Wave Drift Forces and Moments on Two Ships with Side-by-Side Arrangement

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1. Introduction

For the numerical simulation of motions of two ships in close proximity or of a LNG carrier moored to a FPSO, accurate prediction of the wave drift force and moment on each ship is of great importance. In particular, hydrodynamic interactions are expected to be large and complex because of narrow gap between side-by-side two ships, which must be taken into account in the design of the mooring system.

It is well known that the two different methods are available for computing the wave drift forces: the near-field method based on the direct pressure integration and the far-field method based on the momentum-conservation principle. Since the prediction of the forces on each ship is prerequisite in the present study, the near-field method must be used. Although some papers have been published on this topic, it is rather difficult to keep high numerical accuracy. In the present study, a higher-order boundary element method (HOBEM) is applied to solve for the velocity potential (pressure) on the wetted surface of ships directly.

Recently, *Fang et al. (2002)* proposed a far-field method for computing the wave drift force on each ship separately. However this method is not exact, which is demonstrated in this paper by computing with both of the near- and far-field methods and by comparing their results to the experiments newly conducted.

2. Formulation and Solution Method

Assuming the inviscid flow with irrotational motion, the velocity potential is introduced, and the linearized boundary-value problem for two ships is firstly solved. Based on the perturbation method, the wave-induced steady forces (which are of second order in the wave amplitude) can be computed with only the first-order quantities.

The flow of fluid is assumed to be periodical with circular frequency ω, and by linear decomposition the velocity potential is expressed in the form

$$\Phi(x,y,z;t) = \mathrm{Re}\left[\frac{g\zeta_w}{i\omega} \left\{ \varphi_D - Ka \sum_{j=1}^{6} \sum_{\ell=A}^{B} \frac{X_{j\ell}\epsilon_j}{\zeta_w} \varphi_{j\ell} \right\} e^{i\omega t} \right] \tag{1}$$

where

$$\varphi_D = \varphi_I + \varphi_S \tag{2}$$

$$\varphi_I = Z_0(z)\, e^{-ik_0(x\cos\beta + y\sin\beta)} \tag{3}$$

$$Z_0(z) = \frac{\cosh k_0(z-h)}{\cosh k_0 h}\,, \quad K = \frac{\omega^2}{g} = k_0 \tanh k_0 h \tag{4}$$

and g is the gravitational acceleration; ζ_w is the amplitude of the incident wave; a is the characteristic length scale for nondimension; j is the mode number of six degrees of freedom in the radiation problem and the suffix $\ell = A$ and B denote the number of ships; $\epsilon_j = 1$ for $j = 1 \sim 3$ and $\epsilon_j = a$ for $j = 4 \sim 6$; $X_{j\ell}$ is the complex amplitude of the j-th mode of the ℓ-th ship; and h is the water depth which is assumed constant. The diffraction potential φ_D is defined as the sum of the incident-wave potential φ_I plus the scattering potential φ_S. β is the angle of incident wave relative to the positive x-axis, with $\beta = 180^\circ$ defined as the head wave.

The diffraction potential φ_D and the radiation potential $\varphi_{j\ell}$ are sought to satisfy the following body boundary condition

$$\frac{\partial\varphi_D}{\partial n} = 0\,, \qquad \frac{\partial\varphi_{j\ell}}{\partial n} = n_{jk}\,\delta_{k\ell} \quad (j = 1 \sim 6;\ k,\, \ell = A \text{ or } B) \tag{5}$$

where n_{jk} is the j-th component of the normal vector on the k-th skip and $\delta_{k\ell}$ denotes Kroeneker's delta.

