SYNCHRONOUS EXCITATION OF TRAPPED MODES ALONG VENICE STORM BARRIER

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1 Introduction

To protect the islands in the Venice Lagoon against frequent flooding by storm tides from Adriatic Sea, four flood barriers have been designed for the four inlets along the outer islands (Chioggia, Malamocco, Lido-Triporti and Lido-Saint Nicolo). Each barrier consists of a series of 19 or 20 hollow gates which are closely aligned and hinged along a common axis fixed at the seabed, and are not connected otherwise. Each gate is a hollow box about 20 m long and $4 \sim 5$ m thick; its height ranges from 14 m to 25 m depending on the local depth of the inlet. In calm weather, all gates will be filled with enough water and lowered into their housings on the seabed so as to allow normal shipping traffic. Before a storm, the gates will be raised, by compressing air into the hollow chambers, to an inclination of $40° \sim 50°$, and act together as a dam to maintain a water-level difference up to 2 m. In order to minimize the wave force tranmitted to the relatively soft seabed, all gates will be allowed to swing to and fro. Guided by long and parallel jetties normal to the coast, waves are expected to approach each barrier normally, and all gates are expected to oscillate in unison and keep out the high water during the storm (estimated to be $4.5 \sim 5$ hours).

For the multi-gate system, experiments either in a long wave flume or a rectangular basin of finite width have revealed an unexpected phenomenon (Consorzio Venezia Nuova Report, 1988). The gates spanning across the full width of the flume or basin were excited by normally incident long-crested waves of a single frequency. It was found that at certain frequency ω the neighboring gates oscillate in opposite phase at the frequencies $\omega/2$. The physical source of resonance has been identified by Mei et al.,(1994) who used the simplified model of vertical gates. In particular, the observed resonance has been shown to be an eigenmode with energy trapped near the barrier, similar to edge waves on a sloping beach. The phenomenon here owes its origin to the articulated structure, and is somewhat different from wave trapping on a sloping beach or around periodic and rigid bodies (Evans & Linton, 1997, etc.). Later, nonlinear theories have been developed by Sammarco, et al. (1997a,b) for the forced oscillation of vertical gates due to incident waves of twice the frequency, only for the severest mode where each gate moves in opposite phase with its immediate neighbors. For inclined gates in housings of complex geometry, a numerical treatment based on the hybrid finite element method has been developed by Liao & Mei (2000) to predict the two most severe eigenmodes.

Near the Venice inlets incident waves are guided by a pair of long jetties only on the seaward (Adriatic) side, and transmitted waves are radiated to the open space in the Lagoon, much like a two-dimensional horn. Radiation damping must therefore be present, and should enable resonance of an eigenmode linearly by incident waves of the same frequency, similar to the resonant scattering by a harbor with a narrow entrance. In this presentation we first extend the earlier theory to predict trapped modes along a long barrier consisting of arbitrarily many identical vertical gates across the middle of an infinitely long channel (Li & Mei, 2004). It is found that the modal profile is the discrete substitute of a cosine curve. hence the eigenfrequency can be solved simply from the dynamics of one gate within a modal wavelength. Different eigenmodes are found by requiring that an integral number of half cosine curves are fitted along the barrier.

We then develop a linearized theory for the gate responses due a monochromatic incident wave (Adamo & Mei 2004). An energy identity is derived to provide a global check for numerical computations. Gate responses are first carried out for a wide range of incident wave frequencies. Resonances are found at frequencies close to the natural frequencies predicted recently by Li & Mei (2003). Responses to random incident waves described by JONSWAP spectrum are discussed.

