WAVE BREAKING AND CAVITATION AROUND A VERTICAL CYLINDER: EXPERIMENTS AND LINEAR THEORY

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SUMMARY

In linear theory, the free surface can be defined as the position of a sheet of particles, as it evolves in time – this is as rigorous and consistent as the conventional definition. On this particle-sheet definition, the free surface "breaks" when the waves break, as described at the last Workshop.

This paper considers the problem of a vertical cylinder from that viewpoint, and compares the results with experiments. At a cylinder radius c with kc = 0.29, and a wave steepness (based the waveheight $2a$) of ka = 0.42, two small breaking waves form on either side of the cylinder, and propagate around it. The same thing happens in the experiments.

Experimentally, however, additional surface phenomena are seen close (<0.2c) to the cylinder. There is a marked run-up at the front of the cylinder in a crest, and a smaller run-up behind the cylinder in a trough. A jet is also thrown up behind the cylinder after the crest has passed, which behaves ballistically. It appears that the explanation of the jet is the depression on either side of the cylinder, which is seen as well as the run-up, and which propagates round ahead of the breaking wave, to form a cavity behind the cylinder. The collapse of this cavity appears to throws the jet upwards. The resulting impact may explain the "secondary loading cycle" which is a well-known feature of this problem, associated with the phenomenon of "ringing".

Since a depression on either side of the cylinder is a feature of second-order theory, it is conjectured that a particle-sheet simulation with the second-order potential might reveal some of this additional behaviour.

1. BACKGROUND

There is no single "consistent" linear theory for water waves, any more than there is, for example, to the problem of a simple pendulum. There, we can take moments about the pivot and write a linear differential equation for the angular motion θ, viz:

$$ml^2\ddot{\theta} + mgl\theta = 0 \tag{1}$$

where m is the mass of the pendulum bob, and l is the length of the string. Alternatively, we can resolve forces horizontally and write a linear differential equation for the horizontal motion x, viz:

$$m\ddot{x} + mgx/l = 0 \tag{2}$$

These equations respectively have solutions:

$$\theta = \Theta \sin \omega t \tag{3}$$
$$x = X \sin \omega t \tag{4}$$

where $\omega^2=g/l$. Both are entirely consistent linear theories. To first order, they agree – but they have different higher-order errors.

Likewise with water waves, the classical approach of applying a boundary condition on the still-water position z = 0, is not the only approach. We can alternatively observe that the solution to Laplace's equation for the velocity potential must (in the standard notation) be of the form:

$$e^{kz}\sin(kx - \omega t) \tag{5}$$

by separation of variables. Then, we can apply boundary conditions on a sheet of particles, and observe that if $\omega^2=gk$, the pressure is constant there. To first order, there is no difference between adopting this as the free surface, and the conventional definition of the free surface as the pressure head $-g^{-1}\partial\varphi/\partial t$ on z = 0. But the higher-order errors are much less – in regular waves in deep water for example, this method has no second-order error. And in irregular waves (i.e. a combination of many of the above solutions, at different frequencies), the particles "escape" from time to time, with the free surface erupting into the shape of a breaking wave. This is a very natural explanation [1] of the universal phenomenon that irregular waves break – which is readily seen in any photograph of the real ocean.

The same arguments apply to the diffraction of waves around a vertical cylinder. There, separation of variables gives the velocity potential as the real part of:

$$e^{kz}\left\{\sum_{m=0}^{\infty} B_m(kr)\cos(m\theta)\right\}e^{-i\omega t} \tag{6}$$

where B_m is the well-known McCamy-Fuchs combination of Bessel functions (see e.g. [2] p.390) and the polar coordinates r, θ replace the Cartesian coordinates x,y (with θ = 0 the +ve x-axis) The free surface can again be defined as a sheet of particles, and followed numerically to see if the wave breaks.

The vertical-cylinder problem is of interest because it has been widely studied with the conventional definition of the free surface (e.g. [3] takes the analysis to 3rd order), without revealing the complicated free-surface behaviour which is seen experimentally in the region kc, $ka \sim 0.3$.

