Time-domain Calculation of Moored Ship Motions in a Harbour

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Introduction

In the past decades the increase of particularly global container traffic has led to the development of larger ships and several port expansions. These large ships demand for a large water depth in the harbour and a wide entrance. Hence significant wave penetration in the harbour is nearly inevitable, while also the large container carriers still allow limited movements for efficient loading and offloading. Especially in ocean facing ports long waves in the harbour can cause harbour oscillations which result in low-frequency ship motions.

The calculation of waves in the harbour and the calculation of resulting ship motions require different approaches. For the calculation of the wave field an accurate description of the shape of the harbour and the bathymetry is required, while for the calculation of ship motions the shape of the hull is most important. Therefore different methods are used for either problem. Calculation of the propagation of the incident waves is done using a Boussinesq-type wave model. The disturbance of the incident waves by the moving ship hull is treated by a panel-based linear time-domain diffraction model. This time-domain method is a well-established approach for the calculation of the motions of a ship sailing at the ocean, e.g. Bingham et al. (1994) and Korsmeyer and Bingham (1998). Concerning a moored ship the problem becomes easier because all terms proportional to the forward speed can be neglected. On the other hand the influence of nearby quay walls needs to be taken into account.

The superposition principle is used to consider the motions of the body in waves, i.e. the diffraction of the incident waves around the restraint body and the radiation problem are treated separately. This linearized approach is allowed as long as the motions are small, which is allowed for almost any mooring system in a harbour. The calculated incident waves are considered on the hull and the diffraction problem is solved with linear free surface conditions.

The computation procedure is as follows, given a certain wave field at the ocean:

- determine the waves throughout the harbour and at the location of the moored ship;
- determine added mass coefficients and retardation functions by solving the radiation problem of an initial displacement of the ship in the harbour geometry;
- determine the diffraction of the incident waves around the submerged hull;
- integrate the equation of motion and determine the exciting first and second order wave forces and forces in fenders and mooring lines.

Waves in the Harbour Basin

For the determination of the wave field in the harbour, not taking into account the presence of the moored ship, use is made of the Boussinesq-type wave model TRITON (Borsboom et al., 2000). The advantage of the use of Boussinesq equations to solve the wave propagation problem is that all non-linearities are preserved, both high-frequency and low-frequency non-linearities, within 2D equations. The vertical dimension is eliminated by expanding the velocity potential in a polynomial in terms of kh (local wave number times local water depth). Boussinesq theory provides an efficient method to predict wave propagation over a mildly sloping bathymetry and throughout an arbitrarily shaped domain.

For most applications it is important that the non-linear wave profile is well represented. For the calculation of ship motions these high-frequency effects are not normative. Apart from this, Boussinesq equations also show the effect of set-down beneath wave groups, diffraction around breakwaters and possible resonance of the set-down wave in the harbour basin.

The used set of equations conserve both mass and momentum, it shows good correspondence with the dispersion relation and it is relatively compact. In the present form the model is extremely suitable for application in small harbours.
Results from the Boussinesq-type wave model are wave elevations and depth-averaged velocities. For the calculation of the scattering of waves by the ship, the pressure distribution and local particle velocities are required on the hull. To obtain the specific particle velocities and pressures at the collocation points of the panels on the hull, an estimation of the velocity profile is made based on space and time derivatives of the depth-averaged velocities in the polynomial approximation of the velocity potential.

Radiation Problem

With the assumption that the movements are small, a linear approach of the radiated waves can be followed and the forces can be written in terms of the hydrodynamic coefficients, which are included in the equation of motion:

$$(\mathbf{M}+\mathbf{A})\ddot{\vec{X}}(t) \;+\; \mathbf{B}\dot{\vec{X}}(t) \;+\; \mathbf{C}\vec{X}(t) \;+\; \int_0^\infty \mathbf{K}(\tau)\dot{\vec{X}}(t-\tau)d\tau \;=\; \vec{F}(t) \tag{1}$$

in which $\mathbf{M}$ is the inertia matrix for six degrees of freedom. The matrix $\mathbf{B}$ can be used to include (linearized) viscous damping coefficients. $\mathbf{C}$ is the matrix with hydrostatic restoring coefficients. The force $\vec{F}$ consists of the exciting wave forces up to second order and forces in fenders and mooring lines. The hydrodynamic coefficients can be solved by giving the body a small impulsive displacement in the k-mode at $t = 0$. The solution of the velocity potential is divided into an impulsive part and a time-varying part due to the disturbance of the free surface:

$$\phi_k(\vec{x},t) \;=\; \psi_k(\vec{x})\delta(t) \;+\; \chi_k(\vec{x},t) \tag{2}$$

