Horizontal force on a very thin plate in waves

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1. Introduction

In the analysis of the transverse vibration of a floating structure of very thin plate configuration we usually assumme its draft is zero. This assumption is mathematically legitimate when not only the body's horizontal size but the wavelength of sea waves is very large compared with the draft and it leads to accurate prediction as far as the transverse vibration is concerened. For the horizontal wave force acting on the plate structure, however, we have to consider the effect of small but finite draft: we need careful analysis of the flow close to the edge of the plate .

The flow predicted with the assumption of zero draft is interpreted as an outer flow and the local flow within the small distance from the edges matching to the outer flow is to be found. A result of this attempt is presented and applied to compute the drift force on a thin plate vibrating in waves by direct integration of the pressure on the body surface. A result with this idea reported by the present author (Ohkusu 2003) is not satisfactory in that the matching is not complete. In this paper this inadequacy is removed.

We assume a floating thin plate is very long and the flow when it is in beam seas is taken appropriately to be two dimensional with generator paralell to the z axis. The x and z axese are on the mean free surface and the y axis directs vertially upwards; the width of the plate is 2 and the draft d (Fig.1). The gravitaional constant g is 1 in our unit system. Monochromatic wave of the frequency ω and the wave number k is incident at beam; the water depth is deep and $k = \omega^2$. Assumption of our analysis is that the wave length is very small ($k >> 1$) while the draft is small relatively the wavelength ($kd << 1$).

2. The local flow near the side surface

For the flow induced by the transverse vibration of a thin plate floating on the free surface, the outer solution is approximated by the flow of the plate of zero draft because its draft is small compared with other length scales. This flow, when $z = x + \mathrm{j}y$ (j stands for imaginary number) is close to $z = 1$ ($k(x-1) = O(1), ky = 0, x > 1$), is given by

$$\begin{aligned}\phi_R + \phi_D + \phi_e &= i\omega\sqrt{2}A^+ \left\{ \mathrm{Re}\left[\mathrm{j}(z-1)^{\frac{1}{2}}\right] + G_0(k(x-1),0)/\sqrt{k} \right\} \\ &\quad + \frac{i}{\omega}\left\{ \sqrt{\frac{1}{k\pi}}\left[\sqrt{\frac{1+z}{1-z}} - \sqrt{\frac{2}{1-z}}\right] + H(k(x-1),0) \right\} \\ &\quad + \oint_{-1}^{1} \frac{f(\xi)}{\xi - x} d\xi + G_e(k(x-1),0) \end{aligned} \tag{1}$$

where the fist line on RHS is ϕ_R the velocity potential for radiation, the second line ϕ_D the velocity potential for diffraction and the third line ϕ_e an asymptotic form of eigen function as $k >> 1$. The function $G_0(x,y)$ and $H(x,y)$ are functions of a ltttle complicated mathematical form (Ohkusu 2003).

ϕ_R and ϕ_D in eq. (1) were constructed from the asymptotic solutions of Leppington (1972), while ϕ_e is newly introduced for achieving the matching to the local flow solution. A^+ represents the effect of the vibration $\zeta(x)e^{i\omega t}$ of the plate which is computed by

$$A^+ = \frac{1}{\pi}\int_{-1}^{1}\sqrt{\frac{1+\xi}{1-\xi}}\zeta(\xi)d\xi \tag{2}$$

We choose a continuous function $f(x)$ of $f(1) = A$ and $f(-1) = B$, then ϕ_e has a logarithmic singularity of the magnitude A at $x = 1$. A and B are unknown constants to be determined later in the matching to the inner local flow. $G_e(x, y)$ is a solution of

$$\frac{\partial G_e}{\partial y} = 0 \text{ for } x < 0, y = 0 \tag{3}$$

$$kG_e - \frac{\partial G_e}{\partial y} = \oint_{-1}^{1}\frac{f(\xi)}{\xi - x}d\xi \text{ for } x > 0, y = 0 \tag{4}$$

G_e satisfying the radiation condition is obtained assuming an appropriate $f(x)$ (see Appendix)

In order to evaluate the fluid pressure on the side surface of the thin plate, we have to find the local flow in the inner domain of the size $O(d)$ near $x = 1$, near enough for the finite draft d to be felt.