2. COMPUTED FREE-SURFACE POSITION

In this region there is also a slender-body approximation to (6) which is given in [4] as the incident potential (5) plus a diffracted potential:

$$U\frac{c^2}{r}\cos\theta + \frac{E}{4}\left\{-2c^2\ln r + \frac{c^4}{r^2}\cos 2\theta\right\} \tag{7}$$

where U is the horizontal velocity $\partial\varphi/\partial x$, and E is the horizontal velocity gradient $\partial^2\varphi/\partial x^2$, both evaluated on the cylinder axis.

In this paper we therefore compute the water surface around a vertical surface-piercing cylinder in four ways:

I. Conventional free-surface definition, using the slender-body potential (5) + (7). It is necessary to omit the E term because the ln$r$ in (7) implies infinite elevation at infinity – this is a well-known limitation of the theory.

II. Tracking a line of particles, using the slender-body velocity field from (5) + (7). As in [1], the simulation was 2D only, tracking the particles in the plane $y=0$, and on the surface of the cylinder. Details are in [1].

III. Conventional free-surface definition, using the MacCamy-Fuchs potential (6).

IV. Tracking a particle-sheet, using the MacCamy-Fuchs potential (6) to obtain the surface in 3D. As in method II the integration of particle velocities was carried out by fourth-order Runge-Kutta methods, and checks were carried out to ensure that the results were independent of the time step.

The measurements described below were made during the passage of the transient waves at the start of a regular wavetrain, and this situation was readily reproduced in the simulations by taking two equal-amplitude frequency components. The calculations were started with the cylinder at a null in these waves, so that the conventional free-surface was close to the still water level and gave a convenient initial position for the particle sheet. The simulation (method II – it could equally well have been method IV) was first run in the absence of the cylinder, in order to iterate for the amplitudes of the two components. The parameters obtained in this way were:

	1st component	2nd component
amplitude (cm)	2.04	2.04
wavenumber (m^{-1})	5.90	9.30

where the amplitude given is based on the conventional definition $-g^{-1}\partial\varphi/\partial t$ of the free surface. The agreement between simulated and measured water surface elevation is shown in Fig. 1 – note that the mean vertical position in the simulations is considered to be irrelevant, and has been adjusted down by 1.9cm.

The crest-to-trough height $2a$ of the wave under investigation is 11.1cm – together with the average wavenumber of $7.6m^{-1}$, this gives $ka = 0.42$. In the absence of the cylinder the wave was nevertheless well short of breaking.

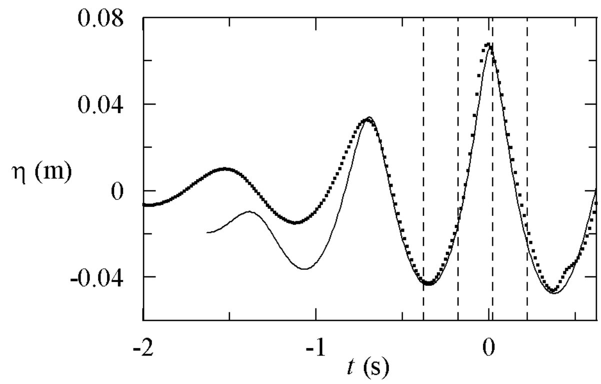

Fig. 1 Water surface histories in the absence of the cylinder, at the position of its axis: measurements (points) and time series used in the simulations (line). The time origin throughout is set to the instant corresponding to the passage of the crest. Images in Fig. 4, at intervals of 0.2s, are at the times shown by the positions of the broken lines.

3. EXPERIMENTAL SET-UP

The experiments were carried out in a wave flume 0.42m wide and 0.7m still water depth, with a vertical hollow perspex cylinder of outer diameter 0.1m. There are undoubtedly blockage effects in these conditions, but they are not thought to introduce major changes, and are discussed in [5]. A fluorescent solution added to the water improved the quality of digital images of the water surface, obtained at 120Hz by a camera positioned on one side of the tank in line with the cylinder's axis. The camera was angled downwards so as to view the water surface surrounding the cylinder, and since the cylinder was transparent, the run-up almost all the way around it could be measured from the images.