2 Trapped modes along a barrier of finite length

We consider a channel of rectangular cross-section, of width L and depth h. N identical gates with negligible gaps are hinged at the bottom along a transverse axis at $x = -0, z = -h$. By using eigen-function expansions the velocity potential on both sides of the vertical barrier $x = \pm b$ are easily found. Their values on one side of the gates $\Phi^+ = \sum_{j=1}^{N} \theta_j \phi_j(b,y,z)\, e^{-i\omega t} + c.c.$ is used to calculate the dynamcial pressure, where

$$\begin{aligned}\phi_j(b,y,z) &= -i\omega \sum_{m=1}^{\infty} \frac{2}{m\pi}\left[\sin(\frac{jm\pi}{M}) - \sin(\frac{(j-1)m\pi}{M})\right]\cos(\frac{m\pi y}{L})\left\{\frac{D_0}{\beta_{m0}C_0}\cosh[k(z+h)]\right. \\ &+ \left.\sum_{n=1}^{\infty}\frac{D_n}{\beta_{mn}C_n}\cos[k_n(z+h)]\right\}, \quad j=1,2,3,...N \end{aligned} \tag{2.1}$$

with

$$\beta_{m0} = \sqrt{(m\pi/L)^2 - k^2}, \quad \beta_{mn} = \sqrt{(m\pi/L)^2 + k_n^2} \tag{2.2}$$

To exclude propagating modes the constants β_{m0} must be real so that all terms decay exponentially with $|x|$. Specifically, if $k < \pi/L$, all terms in (2.1) are kept. If $\pi/L < k < 2\pi/L$, the term associated with β_{10} is excluded. If $2\pi/L < k < 3\pi/L$, the term associated with β_{10} and β_{20} are excluded. If $M\pi/L < k < (M+1)\pi/L$, terms associated with $\beta_{10}, \beta_{20}, \beta_{30}, ...\beta_{M0}$ are excluded.

Invoking the equation of motion for gate j we again get the homogeneous matrix equation for $\{\theta_j\}$,

$$-\omega^2 I\theta_j + C\theta_j - F_{jp}\theta_p = 0, \quad j=1,2,3,...N; \tag{2.3}$$

where

$$\begin{aligned}F_{jp} = 2\rho\omega^2 L \sum_{m=1}^{\infty}\frac{2}{m^2\pi^2}\left[\sin(\frac{jm\pi}{N}) - \sin(\frac{(j-1)m\pi}{N})\right]\cdot \\ \left[\sin(\frac{pm\pi}{N}) - \sin(\frac{(p-1)m\pi}{N})\right]\sum_{n=0}^{\infty}\frac{D_n^2}{C_n\beta_{mn}}\end{aligned} \tag{2.4}$$

Again in the $m-$ series in (2.4) includes only terms with real β_{m0}.

In principle, the unknown eigenfrequencies ω are determined by equating the coefficient determinant of (2.3) to zero; this amounts to applying the condition (2.3) on half of N gates (half because of symmetry). We however find a simpler method by assuming first that each eigenmode is the discrete substitute of the cosine curve $\cos(2K\pi y/L)$, provided that $2K$ is an integer so that there are K half-modal wavelengths across the channel. Equivalently, $2K\pi/L$ is the model wavenumber in the range $(0, L)$. In particular the rotation amplitude of gate j is

$$\theta_j = \theta_o \int_{Y_j} \cos(\frac{2K\pi y}{L})dy = \frac{L\theta_o}{2K\pi}\left(\sin\frac{2Kj\pi}{M} - \sin\frac{2K(j-1)\pi}{M}\right) \quad j=1,2,3,...N; \tag{2.5}$$

where θ_o is an arbitrary constant. With this, one can calculate the eigen-frequency ω by satisfying (2.3) on any single gate j.

For a 20-gate barrier with the gate dimensions comparable to those for the Malamocco Inlet, the dispersion relation between eigen-frequency and the modal wave number or period is shown in Figure 2 for both odd $\mathcal{N}_1, \mathcal{N}_3, ...\mathcal{N}_{19}$ and even $\mathcal{N}_2, \mathcal{N}_4, ...\mathcal{N}_{18}$ modes.

The modal shapes of all even modes $\mathcal{N}_2, \mathcal{N}_4, ...$ which can be resonated by normally incident sea will be shown at the presentation.

