A new absorbing boundary conditions for three-dimensional surface wave simulations

DIDIER CLAMOND, DORIAN FRUCTUS & JOHN GRUE

MECHANICS DIVISION, DEPARTMENT OF MATHEMATICS, UNIVERSITY OF OSLO, NORWAY.

E-Mail: didier@math.uio.no, dorianf@math.uio.no & johng@math.uio.no

1 Introduction

Numerical simulations of non-periodic problems extending to infinity in some horizontal directions, require the artificial truncation of the computational domains. The absorption of outgoing waves, at a truncation boundary, is therefore crucial; a physically and numerically efficient technics must thus be used. Various so-called *open boundary conditions* have long been discussed (see [4, 6] for reviews). For the three-dimensional extension of our numerical model [1], we developed an open boundary condition that belongs to the class of so-called "sponge layers" or "numerical beaches". These methods are inspired from fluid flows in porous media.

Fluid flows in porous media can be modelled by the Darcy law $\vec{v} = -\gamma^{-1}\,\mathrm{grad}\,\mathcal{H}$, where $\vec{v} = (\boldsymbol{u}, v)$ is the velocity vector ($\boldsymbol{u} = (u_1, u_2)$ is the horizontal velocity field and v is the vertical one), γ is the permeability and $\mathcal{H}$ is the hydraulic discharge. For homogeneous media (constant γ), this law implies a potential flow, and Darcy's equation can be integrated into a Bernoulli–Darcy equation (BDE):

$$\phi_t + \gamma\phi + \tfrac{1}{2}(\mathrm{grad}\,\phi)^2 + gy + p = B(t), \tag{1}$$

where ϕ is the velocity potential, g is the acceleration due to gravity, y is the upward vertical coordinate, B is a Bernoulli 'constant' and p is the pressure (per unit of mass). Thus, by analogy with flows in porous media, a damping technics for surface waves consists in modifying the surface dynamic condition by adding a term of the form $\gamma\tilde{\phi}$ ($\gamma \neq 0$ where damping is required), i.e.

$$\tilde{\phi}_t + \gamma\tilde{\phi} + g\eta + \tfrac{1}{2}\tilde{\boldsymbol{u}}\cdot\nabla\tilde{\phi} - \tfrac{1}{2}\tilde{v}V = B(t), \tag{2}$$

where tildes denote the quantities at the surface $y = \eta(\boldsymbol{x}, t)$ ($\boldsymbol{x} = (x_1, x_2)$ are the horizontal Cartesian coordinates), ∇ is the horizontal gradient and $V = \phi_n\sqrt{1+|\nabla\eta|^2}$, ϕ_n being the outward normal derivative of ϕ at the surface. It has been noted that this absorber is not very efficient, and to avoid spurious parasitic effects (reflection, emission) both γ and $|\nabla\gamma|$ must be relatively small. Hence, a significant damping is obtained at the expense of using a large damping area.

An alternative damper has then been proposed where $\gamma\tilde{\phi}$ is replaced by $\gamma\phi_n$. Though better, this low-pass damper is not completely satisfactory and other absorbing methods have been proposed. For example, one can couple this spongious absorber with an active paddle [2]. This method is not so easy to implement, however.

We propose here a new spongious-like absorber that has the same advantages as the orginal $\gamma\tilde{\phi}$ — i.e. it damps all the frequencies with the same intensity and it is easy to implement — but it is more effective.