4. RESULTS

Fig. 4 shows four digital images of the free surface at intervals of 0.2s, with the third image just after the instant when the wave crest passes the cylinder axis. To the right of these images are the corresponding results of the full (method IV) particle-sheet simulations of the free surface. Also shown are measurements of the free surface position taken from the images, plotted against the results of all four calculations (methods I to IV). The results of the particle-sheet simulations (methods II & IV) have been moved down by 1.9cm (see Fig. 1).

5. DISCUSSION

The most striking result is that the breaking wave seen propagating around the cylinder between the second and third images, is also seen in the particle-sheet simulations (with or without the slender-body approximation), despite the fact that they only use linear theory. This is however to be expected, since with the particle-sheet free-surface definition, linear theory predicts wave breaking more generally – see [1].

There is an interesting contrast with the alternative "wavy lid" approximation featured in [5]. That completely suppresses wave breaking, but has the advantage that closed-form expressions for the wave force, including the local effect at the surface, can be derived in the slender-body case.

The thin run-up very close to the cylinder is also striking, especially in the wave crest, although it is also present in the wave trough. This appears to be the only feature which the particle-sheet simulations fail to predict. However, in the last image it may be seen that the water surface is higher immediately behind the cylinder than is predicted by the simulations. Its height has been extracted at all the intermediate images (i.e. at the 120Hz camera speed, which is 24 times faster than the sequence of Fig.4) and is plotted in Fig. 2 below.

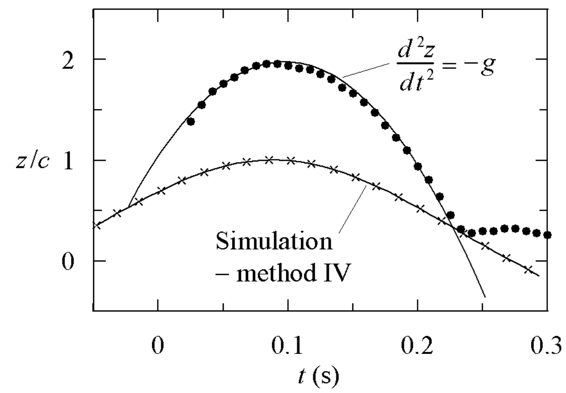

Fig. 2. Elevation of the water surface on the rear face of the cylinder: measurements (shown as points) and simulation.

As may be seen, the water surface in the experiment is behaving ballistically, i.e. the water has been thrown up as a jet, and is now in free-fall. But not that in the simulations.

It is also significant that the simulations completely fail to predict the "secondary loading cycle" first described in [5], which should be strong in the present case, according to [6]. This can be seen in Fig. 3 below, which shows the force on the cylinder predicted by the particle-sheet simulation.

The explanation appears to be a feature of the water surface which cannot be seen in Fig. 4 because of the dye in the water. This is the depression in the surface close to the cylinder, which propagates around ahead of the breaking wave, and is readily seen in the photographs without dye in [5]. This depression ultimately forms a cavity behind the cylinder – it is the collapse of this cavity which appears to be responsible for both for the jet and the "secondary loading cycle".

Since a depression on either side of the cylinder (caused by the dynamic pressure, which is ignored in linear theory) is a feature of the second-order potential, it is conjectured that a particle-sheet simulation with this potential may reveal some of this cavitation behaviour.

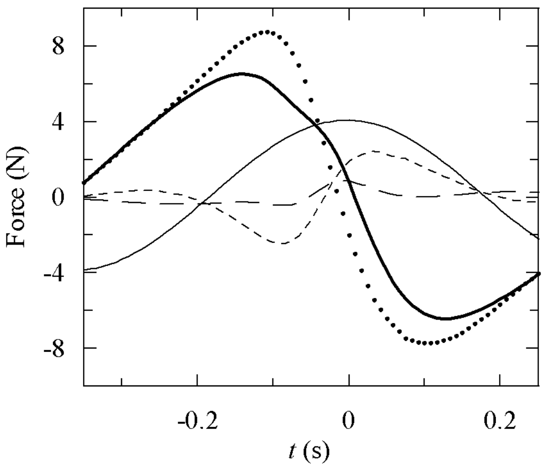

Fig. 3. Force on the cylinder, from simulation method II: total force (thick line); force from transient pressure (dots); force from hydrostatic pressure (short dashes), force from dynamic pressure (long dashes). The undisturbed water surface (by method I) is shown as a thin line.