3 Single-frequency excitation of a 20-gate barrier

We idealize the inlet system as a long channel open to a semi-infinite water body, with the barrier spanning across the junction. The velocity potentials in the channel on the Adriatic side is the sum of a diffraction problem and N radiation problems $\Phi^- = \Phi^D + \sum_\alpha \Omega_\alpha \phi_\alpha^-$, where $\Phi^D = \Phi^I + \Phi^R$, where $\Omega_\alpha = -i\omega\theta_\alpha$ denotes the angular velocity amplitude of gate α. All the normalized ϕ_α^- can be solved by eigenfunction expansions. On the lagoon side only the radiated waves are present, $\Phi^+ = \sum_\alpha \Omega_\alpha \phi_\alpha^+$ where the normalized radiation potentials ϕ_α^+ can be solved by using a Green function.

The angular displcements are then solved from the laws of the conservation of angular momentum

$$-\omega^2 I\theta_\alpha + C\theta_\alpha = -i\omega\rho \iint_{S_\alpha}(\Phi^+ - \Phi^-)(z+h)dS \tag{3.1}$$

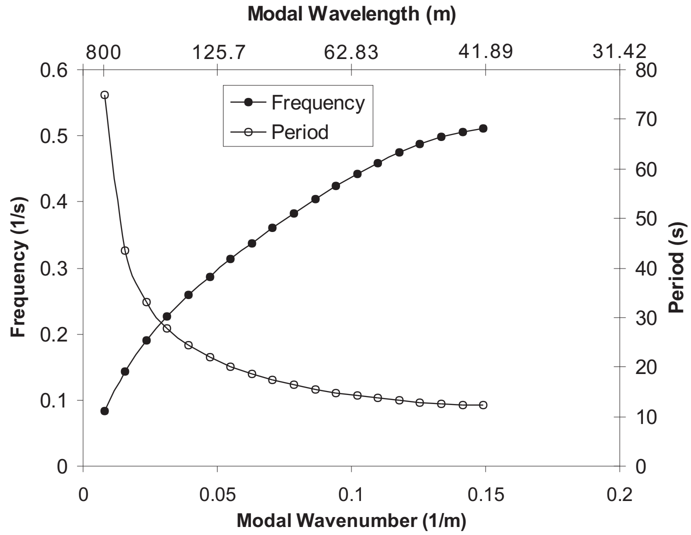

Figure 1: Dispersion relationship for all eigenmodes in a barrier of 20 gates. In the sequence from low to high frequencies, dots correspond to Modes $\mathcal{N}_1, \mathcal{N}_2, ...\mathcal{N}_{19}$.

where I is the moment of inertia of each gate about the bottom hinge. In the prototype the gates are hollow; the inertia I and buoyancy torque C can be determined by measurements; they are assumed to be given here.

For a single floating body in simple harmonic waves there exist many general identities derived by means of Green's theorem (see e.g., Mei 1989). Similar results have been derived for the multi-gate inlet system. For example, the identity on energy conservation is,

$$\begin{aligned} & 2\,\mathrm{Re}\left[gAa\sum_{\alpha}\theta_\alpha A_{00}^{(\alpha)}\right]\left[\frac{\sinh[2kh]+2kh}{4k\cosh(kh)}\right]\sqrt{\frac{2}{h+(g/\omega^2)\sinh^2 k_0 h}} \\ & +\sum_{\alpha}\sum_{\beta}\omega^2 a\,\theta_\alpha\theta_\beta^*\left(\sum_{m=0}^{M}\frac{\alpha_{m0}}{\epsilon_m}A_{m0}^{(\alpha)}A_{m0}^{(\beta)*}\right) \\ & =\mathrm{Im}\sum_{\alpha,\beta}\ \omega^2\theta_\alpha\theta_\beta^*\,|A_0|^2\,\frac{-8i}{k^2\pi}\int_0^\pi e^{ik(\bar{y}_\beta-\bar{y}_\alpha)\cos\varphi}\frac{\sin^2\left(\frac{L}{2}k\cos\varphi\right)}{\cos^2\varphi}d\varphi \end{aligned} \tag{3.2}$$

where α_{m0}, $A_{mn}^{(0)}$ and $A^{(0)}$ are analytically known coefficients. This identity relates all the gate displacements, and is used as a global check of the correctness of the computation.