Based on the boundary element method using the free-surface Green function $G(\mathrm{P};\mathrm{Q})$, the velocity potentials satisfying (5) can be obtained by solving the following integral equation:

$$
\begin{aligned}
C(\mathrm{P})\varphi_m(\mathrm{P}) + \sum_{k=A}^{B} \iint_{S_k} \varphi_m(\mathrm{Q}) \frac{\partial}{\partial n_{\mathrm{Q}}} G(\mathrm{P};\mathrm{Q})\, dS \\
= \begin{cases} \varphi_I(\mathrm{P}) & \text{for } m = D \\ \displaystyle\sum_{k=A}^{B} \iint_{S_k} \frac{\partial \varphi_m(\mathrm{Q})}{\partial n_{\mathrm{Q}}} G(\mathrm{P};\mathrm{Q})\, dS & \text{for } m = j\ell \end{cases}
\end{aligned}
\tag{6}
$$

where $\mathrm{P} = (x, y, z)$ is the field point, $\mathrm{Q} = (x', y', z')$ is the source point on the body surface, and $C(\mathrm{P})$ is the solid angle which can be computed numerically in the HOBEM. It should be noted that, within the framework of linear potential theory, hydrodynamic interactions are exactly taken into account in numerical solutions to be obtained from (6).

Once the velocity potentials on the body surface are obtained, it is straightforward to compute the 1st order hydrodynamic forces as follows:

$$
E_i^k = \iint_{S_k} \varphi_D\, n_{ik}\, dS\,, \qquad \mathcal{F}_{ij}^{k\ell} = A_{ij}^{k\ell} - i\, B_{ij}^{k\ell} = -\iint_{S_k} \varphi_{j\ell}\, n_{ik}\, dS \tag{7}
$$

Here E_i^k is the wave-exciting force in the i-th mode of k-th ship, and $A_{ij}^{k\ell}$ and $B_{ij}^{k\ell}$ are the added-mass and damping coefficient respectively in the i-th direction of k-th ship due to the j-th mode of motion of ℓ-th ship.

With hydrodynamic forces computed above and hydrostatic restoring-force coefficients, the complex motion amplitude $X_{j\ell}$ can be readily computed by solving the coupled motion equations.

3. Calculation of Wave Drift Forces

3.1 Far-field method

Let us denote the nondimensional total velocity potential in braces of (1) as $\varphi(\mathrm{P})$, which is the sum of the incident-wave potential $\varphi_I(\mathrm{P})$ and the disturbance potential due to the presence of ships (denoted as $\psi(\mathrm{P})$).

The asymptotic expression of $\psi(\mathrm{P})$ at far field ($r \to \infty$) can be obtained from (6) by putting $C(\mathrm{P}) = 1$, with the result

$$
\psi(\mathrm{P}) \equiv \varphi(\mathrm{P}) - \varphi_I(\mathrm{P}) = \sum_{k=A}^{B} \psi^k(\mathrm{P}) \tag{8}
$$

$$
\begin{aligned}
\psi^k(\mathrm{P}) &= \iint_{S_k} \Big\{ \frac{\partial \varphi(\mathrm{Q})}{\partial n_{\mathrm{Q}}} - \varphi(\mathrm{Q}) \frac{\partial}{\partial n_{\mathrm{Q}}} \Big\} G(\mathrm{P};\mathrm{Q})\, dS \\
&\simeq \sum_{m=-\infty}^{\infty} \mathcal{A}_m^k\, Z_0(z)\, H_m^{(2)}(k_0 r_k)\, e^{-im\theta_k} \qquad \text{as } r_k \to \infty
\end{aligned}
\tag{9}
$$

where (r_k, θ_k, z) is the cylindrical coordinate system with the origin placed at the center of the k-th ship, and the coefficient $\mathcal{A}_m^k$ is related to the amplitude of outgoing progressive waves, given by

$$
\left.
\begin{aligned}
\mathcal{A}_m^k &= \frac{i}{2} C_0 \iint_{S_k} \Big\{ \frac{\partial \varphi}{\partial n} - \varphi \frac{\partial}{\partial n} \Big\} Z_0(z)\, J_m(k_0 r_k)\, e^{im\theta_k}\, dS \\
C_0 &= \frac{k_0^2}{K + (k_0^2 - K^2)h}
\end{aligned}
\right\}
\tag{10}
$$

