ANALYSIS OF HYDROELASTIC BEHAVIOR OF A VERY LARGE MOBILE OFFSHORE STRUCTURE IN WAVES

by
Ken Takagi and Wataru Yano
Department of Naval Architecture and Ocean Engineering, Osaka University
2-1 Yamadaoka Suita, Osaka 565-0871, Japan
E-mail: takagi@naoe.eng.osaka-u.ac.jp

1.INTRODUCTION

Recent years are showing increasing temperatures in various regions, and/or increasing extremities in weather patterns and, many reports from various scientist show that climate change is here, and further, that human activity has contributed to it. Besides the global warming, exhaustion of the fossil fuels has been on notice since 1970s. Therefore, the renewable clean energy such as solar energy and wind energy are in the limelight recently. The solar energy and the wind energy are in practice in many countries, although their share is very small compared with fossil fuels or nuclear energy. The biggest problem on the practical application of these renewable energies in Japan is that Japan is composed of mountainous islands and has no advantage as a location of renewable energy plant. However, the EEZ (Exclusive Economic Zone) of Japan is broad and has many possibilities of utilization. We proposed a new design concept of a very large floating structure with thrusters (we call it VLMOS; Very Large Mobile Offshore Structure) that is utilized as a carrier of the renewable energy plant (see Takagi et al. [1]). It is apparent that the concept of VLMOS is suitable for the solar energy plant, because it requires a large area while Japan has no such a broad area in the country. The structure is planned to sail in the EEZ of Japan and the generated energy is transformed into hydrogen, which is easily transported by shuttle tankers.

The mobility plays an important role on this concept, since the weather condition affects a lot on the efficiency of power plants. If the structure is mobile, we could move the structure to get the maximum efficiency of the power plant. Mobility gives another benefit that the structure is free from the mooring system and also its cost. In order to get good mobility, VLMOS has a configuration that a broad deck is supported with many slender lower hulls and vertical columns. Since the section of lower hulls is semicircle for minimizing the resistance due to the surface friction, it is supposed that the wave exciting force on lower hulls is not small. Thus we need to analyze the hydroelastic behavior of VLMOS in waves for the design of a early prototype. Green function for hydroelastic analysis of vibrating free-free plates by Eatock Taylor and Ohkus [2] is used for representing the elastic motion of the deck and the conventional modal analysis and the three-dimensional boundary element method are employed for the hydroelastic analysis of the lower hull. Combining these methods, the hydroelastic behavior of VLMOS in waves is analyzed.

2.FORMULATION OF THE PROBLEM

Suppose a VLMOS which has a very large flat deck supported with $2N \times 2M$ columns and $2N$ slender lower hulls as shown in Fig.1. Cartesian coordinate system $(x,\ y,\ z)$ is defined so that the z axis is vertically upward and $x - y$ plane coincides with the calm water surface.

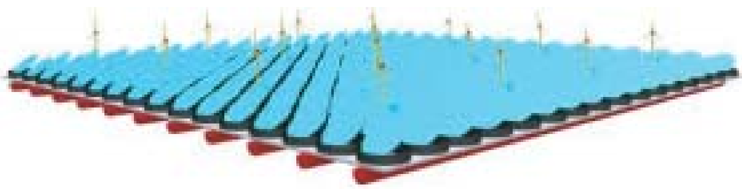

Figure 1: Image of Very Large Mobile Offshore Structure.

The following assumptions are employed regarding the elastic behavior of the hull structure.

- The elastic behavior of the upper deck is represented by the thin elastic plate theory.
- Deformations in the $x - y$ plane of the upper deck are represented by the two-dimensional stress field theory.
- Columns are not deformable, since they are fat and short.
- Lower hulls deflect according to the Euler beam theory in horizontal and vertical plane.

- Lower hulls are compressed or extended in the longitudinal direction.
- Lower hulls twist as a slender axisymmetry beam.

In addition to these assumptions, it is assumed that the fluid is incompressible, inviscid and irrotational. The fluid motion accords with the linear free surface and body surface condition. It is also assumed that all motions are sinusoidal in time with the circular frequency ω.

2.1 Equation of motions

It is very important to define the elastic modes of the hull structure properly, since it strongly affects the efficiency of the computation. We assumed that the shape of VLMOS is symmetry with respect to the x-axis and y-axis. In order to reduce the computational time, we use this symmetry property.