Extensive computations have been carried out for all gate responses for incident wave frequencies from $\omega = 1.6-0.10$ 1/sec (periods from 4 to 60 seconds). Resonance occurs at nine different frequencioes (periods) all of which are very close to the natural frequencies (periods) of the trapped modes which are symmetric with respect to the centerline of the inlet.

4 Response to random incident waves

We assume the incident waves to be a stationary random process with Spectrum $S_{II}(\omega)$. The response spectrum $S_{\alpha\alpha}$ of gate α is computed from,

$$S_{\alpha\alpha} = \frac{|\theta_\alpha|^2}{A^2}S_{II} \tag{4.1}$$

from which the mean-square displacement of gate α can be obtained.

Based on oceanographic records, it has been estimated by Technital Inc., Milan, that the peak periods of Adriatic sea spectra lie between 5.5 sec to 9.8 sec (ω = 1.14 to 0.64 rad/sec.). For the present design,

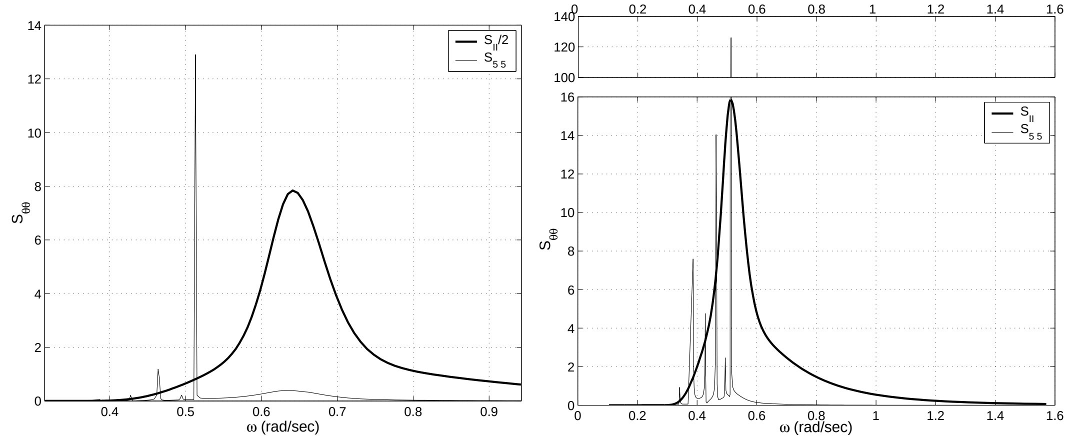

Figure 2: Response spectra of gate 5, (in $rad^2 s$) and the sea spectrum S_{II} (in $m^2 s$). Left: T_p =9.8 sec. Right: $T_p = 12.25$s

the most undesirable mode has the shortest natural period around 12 seconds ($\omega = 0.512$ rad/sec). The statistical likelihood of a sea spectrum with such a long period is estimated to be very low (once in 1000 years).

To provide some preliminary information for assessing the current design, we describe sample predictions based on the present simple model. Results for two incident wave spectra in JONSWAP form will be presented: one with $T_p = 9.8$ sec (ω_p =0.64 rad/sec) which is the longest estimated by Technital, the other with the practically improbable $T_p = 12.25$ sec. (ω_p =0.512 rad/sec). which coincides with the most undesirable trapped mode. The RMS wave height is assumed to be the same : 2.5 m. For each input the spectra and the root-mean-square (RMS) of the gate dispacement are calculated for each gate.

Typical gate response spectra are compared for the two peak frequencies in Figure 2,

We also define the cumulative spectrum by the integral

$$\text{Cum } S(\omega) = \int_0^\omega S_{\alpha\alpha}(\omega) d\omega \tag{4.2}$$

A sudden jump of the cumulative spectrum corresponds to the exclusive contribution by a resonance peak in the single frequency response. The cumulative spectra of all gates are shown in Figure 3. Sudden jumps are clearly seen near $\omega_p = 0.512$ 1/sec which corresponds to the most dangerous mode. From the limit of large ω, the RMS displacements for half of the gates are found.