2 Modified Bernoulli–Darcy equation

An efficient absorber must damp the physical quantities (e.g. velocity). In order to derive a physically consistent absorbing equation, we first compute the horizontal gradient of (2)

$$\boldsymbol{U}_t + \gamma \boldsymbol{U} + \tilde{\phi}\nabla\gamma \;=\; -\left[g\eta + \tfrac{1}{2}\tilde{\boldsymbol{u}}\cdot\nabla\tilde{\phi} - \tfrac{1}{2}\tilde{v}V \right], \tag{3}$$

where $\boldsymbol{U}=\nabla\tilde{\phi}$. In the left-hand side of (3), the expression $\boldsymbol{U}_t+\gamma\boldsymbol{U}$ acts as a purely damping operator, while the expression $\boldsymbol{U}_t + \tilde{\phi}\nabla\gamma$ acts as a wave propagator (hyperbolic operator). Therefore, a more efficient equation (i.e. purely absorbing operator) is obtained cancelling the term $\tilde{\phi}\nabla\gamma$ in (3) and integrating back the resulting equation, i.e.

$$\tilde{\phi}_t + \nabla^{-1}\{\gamma\nabla\tilde{\phi}\} + g\eta + \tfrac{1}{2}\tilde{\boldsymbol{u}}\cdot\nabla\tilde{\phi} - \tfrac{1}{2}\tilde{v}V = B(t), \tag{4}$$

where $\nabla^{-1}=\Delta^{-1}\nabla$. The inverse of the horizontal Laplacian operator Δ can be computed easily via Fourier transform. The equation (4) is the modified Bernoulli–Darcy equation (MBDE) we use for absorbing wave. If γ is constant the original BDE is recovered.

This absorber has the following features: *i*) it is easy to implement; *ii*) it is quickly computable; *iii*) it does not require any information about the wave field; *iv*) it can be easily adapted to complex geometries; *v*) it damps all the frequencies with the same intensity. In particular it is efficient in presence of (local) horizontal current, that is not the case with the damper $\gamma\phi_n$.

3 Implementation and tests

We test the MBDE by simulation of the generation and propagation of an axisymmetric wave absorbed by an axisymmetric damping region and by a squared one.

3.1 Equations

For the direct simulation of three-dimensional surface waves, we have to solve numerically two prognostic equations together with the solution of the Laplace equation obtained from the Green function, i.e.

$$\eta_t - V \;=\; 0, \tag{5}$$

$$\tilde{\phi}_t \;+\; \nabla^{-1}\left\{\gamma\nabla\tilde{\phi}\right\} + \,g\eta \;+\; \tfrac{1}{2}\,\tilde{\boldsymbol{u}}\cdot\nabla\tilde{\phi} - \tfrac{1}{2}\,\tilde{v}\,V \;+\; \tilde{p}_{\mathrm{G}} \;=\; 0, \tag{6}$$

$$\int \frac{V'}{(1+D^2)^{\frac{1}{2}}}\,\frac{\mathrm{d}\boldsymbol{x}'}{R} \;=\; 2\pi\tilde{\phi} \;+ \int \frac{\tilde{\phi}'\,(\boldsymbol{R}\cdot\nabla'\eta' - \eta' + \eta)}{(1+D^2)^{\frac{3}{2}}}\,\frac{\mathrm{d}\boldsymbol{x}'}{R^3}, \tag{7}$$

where $\boldsymbol{R}=\boldsymbol{x}'-\boldsymbol{x}$, $R=|\boldsymbol{R}|$, $D=(\eta'-\eta)/R$ and $\tilde{p}_{\mathrm{G}}$ is a forcing pressure at the surface used to generate an axi-symmetric wave from rest. We take

$$\tilde{p}_{\mathrm{G}} = A\sin(\sigma t)\,\mathrm{e}^{-|\boldsymbol{x}|^2/4\lambda^2} \qquad t \geq 0, \tag{8}$$

where A and λ are constants. To transmit as much energy as possible to the far field, given by

$$\eta \sim a\,(\kappa|\boldsymbol{x}|)^{-\frac{1}{2}}\cos(\kappa|\boldsymbol{x}| - \sigma t - \tfrac{1}{4}\pi) \qquad \text{as} \qquad |\boldsymbol{x}| \to \infty, \tag{9}$$

with $\sigma^2\simeq g\kappa$, the wavemaker parameters must be taken as [7]

$$\lambda = 1/\kappa, \qquad\qquad A = g a\, e/\sqrt{8\pi}. \tag{10}$$

The practical numerical resolution of the equations (5)–(7) is given in [1]. (A generalization for internal waves can be found in [5].)