Furthermore, in terms of Graf's addition theorem, the Hankel function $H_m^{(2)}(k_0 r_k)$ can be expressed with the global cylinderical coordinate system (r, θ, z); thereby the disturbance potential $\psi(\mathrm{P})$ can be expressed as

$$
\psi(\mathrm{P}) = \psi^A(\mathrm{P}) + \psi^B(\mathrm{P}) \simeq \sum_{m=-\infty}^{\infty} \mathcal{A}_m\, Z_0(z)\, H_m^{(2)}(k_0 r)\, e^{-im\theta} \qquad \text{as } r \to \infty \tag{11}
$$

where

$$
\mathcal{A}_m = \sum_{\ell=A}^{B} \sum_{n=-\infty}^{\infty} \mathcal{A}_n^\ell\, J_{n-m}(k_0 L_{\ell o})\, e^{-i(n-m)\alpha_{\ell o}} \tag{12}
$$

Here $L_{\ell o}$ and $\alpha_{\ell o}$ are the distance and azimuth angle, respectively, of the origin of the global coordinate system measured with the local coordinate system of the ℓ-th ship.

$\varphi_I(\mathrm{P})$ given by (3) can also be expressed with the global cylindrical coordinate system, which is added to (11), and the resulting total velocity potential takes the following form as $r \to \infty$:

$$\varphi(\mathrm{P}) = \sum_{m=-\infty}^{\infty} \left\{ \alpha_m J_m(k_0 r) + \mathcal{A}_m H_m^{(2)}(k_0 r) \right\} Z_0(z)\, e^{-im\theta} \tag{13}$$

where $\alpha_m = \exp\left\{im(\beta - \pi/2)\right\}$.

Applying the principle of linear and angular momenta conservation and performing necessary integrals with respect to z and θ at a certain distance of r, the calculation formulae for the wave drift forces $(\overline{F}_x - i\,\overline{F}_y)$ in the horizontal plane and the moment $(\overline{M}_z)$ about the vertical axis can be obtained as follows:

$$\frac{\overline{F}_x - i\,\overline{F}_y}{\frac{1}{2}\rho g \zeta_w^2 a} = \frac{i\,k_0}{C_0 K a} \sum_{m=-\infty}^{\infty} \Big[2\,\mathcal{A}_m \mathcal{A}_{m+1}^* + \alpha_m \mathcal{A}_{m+1}^* + \alpha_{m+1}^* \mathcal{A}_m \Big] \tag{14}$$

$$\frac{\overline{M}_z}{\frac{1}{2}\rho g \zeta_w^2 a^2} = -\frac{2}{C_0 K a^2} \sum_{m=-\infty}^{\infty} m \Big[|\mathcal{A}_m|^2 + \mathrm{Re}\big(\alpha_m \mathcal{A}_m^*\big) \Big] \tag{15}$$

where the asterisk in the superscript stands for the complex conjugate.

These formulae include only the coefficients of the disturbance potential, $\mathcal{A}_m$, and of the incident-wave potential, α_m. Therefore accurate results may be given with these formulae. However it should be noted that these are the total wave drift force and moment acting on both ships. We can also see from (14) and (15) that the drift force and moment consists of quadratic terms in the disturbance wave and cross terms between the incident wave and the disturbance wave.

3.2 Near-field method

Derivation of the calculation equation for the 2nd order steady force on the basis of the near-field method is somewhat lengthy. Referring to the established 2nd-order theory using consistent perturbation scheme, the time averaged 2nd-order steady force in the i-th direction ($i = 1 \sim 3$) of the m-th ship can be computed by