The instantaneous shape of the n-th hull is represented as

$$b_n(x-\tilde{\xi}_{n,1}, y-y_n-\tilde{\xi}_{n,2}+z\tilde{\xi}_{n,4})-(y-y_n)\tilde{\xi}_{n,4}-z-\tilde{\xi}_{n,3}=0, \tag{1}$$

where the centerline of the n-th hull is located on $y=y_n$, $b_n(x,y)-z=0$ is the original n-th hull shape, $\tilde{\xi}_{n,1}(x,t)=\Re[\xi_{n,1}(x)\exp(i\omega t)]$ is telescopic deformation in longitudinal direction, $\tilde{\xi}_{n,2}(x,t)=\Re[\xi_{n,2}(x)\exp(i\omega t)]$ is horizontal deflection $\tilde{\xi}_{n,3}(x,t)=\Re[\xi_{n,3}(x)\exp(i\omega t)]$ is vertical deflection and $\tilde{\xi}_{n,4}(x,t)=\Re[\xi_{n,4}(x)\exp(i\omega t)]$ is the twisting deformation. It is assumed that the deformations of the n-th hull are represented by a linear combination of the vibration modes

$$\xi_{n,1}(x)=\sum_{j=0}^{\infty}(X_{n,8j+1}\pm X_{n,8j+2})\tilde{\psi}_{j+1}(x), \tag{2}$$

$$\xi_{n,2}(x)=\sum_{j=0}^{\infty}(\pm X_{n,8j+3}+X_{n,8j+4})\psi_{j+1}(x), \tag{3}$$

$$\xi_{n,3}(x)=\sum_{j=0}^{\infty}(X_{n,8j+5}\pm X_{n,8j+6})\psi_{j+1}(x), \tag{4}$$

$$\xi_{n,4}(x)=\sum_{j=0}^{\infty}(\pm X_{n,8j+7}+X_{n,8j+8})\tilde{\psi}_{j+1}(x), \tag{5}$$

where $X_{n,j}$ is the amplitude of the j-th vibration mode of the n-th lower full and we take plus sign, if $n\le N$ and we take minus sign, if $n>N$. The vibration modes are given as follows:

$$\tilde{\psi}_j(x)=\sqrt{2}\cos\frac{j-1}{L}\pi\left(x+\frac{L}{2}\right), \tag{6}$$

$$\psi_1(x)=1, \quad \psi_2(x)=\sqrt{2}\frac{x}{L}, \tag{7}$$

$$\psi_j(x)=\frac{\cosh p_jL-\cos p_jL}{\sinh p_jL-\sin p_jL}\left\{\sinh p_j\left(x+\frac{L}{2}\right)+\sin p_j\left(x+\frac{L}{2}\right)-\cosh p_j\left(x+\frac{L}{2}\right)-\cos p_j\left(x+\frac{L}{2}\right)\right\}, \tag{8}$$

where p_j is a root of $\cos p_jL\cosh p_jL=1$ and L is the length of the lower hull. The velocity potential may be represented as

$$\phi=\frac{g}{i\omega}(\phi_I+\phi_D)+\sum_{n=1}^{N}\sum_{j=1}^{\infty}i\omega X_{n,j}\phi_{n,j}, \tag{9}$$

where g is the gravitational acceleration, ϕ_I is the incident wave potential and ϕ_D is the diffraction potential. According to this representation, the linear hull surface conditions for the n-th velocity potential may be

$$\frac{\partial}{\partial n}\phi_{n,8j+1}=\begin{cases}\tilde{\psi}_{j+1}(x)n_x & \text{on the } n\text{-th and } n'\text{-th hull surface,}\\ 0 & \text{on the other hulls,}\end{cases} \tag{10}$$

$$\frac{\partial}{\partial n}\phi_{n,8j+2}=\begin{cases}\tilde{\psi}_{j+1}(x)n_x & \text{on the } n\text{-th hull surface,}\\ -\tilde{\psi}_{j+1}(x_n)n_x & \text{on the } n'\text{-th hull surface,}\\ 0 & \text{on the other hulls,}\end{cases} \tag{11}$$

$$\frac{\partial}{\partial n}\phi_{n,8j+3} = \begin{cases} \psi_{j+1}(x)n_y & \text{on the } n\text{-th hull surface,} \\ -\psi_{j+1}(x)n_y & \text{on the } n'\text{-th hull surface,} \\ 0 & \text{on the other hulls,} \end{cases} \tag{12}$$

$$\frac{\partial}{\partial n}\phi_{n,8j+4} = \begin{cases} \psi_{j+1}(x)n_y & \text{on the } n\text{-th and } n'\text{-th hull surface,} \\ 0 & \text{on the other hulls,} \end{cases} \tag{13}$$

