# FORCED VIBRATIONS OF FLOATING ELASTIC PLATE

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## SUMMARY

Plane problem of finite plate behavior under periodic external load is considered. There are many numerical methods for VLFS problems. But there are only few papers on plate behavior under periodic external loads [1-2]. The numerical method based on the Wiener-Hopf technique, that was presented earlier for the diffraction problem [3], is developed. The short-wave approximation of the solution is presented in explicit form. Numerical analysis is performed for amplitudes of the plate deflection and the free surface elevation depending on frequency, fluid depth and the character of external load. It is found that for low frequencies deflection of the plate is maximal near the load region. For high frequencies waves reflected from edges become significant and resonance amplification is found on the scattering frequencies. Existence and conditions of localized vibrations were revealed when the fluid is at rest and plate vibrations of the plate are localized near the load region.


## 1. FORMULATION OF THE PROBLEM

We assume that the liquid is ideal incompressible and occupies the region $-H_0 < y < 0$. The liquid surface is covered partly with a thin homogeneous plate ($y = 0,\ 0 < x < L_0$) of thickness $h$. The edges of the plate are free. The external periodic pressure of the form $q(x)e^{-i\omega t}$ acts on the plate surface. In the linear approach the fluid motion is described by the velocity potential $\varphi$, satisfying the Laplace equation. We assume also that a wave length is much greater than the plate thickness.

First we consider the case of concentrated load $q(x,t) = q_0\delta(x - x_0)e^{-i\omega t}$. The time dependence of all functions is expressed by the factor $e^{-i\omega t}$. We put $\varphi = \phi(x,y)e^{-i\omega t}$. To reduce the number of free parameters, we introduce scaled variables as follows $\phi' = \phi\omega\rho/q_0$, $w' = w\rho g/q_0$, $p' = q_0 p$, $x' = x/l$, $y' = y/l$, $t' = \omega t$, $l = g/\omega^2$, where $l$ is the characteristic length, $g$ is the gravity acceleration, $w$ is the displacement, $p$ is hydrodynamic pressure, $\rho$ is fluid density. Hereafter all primes will be omitted.

In non-dimension variables we derive the following boundary-valued problem

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0,\ (-H < y < 0), \qquad \frac{\partial\phi}{\partial y} = 0,\ (y = -H) \tag{1}$$

$$\frac{\partial\phi}{\partial y} - \phi = 0,\ (y = 0,\ x \in (-\infty, 0) \cup (L, \infty)) \tag{2}$$

$$(\beta\frac{\partial^4}{\partial x^4} + 1 - d)\frac{\partial\phi}{\partial y} - \phi = -i\delta(x - x_*),\ (y = 0,\ 0 < x < L) \tag{3}$$

$$\frac{\partial^2}{\partial x^2}\frac{\partial\phi}{\partial y} = \frac{\partial^3}{\partial x^3}\frac{\partial\phi}{\partial y} = 0,\ (y = 0,\ x = 0, L) \tag{4}$$

Here $L = L_0/l$, $H = H_0/l$, $x_* = x_0/l$, $\beta = D/(\rho g l^4)$, $d = d_0/l$, $D$ is the flexural rigidity of the plate, $d_0$ is the plate draught. Furthermore, the radiation condition as $|x| \to \infty$ and the regularity condition in a vicinity of the plate edge should be satisfied. The last condition means that the fluid energy in vicinities of the plate edges is limited. According to the above assumptions the parameter $d << 1$.

## 2. THE SYSTEM OF INTEGRAL EQUATIONS

The problem (1)-(4) is solved by the Wiener-Hopf technique. We introduce the functions of the complex variable $\alpha$ as follows

$$\Phi_+(\alpha, y) = \int_L^\infty \mathrm{e}^{i\alpha(x-L)}\phi(x,y)dx, \quad \Phi_-(\alpha, y) = \int_{-\infty}^0 \mathrm{e}^{i\alpha x}\phi(x,y)dx, \quad \Phi_1(\alpha, y) = \int_0^L \mathrm{e}^{i\alpha x}\phi(x,y)dx, \tag{5}$$

$$\Phi(\alpha, y) = \Phi_-(\alpha, y) + \Phi_1(\alpha, y) + e^{i\alpha L}\Phi_+(\alpha, y).$$

We denote by $D_\pm(\alpha), D_1(\alpha)$ integrals of the form (5), where the integrand is the left side of (3), and by $F_\pm(\alpha), F_1(\alpha)$ analogous integrals where the integrand is the left side of (4). The functions with subscript $+$ and $-$ are regular in the upper and lower semi-planes respectively. The function $\Phi(\alpha, y)$ is the usual Fourier transform. From (1) we have $\Phi(\alpha, y) = C(\alpha)\mathrm{ch}(\alpha(y + H))/\mathrm{ch}(\alpha H)$.