$$\begin{aligned}
\frac{\overline{F}_i^m}{\frac{1}{2}\rho g \zeta_w^2 a} = \frac{1}{2Ka} & \iint_{S_m} |\nabla\varphi|^2 n_i\, dS \\
& - \frac{1}{2} \int_{WL} \left| \varphi - \left\{ \frac{X_3}{\zeta_w} + \frac{X_4 a}{\zeta_w} y - \frac{X_5 a}{\zeta_w}(x - x_G) \right| \right.^2 \frac{n_i}{\sqrt{1 - n_3^2}}\, d\ell \\
& + \mathrm{Re} \iint_{S_m} \left\{ \frac{X_j}{\zeta_w} + \epsilon_{jk\ell} \frac{X_{k+3} a}{\zeta_w}(x_\ell - x_{\ell G}) \right\} \frac{\partial \varphi^*}{\partial x_j}\, n_i\, dS \\
& - Ka\, M\, \mathrm{Re}\Big[\epsilon_{ijk} \frac{X_{j+3} a}{\zeta_w} \frac{X_k^*}{\zeta_w} \Big] - \frac{A_W}{a^2} \mathrm{Re}\Big[\frac{X_6 a}{\zeta_w} \frac{X_4^* a}{\zeta_w} (x_F - x_G) \Big] \delta_{i3}
\end{aligned} \tag{16}$$

where x_ℓ ($\ell = 1, 2, 3$) is used to mean (x, y, z); ϵ_{ijk} is the alternating tensor; M and A_W are respectively the mass and the water-plane area of the ship to be computed; and x_F and x_G are the x-ordinates of the centers of floatation and gravity respectively.

The total wave drift force on both ships, corresponding to (14), can be readily computed by simple summation of the force on each ship.

4. Numerical Computation and Experiment

The integral equation (6) was solved by the HOBEM using 9-point quadratic representations for both the surface geometry and the velocity potential. With the HOBEM, it can be relatively accurate in computing the spatial derivatives of the velocity potential, $\nabla\varphi$, and the relative wave elevation along the intersection between the body and free surfaces which are necessary in the near-field method as described in (16).

Numerical computations and experiments were carried out for the side-by-side arrangement of a modified Wigley model (*Ship-A*) and a rectangular barge model (*Ship-B*). Here, the modified Wigley model used in the experiment is expressed mathematically as

$$\left. \begin{array}{c} \eta = \big(1 - \xi^2\big)\big(1 - \zeta^2\big)\big(1 + 0.2\xi^2\big) + \zeta^2\big(1 - \zeta^8\big)\big(1 - \xi^2\big)^4 \\ \xi = \dfrac{2x}{L}\,, \quad \eta = \dfrac{2y}{B}\,, \quad \zeta = \dfrac{z}{d} \end{array} \right\} \tag{17}$$

Both models are $L = 2.0$ m in length, $B = 0.3$ m in breadth, and $d = 0.125$ m in draft. They were set in the beam-wave condition ($\beta = 90°$), with the separation distance between the longitudinal centerlines of each ship (which is denoted as $S = |x_{OA} - x_{OB}|$) set equal to $S = 1.097$ m and $S = 1.797$ m. Experiments were carried out for (1) the forced heave oscillation tests (with *Ship-A* oscillated and *Ship-B* fixed, and vice versa) and (2) the measurement of the wave-exciting force and the 2nd-order steady force with both ships fixed, corresponding to the diffraction problem.

5. Results

Because of limited space of the paper, only a few of the results are shown here, and others including the comparison with experiments will be presented at the Workshop.

Figures 1 and 2 are the wave-induced steady force in the horizontal (sway) direction for the case of $S = 1.797$ m and $\beta = 90°$, (*Ship-A* in Fig. 1 is located in the weather side and *Ship-B* in Fig. 2 is in the lee side). Figure 3 shows the summation of the steady sway forces acting on both ships. The results by the near-field method (solid line) is in good agreement with the experiments and the result by the far-field method (dashed line). The dotted lines are the results based on the approximated far-field method proposed by *Fang et al. (2002)*, which were computed from (14), with $\mathcal{A}_m$ and α_m replaced by corresponding values of the k-th ship. These results are obviously not correct but may give a rough estimation without using rather complicated near-field method. Figure 4 shows the wave-induced steady force in the vertical (heave) direction on the modified Wigley model (*Ship-A*) located in the weather side. Good agreement is found in this case, too.

References

Fang, M. C. and Chen, G. R.: On Three-Dimensional Solutions of Drift Forces and Moments Between Two Ships in Waves, *Journal of Ship Research*, Vol. 46, No. 4, pp. 280–288, 2002.