For $T_p = 9.8$ sec., the resonance peaks of all gates are low. Contributed by all frequencies, gate 5 has the largest RMS displacement of all gates. It cumulative spetrum has the largest jump at $\omega_p = 0.512$ 1/sec. Since the jump is about 1/3 of the total, the root-mean square amplitude of mode $\mathcal{N}_{18}$ is roughly $0.120/\sqrt{3} = 0.069$ radian = 3.97 degrees. This is likely too small to cause significant separation of neighboring gates.

For $T_p = 12.25$ sec., the resonance peaks of all gates are much higher. Gate 5 has of course the largest RMS displacement equal to 0.295 radian= 16.9 degrees. Since the jump of cumulative spectrum at $\omega_p = 0.5129$ /s contributes nearly a half of the total, the RMS amplitude associated with the most undesirable mode $\mathcal{N}18$ is roughly $16.9/\sqrt{2}$=11.7 degrees. For absolute safety, the gate dimensions and design can certainly be modified to reduce the danger of this most detrimental mode.

5 Conclusions

Although the amplification at resonance frequencies according to our linearized single-frequency theories is very high so that nonlinearity should be included in principle, the results for random incident waves are reasonable and indicate that the worse-case scenario is still mangeable, and can be taken care of by the designers without fundamental changes.

For more refined guidance of design and future operations of the barriers, further studies accounting for gate inclination, nonlinearity, vortex shedding , finite inlet jetties and local bathymetry are useful. In particular, the narrow resonant peaks in the single-frequency responses indicate that radiation damping due

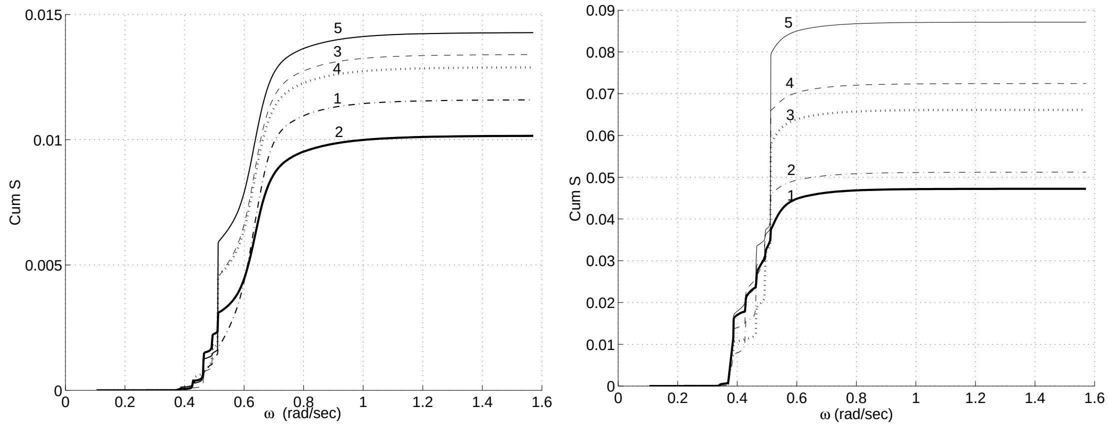

Figure 3: Cumulative response spectra of gates 1-5. Left:T_p=9.8 sec. Right: T_p=12.25 sec.

to the open space on the lagoon side is not very small. It then follows that the mechanism studied before by Sammarco et al (1997 a,b) may be effective so that spectra with peak period near 5 to 6 sec. can still induce subharmonic resonance nonlinearly. A new nonlinear theory accounting for the open space in the lagoon is needed and will be reported in the future.

Acknowledgements

This work has been funded by Magistrato alle Acque di Venezia and Conserzio Venezia Nuova. Insights gained from discussions with Prof. Attilio Adami, and Ing. Alberto Venuti of Protechno. Inc. Padua, Italy, Ing. A. Passadando and Ing Giulia Pellegrini of Technital Inc. Milan, Italy, are recorded with pleasure.

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