$$\frac{\partial}{\partial n}\phi_{n,8j+5} = \begin{cases} \psi_{j+1}(x)n_z & \text{on the } n\text{-th and } n'\text{-th hull surface,} \\ 0 & \text{on the other hulls,} \end{cases} \tag{14}$$

$$\frac{\partial}{\partial n}\phi_{n,8j+6} = \begin{cases} \psi_{j+1}(x)n_z & \text{on the } n\text{-th hull surface,} \\ -\psi_{j+1}(x)n_z & \text{on the } n'\text{-th hull surface,} \\ 0 & \text{on the other hulls,} \end{cases} \tag{15}$$

$$\frac{\partial}{\partial n}\phi_{n,8j+7} = \begin{cases} \tilde{\psi}_{j+1}(x)(yn_z + zn_y) & \text{on the } n\text{-th hull surface,} \\ -\tilde{\psi}_{j+1}(x)(yn_z + zn_y) & \text{on the } n'\text{-th hull surface,} \\ 0 & \text{on the other hulls,} \end{cases} \tag{16}$$

$$\frac{\partial}{\partial n}\phi_{n,8j+8} = \begin{cases} \tilde{\psi}_{j+1}(x)(yn_z + zn_y) & \text{on the } n\text{-th and } n'\text{-th hull surface,} \\ 0 & \text{on the other hulls,} \end{cases} \tag{17}$$

where $n' = 2N + 1 - n$, $\partial/\partial n$ denotes the partial derivation with respect to the normal direction to the hull surface, (n_x, n_y, n_z) is components of the normal vector. Velocity potentials are obtained by means of the three-dimensional boundary element method with the linear free surface Green function. The radiation forces and the diffraction forces are obtained by integrating the velocity potential on the hull surface. Using the radiation forces and the diffraction forces, we obtain the equations of motions of the n-th hull. However, it is lengthy to show all equations of motions here. Thus, only the equations of motions in the longitudinal direction are shown here. The equations are categorized into two groups and the symmetric motion is represented as

$$-\left(\omega^2 M_h + \left(\frac{k\pi}{L}\right)^2 AE_x L\right) X_{n,8k+1} - \omega^2 \sum_{m=1}^{N}\sum_{j=0}^{\infty}\sum_{l=1}^{3} AS_{m,n,k+1,8j+2l-1} X_{m,8j+2l-1} \tag{18}$$

$$= \frac{1}{2}\sum_{m=1}^{2M}(f_{h,m,n,1} + f_{h,m,n',1})\tilde{\psi}_{k+1}(x_m) + gE_{n,8k+1}, \tag{19}$$

where $k = 0, 1, 2, \cdots$, $n = 1, 2, 3, \cdots$, $n' = 2N+1-n$, x_m is the longitudinal position of the center of the column, M_h is the mass of the n-th lower hull and AE_x is the rigidity of the longitudinal direction of the lower hull. Similarly, the equation of anti-symmetric motion is given as

$$-\left(\omega^2 M_h + \left(\frac{k\pi}{L}\right)^2 AE_x L\right) X_{n,8k+2} - \omega^2 \sum_{m=1}^{N}\sum_{j=0}^{\infty}\sum_{l=1}^{3} AS_{m,n,k+1,8j+2\ell} X_{m,8j+2\ell} \tag{20}$$

$$= \frac{1}{2}\sum_{m=1}^{2M}(f_{h,m,n,1} - f_{h,m,n',1})\tilde{\psi}_{k+1}(x_m) + gE_{n,8k+2}, \tag{21}$$

where the radiation coefficient for the longitudinal direction $AS_{m,n,k,j}$ is defined as

$$AS_{m,n,k,8j+\ell} = \begin{cases} 0 & \text{when } k \text{ and } j \text{ are even number} \\ & \quad \text{or } k \text{ and } j \text{ are odd number,} \\ 2\rho\displaystyle\int_{S_h} \tilde{\psi}_k(x)\phi_{m,8j+\ell} n_x \, dS & \text{other cases,} \end{cases} \tag{22}$$

where S_h denotes the surface of the n-th lower hull.