The solution of the velocity potentials at $t = 0$ lead to a formulation of the added mass coefficient A_{kj} and the solution at $t = \tau$ lead to the retardation function $K_{kj}(\tau)$:

$$A_{kj} \;=\; \rho \iint_{\mathcal{H}_0} \psi_k(\vec{x}) n_j dS \tag{3}$$

$$K_{kj}(t) \;=\; \rho \iint_{\mathcal{H}_0} \frac{\partial \chi_k(\vec{x},t)}{\partial t} n_j dS \tag{4}$$

where $\mathcal{H}_0$ is the mean submerged hull. The velocity potentials are calculated with a time-domain boundary-integral method with panels on the hull and the quay walls and with the linearized free surface conditions included in the formulation of the Green function.

Diffraction Problem

In order to solve the diffraction problem, pressures and normal velocities of the incident waves need to be prescribed on the submerged hull. The diffraction problem is imposed on the restrained ship. Hence the normal velocity of the scattered wave on the hull is simply the opposite of the normal velocity of the incident wave. The normal velocity on the quay walls is equal to zero. The incident wave does not need to be prescribed here, because reflections against the quay are included in the Boussinesq model. The problem is solved using the scattered potential formulation ϕ_S of the method for the transient diffraction problems:

$$\begin{aligned} 2\pi\phi_S(\vec{x},t) \;&+\; \iint_{\mathcal{H}_0} \left\{\phi_S G_n^{(0)} \;+\; (\phi_I)_n G^{(0)}\right\} dS \;+\; \iint_{\mathcal{W}_0} \phi_S G_n^{(0)} dS \\ &+\; \int_0^t d\tau \iint_{\mathcal{H}_0} \{\phi_S G_n^w \;+\; (\phi_I)_n G^w\} dS \;+\; \int_0^t d\tau \iint_{\mathcal{W}_0} \phi_S G_n^w dS \;=\; 0 \end{aligned} \tag{5}$$

where $(\phi_I)_n$ is the normal velocity of the incident wave, $G^{(0)}$ and G^w are the impulsive and wave part of the Green function respectively and $\mathcal{W}_0$ is the mean area on the quay walls.
The first order exciting wave forces follow directly from integration of the pressures due to the incident and the scattered wave over the submerged hull.

Integration of the Equation of Motion

After solving the radiation and diffraction problem, the equation of motion (Eq. 1) can be integrated directly in time. The second order wave forces and the forces in fenders and mooring lines are evaluated during the numerical integration. It is well-possible to include non-linear characteristics of fenders and mooring lines and to include fender friction. The second order wave forces are calculated during the time integration using the direct pressure integration method. Concerning a moored ship the body is neither free floating nor restrained from moving, so that it is most accurate to use the actual body motions and radiation potentials in the formulation of the second order force. The contribution of the second order potential is omitted in the formulation of the second order force, because the second order wave profile is included in the incident wave as obtained by the non-linear Boussinesq method. These non-linearities are treated as linear waves in the diffraction problem. This assumption is allowed, because the diffracted non-linear waves can be considered as very small perturbations on the incident wave.
The results of the integration are time records of ship motions and forces in mooring lines and fenders. The results can be compared with values of allowable movements for safe and economic loading and allowable forces in mooring lines and fenders.

Discussion

A method has been described which effectively treats the input of arbitrarily prescribed waves at the location of the ship. The scattered and radiated waves are calculated as accurate as possible using the principle of linear superposition. The total wave forces are calculated up to second order. Bingham (2000) has developed an efficient method to calculate the behaviour of a ship in a complex geometry by combining Boussinesq theory and a frequency-domain panel method. Using the Haskind relations he was able to calculate the first order wave force with a limited number of considered frequencies. Pinkster (2003) showed a more accurate method to determine the effect of passing ships and used all relevant Fourier components to calculate the wave force up to second order. The advantage of the time-domain approach presented in this paper over a frequency-domain approach is that an accurate description of the distribution of the wave potentials over the hull is obtained without the disadvantage of the inclusion of lots of Fourier components to achieve the same accuracy. In principle the time-domain approach includes a long convolution integral for long time records. However the influence of waves and ship motions in the past will decay, so that the convolution integral can be truncated. Furthermore it is possible to include the calculation of second order forces in the equation of motion, which provides a more accurate formulation than determination beforehand, based on a free floating ship.

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