We introduce also dispersion functions $K_1(\alpha) = \alpha \tanh(\alpha H) - 1$ for open water and $K_2(\alpha) = (\beta\alpha^4 + 1 - d)\alpha \tanh(\alpha H) - 1$ for the liquid under the plate. The function $K_1(\alpha)$ has two real roots $\pm\gamma$ and a countable set of imaginary roots, $K_2(\alpha)$ has two real roots $\pm\alpha_0$, a countable set of imaginary roots $\alpha_n,\ n = 1, 2, ...$ and four complex roots $\pm\alpha_{-1}$ and $\pm\alpha_{-2}$.

From boundary conditions (2) and (3) we have $D_-(\alpha) = D_+(\alpha) = 0, \quad F_1(\alpha) = -ie^{i\alpha x_*}$. Then

$$D_1(\alpha) = D(\alpha) = C(\alpha)K_1(\alpha), \quad F_-(\alpha) + F_1(\alpha) + e^{i\alpha L}F_+(\alpha) = C(\alpha)K_2(\alpha)$$

Hence we obtain the equation

$$F_-(\alpha) - ie^{i\alpha x_*} + e^{i\alpha L}F_+(\alpha) = D_1(\alpha)K(\alpha), \quad K(\alpha) = K_2(\alpha)/K_1(\alpha) \tag{6}$$

We factorize the function $K$ in the form $K(\alpha) = K_+(\alpha)K_-(\alpha)$. Multiplying (6) by $e^{-i\alpha L}[K_-(\alpha)]^{-1}$, we transform it to the form

$$\frac{F_+(\alpha)}{K_+(\alpha)} + U_+(\alpha) + M_+(\alpha) = D_1(\alpha)K_-(\alpha)e^{-i\alpha l} - M_-(\alpha) - U_-(\alpha) \tag{7}$$

$$U_+(\alpha) + U_-(\alpha) = \frac{e^{-i\alpha L}F_-(\alpha)}{K_+(\alpha)}, \quad M_+(\alpha) + M_-(\alpha) = -\frac{ie^{i\alpha(x_* - L)}}{K_+(\alpha)}$$

Now we divide (6) by $K_-(\alpha)$ and transform it to the form

$$\frac{F_-(\alpha)}{K_-(\alpha)} + V_-(\alpha) + N_-(\alpha) = D_1(\alpha)K_+(\alpha) - V_+(\alpha) - N_+(\alpha) \tag{8}$$

$$V_+(\alpha) + V_-(\alpha) = \frac{e^{i\alpha L}F_+(\alpha)}{K_-(\alpha)}, \quad N_+(\alpha) + N_-(\alpha) = -\frac{ie^{i\alpha x_*}}{K_-(\alpha)}$$

Using the analytic continuation onto the whole complex plane and the Liouville theorem, we obtain from (7) and (8)

$$\frac{F_+(\alpha)}{K_+(\alpha)} + \frac{1}{2\pi i}\int\limits_{-\infty+i\sigma}^{\infty+i\sigma} \frac{e^{-i\zeta L}F_-(\zeta)d\zeta}{(\zeta - \alpha)K_+(\zeta)} = a_1 + a_2\alpha - M_+(\alpha)$$

$$\frac{F_-(\alpha)}{K_-(\alpha)} - \frac{1}{2\pi i}\int\limits_{-\infty-i\sigma}^{\infty-i\sigma} \frac{e^{i\zeta L}F_+(\zeta)d\zeta}{(\zeta - \alpha)K_-(\zeta)} = b_1 + b_2\alpha - N_-(\alpha)$$

where $a_1, a_2, b_1, b_2$ are unknown constants to be defined from conditions (4). This problem was solved both with account for the structural inertia and without it.

The solution is essentially simpler if the structural inertia is neglected, i.e. $d = 0$. In this case constants $a_j, b_j$ are determined in explicit form. It is found that

$$a_1 = \frac{1}{2\pi i}\int\limits_{C_-} \frac{[e^{-i\zeta L}F_-(\zeta) - ie^{-i\zeta(L - x_*)}]d\zeta}{\zeta K_+(\zeta)}, \quad a_2 = \frac{1}{2\pi i}\int\limits_{C_-} \frac{[e^{-i\zeta L}F_-(\zeta) - ie^{-i\zeta(L - x_*)}]d\zeta}{\zeta^2 K_+(\zeta)}$$

$$b_1 = -\frac{1}{2\pi i}\int\limits_{C_+} \frac{[e^{i\zeta L}F_+(\zeta) - ie^{i\zeta x_*}]d\zeta}{\zeta K_-(\zeta)}, \quad b_2 = -\frac{1}{2\pi i}\int\limits_{C_+} \frac{[e^{i\zeta L}F_+(\zeta) - ie^{i\zeta x_*}]d\zeta}{\zeta^2 K_-(\zeta)}$$