3.2 Numerical experiment

For the simulations, we take $\kappa = 1$, $a = 0.2$ for the wavemaker, and the periodic computational box is squared with side lengths $20\pi/\kappa$. The shapes of the two dampers tested are depicted on figure 1.

Starting from rest the pneumatic wavemaker is activated. The wave field develops and the total energy in the basin increases. When the waves reach the damping zone, they totally absorbed and the energy remains constant forever (figure 2).

The computational domain being periodic, if the waves were not completly absorbed they would leave the domain on one side and re-enter on the other side; the energy would thus increases. Similarly, if the waves were reflected the energy would increase.

4 Conclusion

We have derived a simple absorbing boundary condition that is easy to implement in most of the numerical wave basins and that is efficient for absorbing three-dimensional surface waves. This damper absorbs all the wavelength with the same intensity. If a stronger damping is required for the high frequencies the viscous-like absorber of [3], for example, can be added to the MBDE.

Futher examples, more insights of the method and comparisons with other absorbers will be presented at the workshop.

References

[1] Clamond, D. & Grue, J. 2001 A fast method for fully nonlinear water wave computations. *J. Fluid Mech.* **447**, 337–355.

[2] Clément, A. H. 1996 Coupling of two absorbing boundary conditions for 2D time-domain simulations of free surface gravity waves. *J. Comp. Phys.*, **126**, 139–151.

[3] Israeli, M. & Orszag, S. 1981. Approximation of radiation boundary conditions. *J. Comp. Phys.*, **41**, 115–135.

[4] Givoli, D. 1991. Non-reflective boundary conditions. *J. Comp. Phys.*, **94**, 1, 1–29.

[5] Grue, J. 2002. On four highly nonlinear phenomena in wave theory and marine hydrodynamics. App. Ocean Res., **24**, 5, 261–274.

[6] Romate, J. E. 1992. Absorbing boundary conditions for free surface waves. *J. Comp. Phys.*, **99**, 1, 135–145.

[7] Wehausen, J. V. & Laitone, E. V. 1960. Surface waves, *Handbuch der Physik* **9**, 3, 446–778. Springer-Verlag.

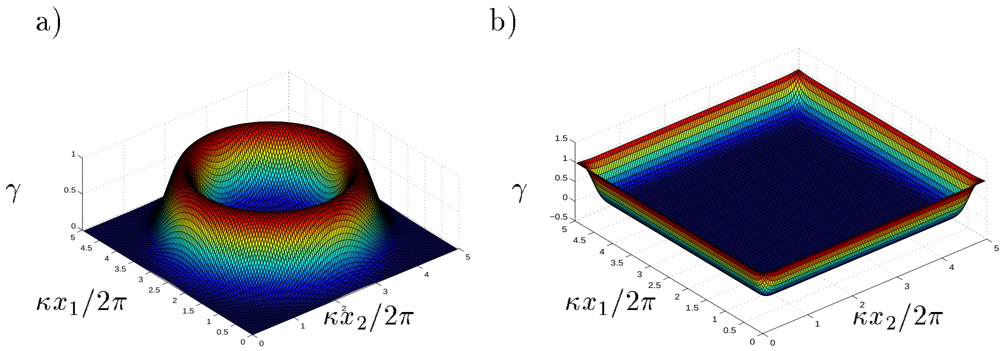

Figure 1: Damper's shapes for a) circular damper and b) squared damper.