First we obtain $\phi_R + \phi_D + \phi_e$ at $z = 1 + O(kd)$. It requires somewhat lengthy algebra but the result is

$$\begin{aligned}\phi_R + \phi_D + \phi_e &= \sqrt{\frac{2}{k}}e^{i(k+\pi/8)} + i\omega\sqrt{\frac{2}{k}}A^+G_0(0,0) \\ &- \left(\omega\sqrt{\frac{1}{k\pi}}A^+ + 2\sqrt{\frac{2}{\pi}}e^{i(k+\pi/8)}\right) X\cdot kd\log kd \\ &+ \left(A^*\log|X| + A^*G_e^1(0,0) + B^*G_e^2(0,0)\right) kd\log kd + O(kd)\end{aligned} \tag{5}$$

where the coordinate is rescaled as $X = k(x-1)/kd$. A and B are replaced by $A^* kd\log kd$ and $B^* kd\log kd$ respectively for the convenience of matching to be shown later.

For the local flow in the inner domain near the plate edge, the free surface condtion is approximated by the rigid surface. The local flow must match to eq.(5) at $X \to \infty$. Considering that eq.(5) is composed of the constant terms and the uniform flow term except for the logarithmic singularity, we come up with the local flow in the form of

$$\begin{aligned}\phi &= i\sqrt{\frac{2}{k}}e^{i(k+\pi/8)} + \sqrt{\frac{\pi}{2}}A^+e^{i\pi/8} + [A^*C_1 + B^*C_2]kd\log kd \\ &+ \left(2\sqrt{\frac{2}{\pi}}e^{i(k+\pi/8)} + \sqrt{\frac{1}{\pi}}A^+e^{i\pi/8}\right) kd\log kd\cdot\psi(X,Y)\end{aligned} \tag{6}$$

where ψ is the flow for unifrom flow on a step as shown in Fig.2

$$\psi = \mathrm{Re}\left[\frac{1}{\pi}\cosh\xi\right],\ X + \mathrm{j}Y = -\frac{1}{\pi}(\xi + \sinh\xi) \tag{7}$$

Since

$$\psi \sim -X - \frac{1}{\pi}\log X + \text{const. at } X \to \infty \tag{8}$$

it is straightforward to complete the matching by putting

$$A^* = -\frac{1}{\pi}\left(2\sqrt{\frac{2}{\pi}}e^{i(k+\pi/8)} + \sqrt{\frac{1}{\pi}}A^+ e^{i\pi/8}\right) \tag{9}$$

B^* is determined similarly by analysis of the flow at the left edge of the plate at $x = -1$.

3. Drift force

Steady force acting on the surface RR' (its sign is negative) is computed by

$$\begin{aligned} D_{SR} &= -\overline{\int_{\zeta e^{i\omega t}-1}^{\eta e^{i\omega t}} P(Y)dY} \\ &= -\frac{\rho}{4}\left|\eta(1)-\zeta(1)\right|^2 \\ &\quad +\frac{\rho}{2}\,\mathrm{Re}\left[\zeta(1)^* i\omega(\phi(0,0)-\phi(0,-1))\right] \\ &\quad +\frac{\rho}{4}\int_{-1}^{0}\left(|\phi_X|^2+|\phi_Y|^2\right)dY + \text{higher order} \end{aligned} \tag{10}$$

Here $\eta(1)e^{i\omega t}$ and $\zeta(1)\ e^{i\omega t}$are the wave elevation at the right edge of the plate $x = 1$ and the vibration of the plate at this location respectively; the bar on the integral stands for the mean over a period and the asterisk the complex conjugate; ϕ_X and ϕ_Y are X and Y component of the fluid velocity on the surface RR'.

In eq.(10) the lowest order term given by the first term will be understood as the relative wave elevation effect resulting from the pressure above $y = 0$ and the second lowest part coming mainly from the second term is of $O(\ kd\log kd\ )$ higher order than the first term. The third term is of much higher order and neglected. D_{SR}, if we retain the terms to the order $O(\ kd\log kd\ )$, is written as:

$$\frac{D_{SR}}{\rho g\delta^2} = -\frac{1}{4}\left|\sqrt{2}e^{i(k+\pi/8)} - i\sqrt{\frac{k\pi}{2}}A^+e^{i\pi/8} - \zeta(1)\right|^2 + D_1\cdot kd\log kd \tag{11}$$

Here δ is the amplitude of incident wave and $A^+$and ζ are normalized by δ. And

$$D_1 = -\frac{1}{2}\,\mathrm{Re}\left[\alpha\left(\sqrt{2}e^{i(k+\pi/8)} - i\sqrt{\frac{k\pi}{2}}A^+e^{i\pi/8} - \zeta(1)\right)^* + 2\left(\frac{i\sqrt{k}}{\pi}\zeta(1)\right)\right] \tag{12}$$