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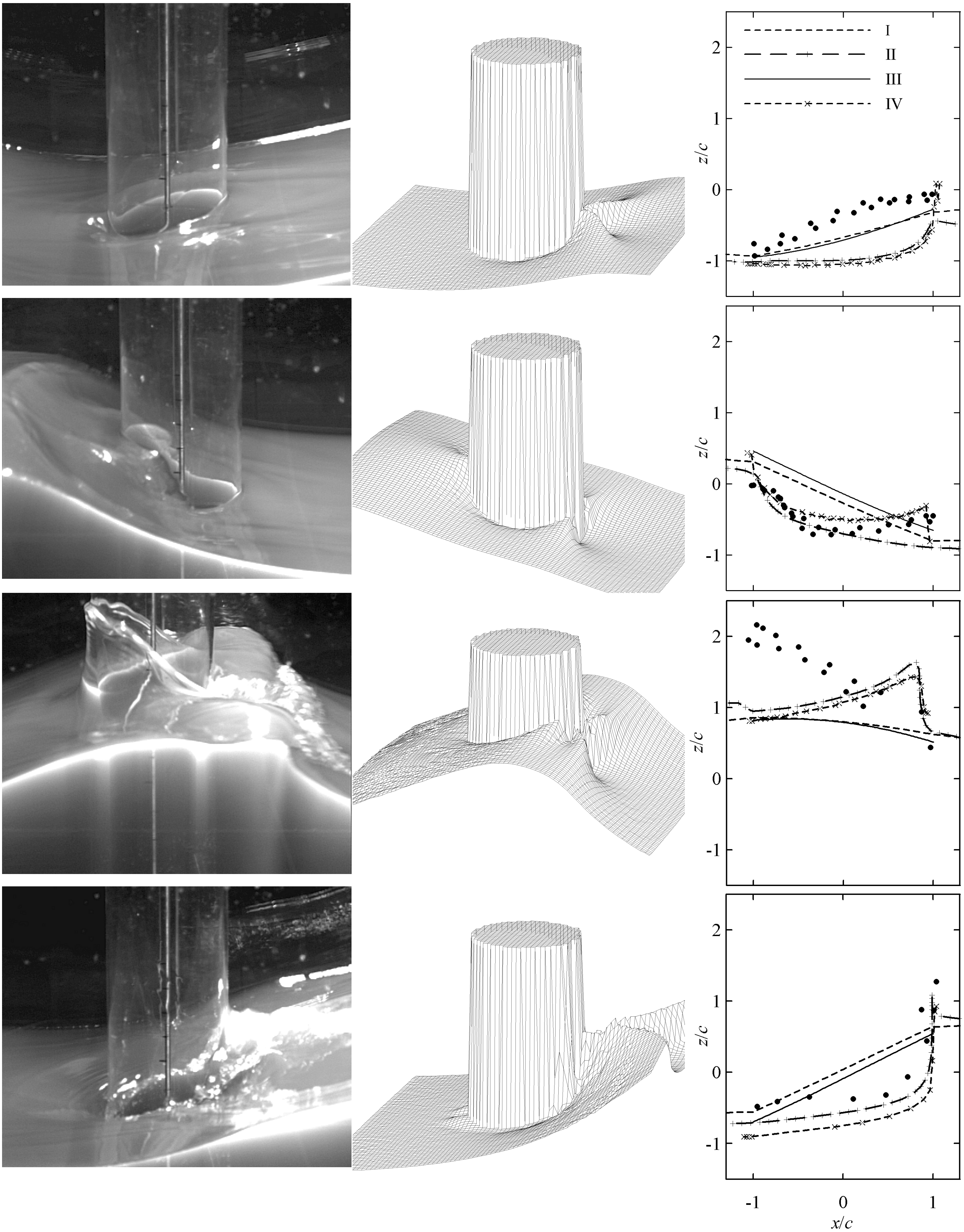

Fig. 4. Laboratory observations (left column) and simulations (middle column) of the water surface around the cylinder at times indicated in Fig. 1. The simulations were obtained by method IV, i.e. updating surface particles with velocities computed directly from the MacCamy-Fuchs linear potential. The viewing angle of the simulations is shifted by 15^o in yaw relative to that of the laboratory observations. The right column shows the corresponding water surface elevations alongside the cylinder computed by all four methods. Measurements are shown as points.