2.2 Elastic motion of the upper deck

On the other hand, elastic motion of the upper deck is represented by Green functions. Elastic motions of the deck in $x - y$ plane are represented by Green functions $g_u(x, y, x', y')$ and $g_v(x, y, x', y')$ which satisfy equations of vibration in $x - y$ plane

$$-\omega^2 m_d g_u + G\bar{t}\left\{\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) g_u + \frac{1+\nu}{1-\nu}\left(\frac{\partial^2}{\partial x^2} g_u + \frac{\partial^2}{\partial x \partial y} g_v\right)\right\} = \delta(x - x')\delta(y - y'), \tag{23}$$

$$-\omega^2 m_d g_v + G\bar{t}\left\{\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) g_v + \frac{1+\nu}{1-\nu}\left(\frac{\partial^2}{\partial y^2} g_v + \frac{\partial^2}{\partial x \partial y} g_u\right)\right\} = \delta(x - x')\delta(y - y'), \tag{24}$$

where m_d is the unit mass of the upper deck, G is the modulus of torsion, $\bar{t}$ is the equivalent thickness of the upper deck and ν is Poisson's ratio. g_u and g_v are obtained by means of the energy method with the free-free boundary condition. The elastic vibration of the upper deck in z direction is represented by a Green function $g_w(x,y,x',y')$ which is presented by Eatock Taylor and Ohkus [2]. It satisfies the equation of the vibration of the thin elastic plate

$$\left[-\omega^2 m_d + D\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right] g_w = \delta(x-x')\delta(y-y') \tag{25}$$

where D denotes the flexural rigidity of the upper deck.

Using these Green functions, deformation of the deck is represented as linear summation of the inner forces acting at connection points of columns

$$d_{m,n,i} = \sum_{k=1}^{2M}\sum_{\ell=1}^{2N}\sum_{j=1}^{6} f_{d,k,\ell,j} G_{i,j}(x_m,y_n,x_k,y_\ell), \tag{26}$$

where $d_{m,n,i,j}$ is the deformation of the deck at the point (x_m,y_n) (where the m,n-th column is located), $f_{d,k,\ell,j}$ is the j-th internal force at the k,ℓ-th column and G_{ij} is the Green function for the elastic deformation of the deck. It is noted that G_{ij} is the general notation of g_u, g_v and g_w and it also represents gradients of g_u,g_v and g_w.

(26) is transformed to represent the displacement of the lower hull $h_{m,n,i}$ at the base of the m,n-th column as a linear summation of internal forces acting on the lower hull

$$h_{m,n,i} = -\sum_{k=1}^{2M}\sum_{\ell=1}^{2N}\sum_{j=1}^{6}(f_{d,k,\ell,j}+\epsilon_j f_{h,k,l,6-j})(G_{i,j}(x_m,y_n,x_k,y_\ell)+\epsilon_{6-j}G_{i,6-j}(x_m,y_n,x_k,y_\ell)), \tag{27}$$

where

$$\epsilon_j = \begin{cases} h_d & \text{when } j{=}4 \\ -h_d & \text{when } j{=}5 \\ 0 & \text{other cases.} \end{cases} \tag{28}$$

The displacement of the lower hull has a relationship with the deformation of the lower hull

$$h_{m,n,i} = \xi_{n,i}(x_m), \qquad i = 1 \sim 4, \tag{29}$$

$$h_{m,n,5} = \frac{\partial}{\partial x}\xi_{n,3}(x_m) \quad \text{and} \quad h_{m,n,6} = \frac{\partial}{\partial x}\xi_{n,2}(x_m). \tag{30}$$

Combining these equations, we obtain the final solution.

3. DISCUSSION

The procedure of the solution is straight forward. However this problem contains a large number of unknowns. The equation of motions contain $24MN$ unknowns for representing the displacement and force at columns and $8JN$ unknowns for representing the elastic motion of lower hull. The equation of motions contains $32J^2MN$ radiation coefficients and $8JN$ exciting force coefficients. In this work, we take $M = N = 5$ and $J = 20$ for the typical case. However, number of modes may not be large enough to ensure the perfect convergence of the solution and increase in the number of modes may leads a problem on CPU time.

Some numerical results will be presented at the workshop and the accuracy of the solution will be discussed.

References

[1] Takagi, K., Yamamoto, K., Kondo, M., Funaki, T. and Kawasaki, Z.:Conceptual Design of a Very Large Mobile Structure for the Renewable Energy Plant, Proceedings of the Asia Pacific Workshop on Marine Hydrodynamics, Kobe, 2002, pp.239-244.

[2] Eatock Taylor, R. and Ohkus, M. :Green Functions for Hydroelastic Analysis of Vibrating Free-Free Beam and Plate, J Applied Ocean Research, vol. 22, 2000, pp.295-314.

Question by : J.N. Newman
Is your plot of the surge exciting force for the entire structure? (the near-trapping theory for large arrays suggest that the peaks will be much larger for the latter component, near the middle of the platform)

Author's reply:
My plot of the surge exciting force is for the entire structure. However, it seems necessary to investigate the force acting on each hull as you suggest.