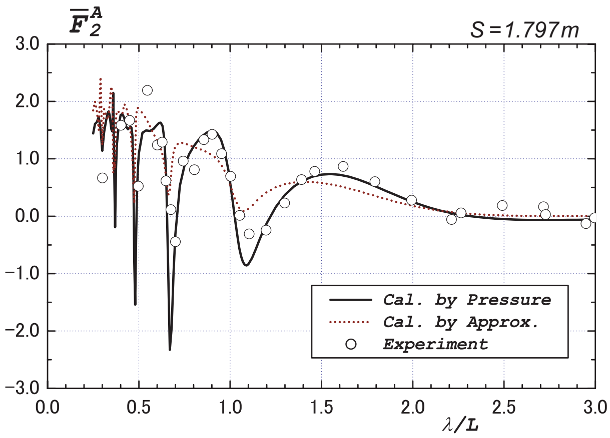

Fig. 1 Steady sway force on *Ship-A* in the weather side of beam wave ($\beta = 90°$), $S = 1.797$ m

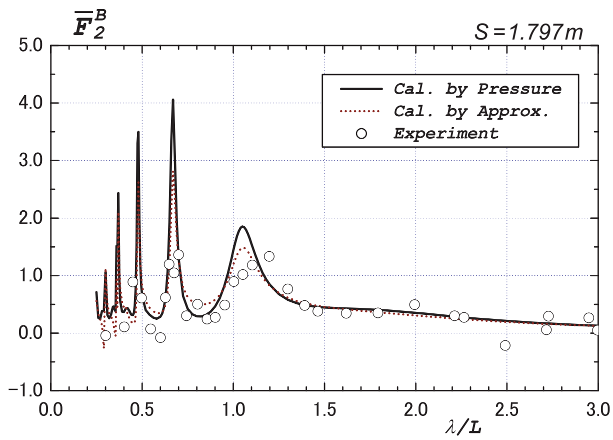

Fig. 2 Steady sway force on *Ship-B* in the weather side of beam wave ($\beta = 90°$), $S = 1.797$ m

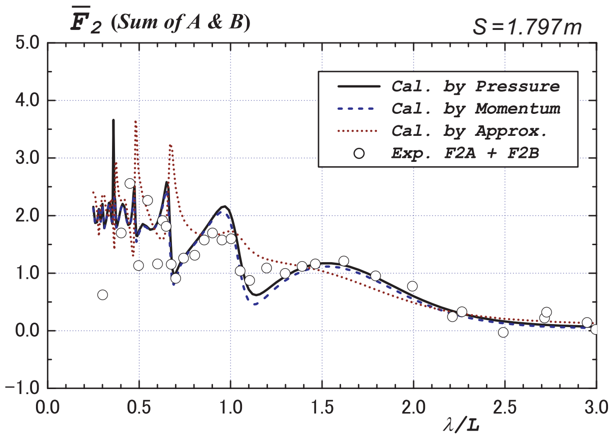

Fig. 3 Total steady sway force on two ships in the beam wave ($\beta = 90°$), $S = 1.797$ m

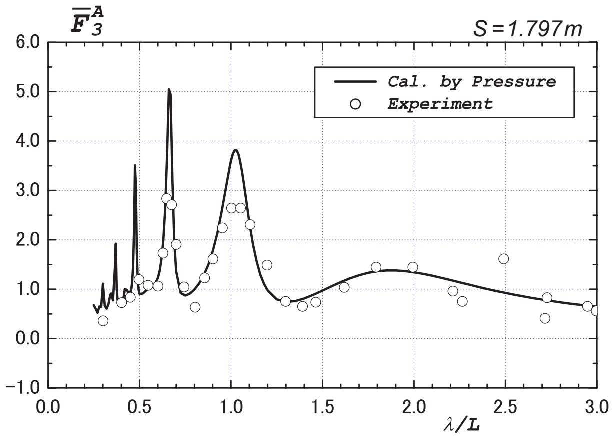

Fig. 4 Steady heave force on *Ship-A* in the weather side of beam wave ($\beta = 90°$), $S = 1.797$ m