Here $C_-, C_+$ are contours along the real axis from $-\infty$ to $\infty$ passing around points $-\alpha_0, -\gamma$ above and points $\alpha_0, \gamma$ down, $C_-/C_+$ passes around zero down/above. Then we obtain the system

$$\frac{F_+(\alpha)}{\alpha^2 K_+(\alpha)} + \frac{1}{2\pi i}\int\limits_{C_-} \frac{e^{-i\zeta L} F_-(\zeta) d\zeta}{\zeta^2 K_+(\zeta)(\zeta-\alpha)} = \frac{1}{2\pi}\int\limits_{C_-} \frac{e^{-i\zeta(L-x_*)} d\zeta}{\zeta^2 K_+(\zeta)(\zeta-\alpha)}$$

$$\frac{F_-(\alpha)}{\alpha^2 K_-(\alpha)} - \frac{1}{2\pi i}\int\limits_{C_+} \frac{e^{i\zeta L} F_+(\zeta) d\zeta}{\zeta^2 K_-(\zeta)(\zeta-\alpha)} = -\frac{1}{2\pi}\int\limits_{C_+} \frac{e^{i\zeta x_*} d\zeta}{\zeta^2 K_-(\zeta)(\zeta-\alpha)}$$

## 3. Numerical solution

Integrals in the system can be evaluated with the help of the residue theory. Then we have the infinite linear algebraic system with respect to the new unknown quantities $\xi_j$ and $\eta_j$

$$\xi_j - \sum_{m=-2}^{\infty} c_{jm}\eta_m = f_j^{(1)}, \quad \eta_j - \sum_{m=-2}^{\infty} c_{jm}\xi_m = f_j^{(2)}$$

$$\xi_j = \frac{F_+(\alpha_j)}{\alpha_j^2 K_+(\alpha_j)}, \quad \eta_j = \frac{F_-(-\alpha_j)}{\alpha_j^2 K_-(-\alpha_j)}, \quad c_{jm} = \frac{e^{i\alpha_m L} K_+^2(\alpha_m) K_1(\alpha_m)}{(\alpha_m+\alpha_j) K_2'(\alpha_m)},$$

$$f_j^{(1)} = i\sum_{m=-2}^{\infty} \frac{e^{i\alpha_m(L-x_*)} K_+(\alpha_m) K_1(\alpha_m)}{\alpha_m^2 K_2'(\alpha_m)(\alpha_m+\alpha_j)}, \quad f_j^{(2)} = i\sum_{m=-2}^{\infty} \frac{e^{i\alpha_m x_*} K_+(\alpha_m) K_1(\alpha_m)}{\alpha_m^2 K_2'(\alpha_m)(\alpha_m+\alpha_j)},$$

We can explicitly express $\xi_j$ and obtain the matrix equation for the vector $\boldsymbol{\eta}$: $(E - C^2)\boldsymbol{\eta} = \boldsymbol{f}$ where $E$ is the unit matrix, $C$ is the matrix with elements $c_{jm}$, $\boldsymbol{f} = C\boldsymbol{f}^{(1)} + \boldsymbol{f}^{(2)}$.

The pressure and plate deflection are given as

$$p(x) = \sum_{j=-2}^{\infty} \frac{K_1(\alpha_j)}{K_2'(\alpha_j)}\Big[-ie^{i\alpha_j|x-x_*|} + \alpha_j^2 K_+(\alpha_j)\Big(\eta_j e^{i\alpha_j x} + \xi_j e^{i\alpha_j(L-x)}\Big)\Big]$$

$$w(x) = -\sum_{j=-2}^{\infty} \frac{\alpha_j \mathrm{th}(\alpha_j H)}{K_2'(\alpha_j)}\Big[-ie^{i\alpha_j|x-x_*|} + \alpha_j^2 K_+(\alpha_j)\Big(\eta_j e^{i\alpha_j x} + \xi_j e^{i\alpha_j(L-x)}\Big)\Big]$$

We consider now the case of distributed load $q(x) \neq 0$ for $x \in [x_1, x_2]$. If we multiply the obtained solution by $q(x_*)$ and integrate the result with respect to $x_*$ over interval $[x_1, x_2]$, then we find the solution for the general case.