Similarly the drift force acting on the plate at lee-side surface is given by

$$\frac{D_{SL}}{\rho g\delta^2} = \frac{1}{4}\left|-i\sqrt{\frac{k\pi}{2}}A^-e^{i\pi/8} - \zeta(-1)\right|^2 + D_2\cdot kd\log kd \tag{13}$$

Other contribution D_B to the drift force will come from the pressure acting on the bottome surface; the pressure of the lowest order is computed by the sum of ϕ_R given by (6) and ϕ_R given by (7):

$$\frac{D_B}{\rho g\delta^2} = -\frac{1}{2}\int_{-1}^{1}\mathrm{Re}\left[(-i\omega\phi_R - i\omega\phi_D - \zeta)\frac{d\zeta^*}{dx}\right]dx + D_3\cdot kd\log kd \tag{14}$$

Here the detailed expressions for D_2 and D_3 are omitted for the sake of brevity.

The drift force acting on a thin but a finite draft plate is given by the summation of eqs.(12), (13) and (14).

3. Numrical computation

Once the vibration ζ is computed, the drift force is readily evaluated. The hydrodynamic force to induce the plate transverse vibration is evaluated by a mathematical expression of ϕ_R and ϕ_D corresponding eq. (1). Formulating equation of the vibration with this force is straightforward. In solving the vibration equation, however, the analytical expression of ϕ_R does not necessarily produce so much computational advantage since ϕ_R is given in the form of a functional of ζ and the main part of equation of the vibration is the fourth derivative of ζ. We employ a numerical method proposed by the present author (Namba and Ohkusu 1999) to compute the vibration ζ, with which one is able to compute the vibration directly without two-step process of computing hydrodynamic force first and then solving the vibration equation.

Numerical results of the drift force with this ζ substituted into eqs.(12), (13) and (14) will be presented at the Workshop.

References

(1) Leppington,FG: On the radiation and scattering of short surface waves, Part 1, J.F.M vol.56 (1972)
(2) Ohkusu,M: Drift force on a thin plate in waves, Proc. Hydroelasticity03 (2003)
(3) Namba,Y. and Ohkusu,M. :Hydroelastic behavior of floating artificial islands in waves, Int.J. of Offshore and Polar Engineering, no.2. (1999)

Appendix

$$G_e(x,y) = A^* G_e^1(x,y) + B^* G_e^2(x,y) \tag{15}$$

$$\Lambda(s) = s^{\frac{1}{2}}(1+s^2)^{\frac{1}{4}} \exp\left[-\frac{1}{\pi}\int_0^s \frac{\log u}{1+u^2}du\right] \tag{16}$$

$$\begin{aligned} G_e^n(x,y) = & -\frac{i}{\sqrt{2}}e^{-i(x-\pi/8)+y}\int_0^\infty \Theta_n(s)\frac{\Lambda(s)}{(1+s^2)}ds \\ & -\frac{1}{2}\int_0^\infty \Theta_n(s)\frac{\cos sy - s\sin sy}{(1+s^2)}e^{-sx}ds \\ & -\frac{1}{2\pi}\int_0^\infty ds\int_0^\infty \Theta_n(s)\frac{\Lambda(s)(\tau\cos\tau y+\sin\tau y)e^{-\tau x}}{(s+i)(\tau-s)(\tau-i)\Lambda(\tau)}d\tau \end{aligned} \tag{17}$$

$$\Theta_n(s) = \left(\delta_{1n} + \frac{(-1)^{n+1}}{2}\frac{d}{ds}\right)(e^{-2s}-1) \tag{18}$$

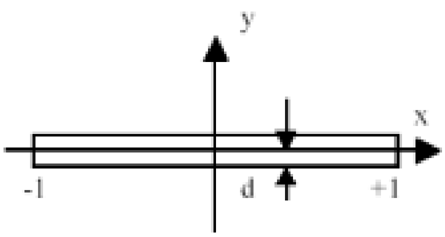

Fig.1

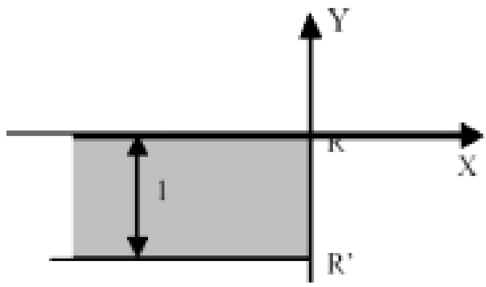

Fig.2