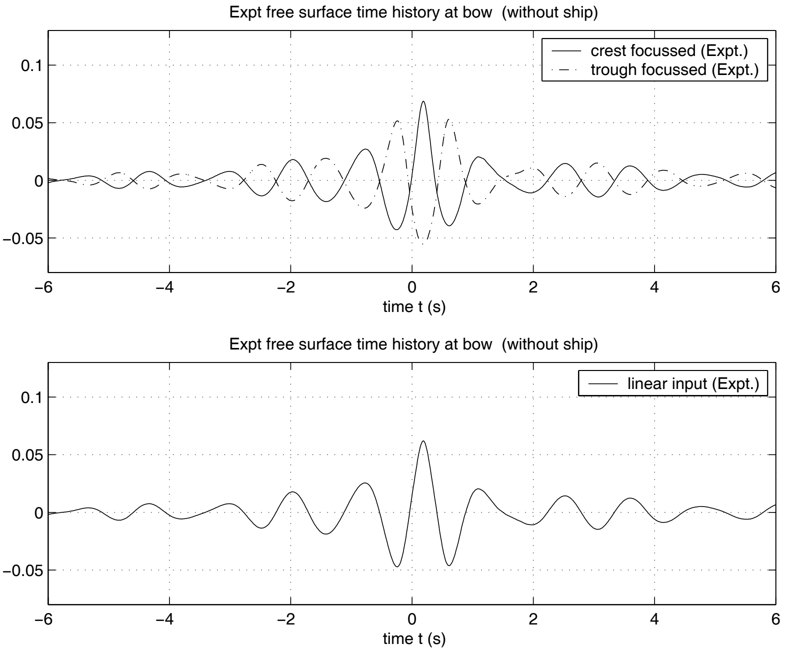

Fig. 1 Measured free surface time histories (without ship) and linearised incoming wave

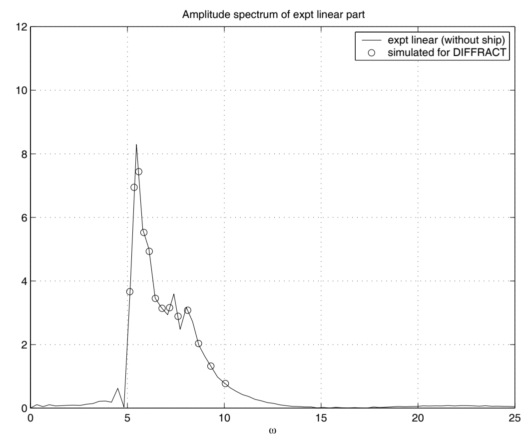

Fig.2 Amplitude spectrum (without ship)

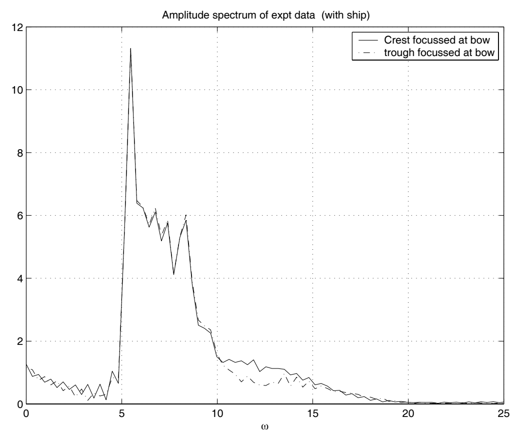

Fig.3 Amplitude spectrum (with ship)

The comparison of the numerical results and the experimental measurements for non-linear free surface elevation around the bow are shown in Fig.5, and the individual second order difference contribution to the free surface at gauge 16 (10*mm* in front of bow) and second order sum contribution to the free surface at gauge 4 (510*mm* in front of bow) are represented in Fig.6 and Fig.7. The agreement in all cases is remarkably good - numerical predictions and experiments giving the same non-linear crest elevation around the bow. The second order diffraction effects are significant as can be easily seen in the Fig.5: the crest elevation around the bow is increased by ~35% by second order scattering on top of the linear diffracted value (itself ~40% higher). For steeper ingoing waves, this second order enhancement will increase rapidly. What is clear from the work so far is the complex local interaction between the incoming and diffracted waves and the resulting greatly increased water surface elevations local to the body. Furthermore, this interaction drives free waves, which are radiated outwards from the body and have frequencies considerably higher than in the incoming group. This behaviour can been easily seen from Fig.7 for the point at 510*mm* in front of the bow, the radiated free waves already having separated from the incoming wave group by ~2.5*s* (and also propagating in the opposite direction). Similar behaviour has also been observed in previous experiments for wave scattering by a model of a gravity base platform (Swan, Taylor and Van Langen, Appl. Ocean Res. 19,309 (1997)), and explicitly calculated and discussed by Buldakov et al. (2004, Ocean Eng., accepted).