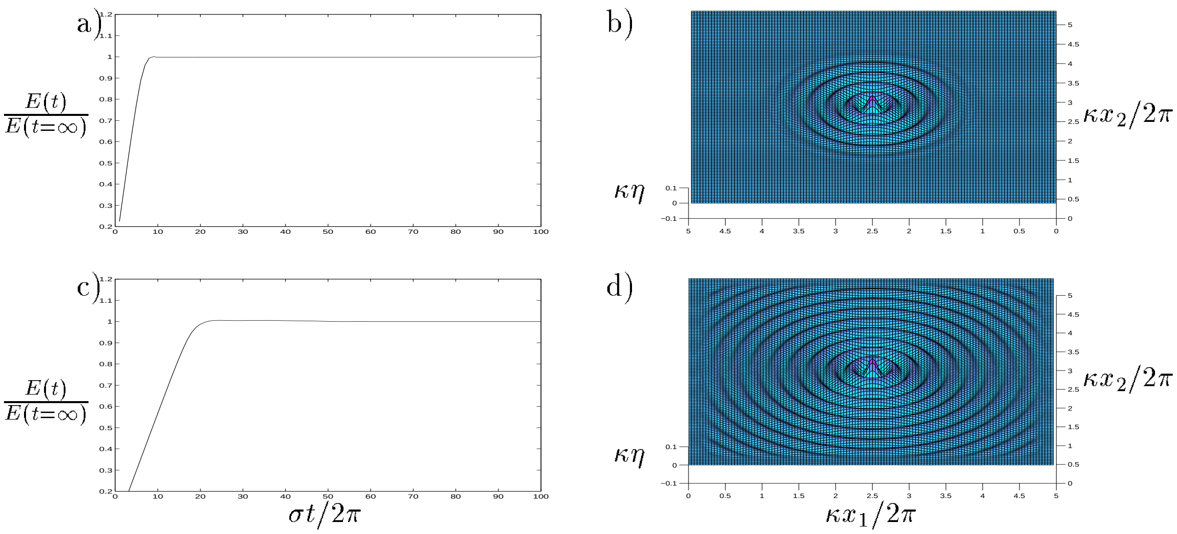

Figure 2: Relative energy evolution and free surface profile at $\sigma t/2\pi = 100$ for two different dampers: a-b) circular damper, c-d) squared damper.

Discusser: H. Bredmose

This looks very interesting. Would you consider it possible to use a similar method to mimic walls at the boundaries instead of damping?

Author's reply:

To model impermeable obstacles, you have to increase damping in order to increase the reflection. This will result in terrible numerical instabilities. I do not think that this method can be used for realistic walls. To do so, you must adapt it.

Discusser: A.H. Clement

What is the typical length of absorption compared to wavelength in your method? How does it compare with other method with regards to this parameter?

Author's reply:

About one characteristic wavelength. Of course, it depends on the wave field. For moderatly steep waves, the damper can be shorter. To avoid reflection, conventional dampers must be slowly varying in space and must thus be quite long, much longer than the new damper.

Discusser: R.W. Yeung

I found your condition rather interesting, certainly it is appealing to dealing with nonlinear problems. In our recent work, Hamilton and Yeung (J. OMAE, vol. 125, p.9-16, 2003) we were able to achieve a "perfectly transparent" condition in three dimensions within the context of linear theory. This was later extended to allow for viscous internal flow. Within the context of linear theory, we have noticed in much of our previous experience that the energy flux is not always directed outward; as a matter of fact, it is often momentarily towards the center of the disturbance, even though the time-averaged value is outward. It is not too clear that your proposed condition has this "bi-directional" transmission property even just for the case of linear waves. If that is the case, then only an approximate state of the linear solution will be achieved. Can you comment? Thank you.

Author's reply:

Our damper is not "perfectly transparent" in theory. If too strong, the waves are partially reflected.

The problem with the "perfectly transparent" conditions we tested, is that they damp very little. This is a problem for periodic domains. Therefore, very large dampers are required for a significant damping, that is computationally expensive.

If the new damper is properly designed (that depends of course on the wave field), the damping effect is strong and the reflection is not significant. Our damper is thus "perfectly transparent" in practice.