## 4. Short-wave approximation

We consider the case when $L >> 1$. Then all elements of the matrix $C$ are exponentially small except for the column $m = 0$ corresponding to the real root $\alpha_0$. Keeping only the distinguished elements and replacing others with zero, we obtain the following explicit formulae

$$\eta_j = c_{j0}\frac{f_0^{(1)} + c_{00} f_0^{(2)}}{1 - c_{00}^2} + f_j^{(2)}, \quad \xi_j = c_{j0}\frac{c_{00} f_0^{(1)} + f_0^{(2)}}{1 - c_{00}^2} + f_j^{(1)}, \quad |c_{00}| = \frac{|\gamma - \alpha_0|}{\gamma + \alpha_0}.$$

So we can expect that the amplitudes $\xi_j$ $\eta_j$ are maximal when $c_{00}$ is real. This occurs when $\alpha_0 L + Arg(K_+^2(\alpha_0)) = \pi k, \quad k = 1, 2, ...$ This condition corresponds to the zero values of the reflected coefficients in the problem of diffraction of surface waves. Meylan [4] has shown that corresponding frequencies are the scattering frequencies.

## 5. Numerical results

Calculations were performed for the plate used in the experiments [5]. External load has the form

$$q(x) = \begin{cases} q_0[1-(x-x_0)^2/s^2], & |x-x_0| < s, \\ 0, & |x-x_0| > s, \end{cases} \tag{9}$$

Fig. 1,a presents the dependence of reflection and transmission coefficients for the diffraction problem on the wave period $T_p$. Fig. 1,b shows the dependence on $T_p$ of forced vibrations of the plate and free surface under the load (9), where $x_0 = L/4$, $s = 0,5$ m. Dashed lines 1, 2 correspond to the free surface elevation in the far field on the left and right from the plate. Solid lines 1, 2, 3 show the plate vibration amplitudes at the left and right edges and at the load center point. We can see the resonance amplification of vibrations of plate and fluid at high values of scattering frequencies. For small frequencies plate deflection is concentrated near the load region, waves reflected from the edges are small.

Performed calculations have shown that for high values of frequency the structural inertia influence is small, and the short-wave approximation is very close to the general solution. The depth influence is essential for low frequencies. For high frequencies the dependance of the vibration amplitudes is weaker.

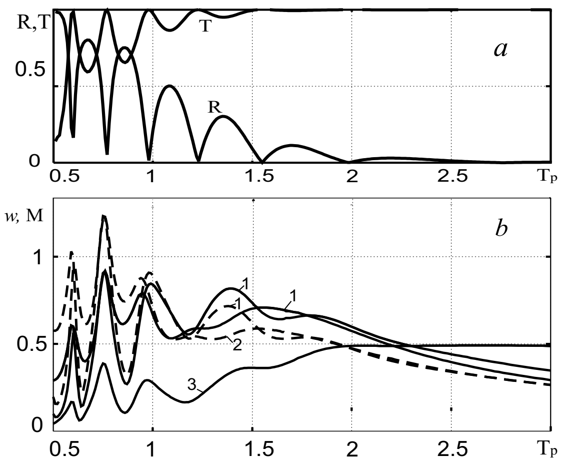

Fig.1.

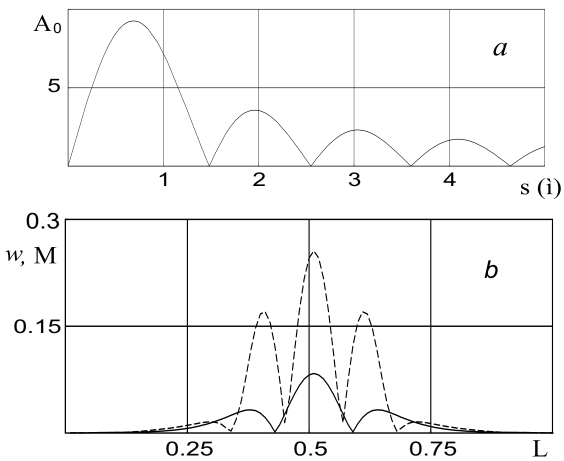

Fig.2.

The existence and conditions of realization of localized vibrations were revealed when the fluid is at rest and vibrations of the plate are localized near the load region. These conditions are

$$A_0 = \int_{x_1}^{x_2} \mathrm{e}^{i\alpha_0 x} q(x)dx = 0$$

and the load domain is far enough from the edges, so that edges are out of the region where decaying modes from the load are essential. Fig.2,a shows the dependance of the value $A_0$ on s. The example of localized vibrations is presented in the fig.2,b for $s = 1.4821$ m, $x_0 = L_0/2$. Solid lines correspond to the plate deflection, dashed lines correspond to the non-dimensional bending moment $M(x) = \beta l|w''(x)|/(Ld)$.

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