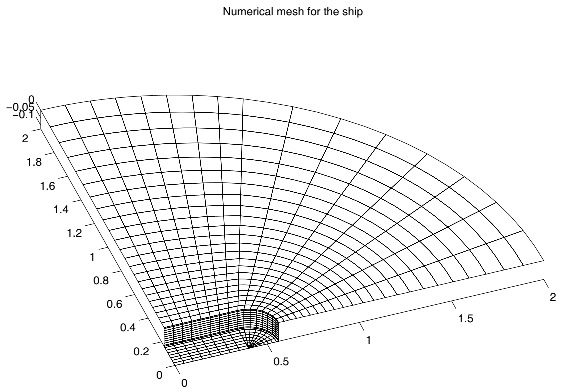

Fig. 4 Numerical mesh for simplified FPSO

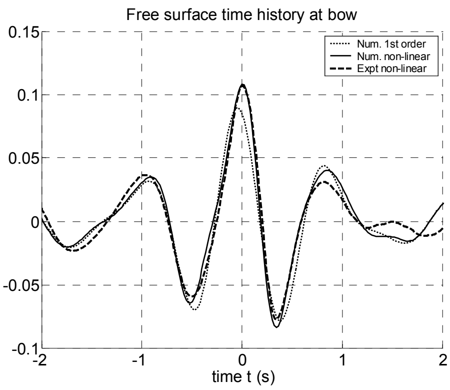

Fig. 5 Non-linear free surface around bow

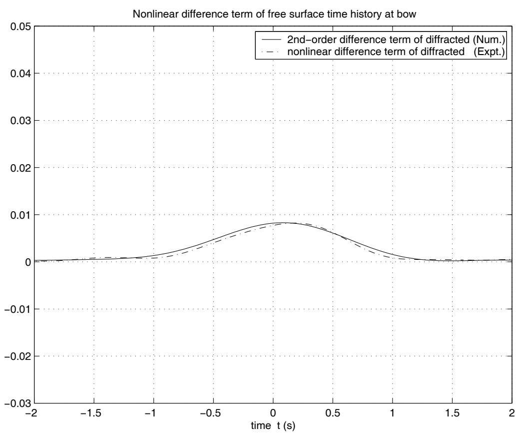

Fig. 6 Second order difference frequency term of free surface at gauge 16

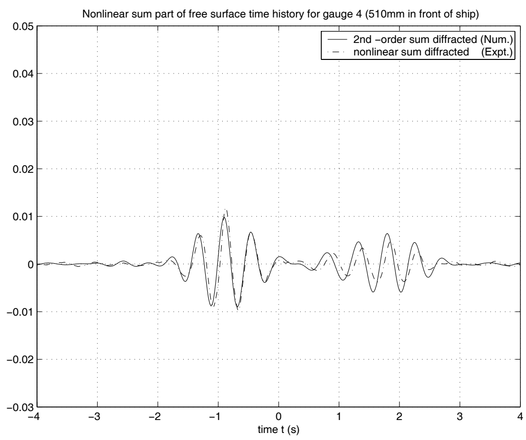

Fig. 7 Second order sum frequency term of free surface at gauge 4

Our overall conclusion from the comparison of the experiments with the calculations is that 2nd order theory based on potential flow works remarkably well for this ship configuration.

DISCUSSION

There has been much work both experimental and computational on wave scattering by surface-piercing cylinders. For example, a recent paper by Trulsen and Teigen (OMAE2002-28178 Wave scattering around a vertical cylinder: fully non-linear potential flow calculations compared with low order perturbation results and experiment) concludes that neither 2nd order nor a fully non-linear scheme quantitatively capture the details of the wave-body interaction in steep regular waves. Qualitatively, wave interaction with the cylinder produces relatively large components at many harmonics of the wave input frequency (2×, 3×, 4×…). The ship configuration produces strong 2nd order scattering and virtually nothing at higher harmonics. What is different for these two cases?

The width of the body in the ship experiments is D/λ=0.21, and 0.13 for the cylinder.
The (peak) wave steepness kH/2=0.27 for the ship, and 0.21 and 0.39 for the cylinder.
The draft for the ship is d/λ=0.080 and 0.19 for the cylinder experiments.

Although these parameters are not identical, they are not sufficiently different to suggest any obvious explanation. However, there is one obvious difference: the shape of the bodies in the downstream direction. The ship-shaped body is almost one wavelength long, so the source of any scattered waves (particularly sum harmonics) from the downstream region is located far from the front stagnation point, where the highest diffracted effects are felt for the incident waves.

In contrast, the cylinder is compact – scattered waves are produced near to the downstream half of the cylinder here as well as off the front of the body. These waves from the region close to the rear stagnation point propagate upstream around the body. These short waves propagate past the shoulders of the body – where the linear fluid velocity components are largest – and where there are very high spatial gradients. Grue and Huseby (2002, 17th IWWWFB, On higher harmonic wave loads on vertical cylinders and ringing of offshore structures) have remarked on the appearance in this region of very steep gradients and breaking that resembles small hydraulic jumps moving upstream for incident waves that are sufficiently steep. This is likely to be the region when the assumption of a simple mathematically well-behaved asymptotic expansion first starts to break down first. In particular, as a large crest passes the fluid particle velocities at the shoulders of the cylinder can easily exceed both the group and phase velocities of short waves (double and triple frequency) scattered in the upstream direction. Obviously the modulation of a short wave, of amplitude proportional of the amplitude of the long wave squared, with the long wave cannot be captured in a 2nd order expansion in the long wave amplitude.

Trulsen and Teigen suggest that viscous effects may explain the discrepancies between their modelling and the experimental results. In this context, it should be observed that the draft of the ship is small and for the trough focus wave group the lower edge of the semi-circular front and the underside of the ship is exposed to strong wave-induced flow. Since the angle of this junction is 90°, large-scale flow separation is inevitable. Of course the 2nd order potential flow calculations ignore this but predict the surface elevation variation both locally and globally very well.

In the talk we will also discuss a range of other diffraction calculations for shapes in between the box and the cylinder, showing that the influence of waves scattered from the stern on elevation at the bow is dramatic for a cylinder but decreases quickly as the downstream length of the body is increased. We also show that a similar effect can be produced by artificially suppressing the 2nd order sum source terms in the region behind the cylinder, revealing how much of the surface elevation at the bow is influenced by short waves propagating around the body from astern. This work also suggests new geometries to be tested experimentally.

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