Question by : J. Grue
The secondary loading cycle is merely caused by a low pressure due to the square term in the Bernoulli equation, rather than the direct effect of the free surface elevation, I think. My group was the first to measure the secondary loading cycle (J.Grue, G.Bjorshol & O.Strand, 9th IWWWFB, Oita, Japan,1994, see also J.Grue & M.Huseby, Appl. Ocean Res. 2002, vol 24). In the measurements we identified a considerable low pressure behind the cylinder from free-surface elevation measurements. In your work (JFM vol 350 1997), the low pressure behind the cylinder, during the period of secondary loading cycle, was experimentally confirmed using pressure gauges.

Author's reply:
The JFM paper you refer to was primarily due to my co-author John Chaplin, who unfortunately could not come to the workshop. He points out that the square term in the Bernoulli equation is insufficient to explain the large run-up ahead of the cylinder, and the deep depression behind it. In the case shown in Fig.12 of our JFM paper, these are about 10cm and 5cm respectively. But the maximum velocity in the incident wave is about 0.7 m/s, which corresponds to a Bernoulli pressure head of only 2.5cm.

We are both mortified at having failed to acknowledge in the present paper that your group was the first to measure the secondary loading cycle. We certainly did so in our JFM paper, where your measurements are discussed in detail, citing an even earlier 1993 University of Oslo Report, by the same authors as your 1994 IWWWFB paper.

Question by : H. Bredmose
As I understand it, a linear model is here reported to throw particles to infinite heights. Since the model is linear, the result should then also be valid for a much smaller wave amplitude. How can this be? Is the strength of the singularity dependent on the amplitude chosen?

Author's reply:
You have a point there. I only get "escape" of water particles above a certain wave height, but I claim the theory is still linear.

Conventionally, however, a "linear" solution is often taken as a "scaled-infinitesimal" solution, i.e. the limiting case of small amplitude, times a scalar multiplier. It then has the property you cite, that it is equally valid (indeed more so) if we consider a much smaller value of the scalar multiplier.

I have in fact two separate comments on that:

1) this "scaled infinitesimal" solution is not unique, but depends on the choice of coordinate system. For example, in the pendulum problem, the scaled-infinitesimal solution $\theta = A\sin\omega t$ appears, in Cartesian coordinates, as

$$X = L\sin\{A\sin\omega t\} = L\{A\sin\omega t + (1/3)(A\sin\omega t)^2 + \dots\}$$

which is clearly not the Cartesian scaled-infinitesimal solution $X = B\sin\omega t$. So I believe "linear solution" has a more general meaning, which is the one I am using, namely that the errors are 2nd order.

How big can we allow the scalar multipliers A or B to be? Everyone would agree that it would be misleading to scale up the traditional pendulum solution above $A = \pi$, because this would conceal the way the pendulum goes "over the top" and finds a new equilibrium at $\theta = 2\pi$. I am arguing that it is equally misleading to adopt scalar multipliers for water waves that are large enough to permit an “escape” of the water particles. This is because the underlying argument in the classical theory considers the constant-pressure surface – but it could equally well consider the kinematically-exact surface, i.e. a sheet of particles. If particles have “escaped”, I believe it should be interpreted as wave breaking.

Question by : M. Tulin

1) Have you validated your “alternative method” by using it to predict the propagation of a wave train through to breaking in the case where exact numerical predictions exist?

2) Was the damage to the tanker “Prestige” caused by an ocean wave in the process of breaking?

Author’s reply:

1) This comparison is underway at this moment – I am indebted to D.H.Peregrine for the loan of the latest version his exact numerical code (JFM vol 150 pp 233-251 and many later papers) for the purpose. Of course the agreement is quite close – at the end of the discussion of my paper at last year’s workshop I cite a simple case where the breaking threshold by my method is $ka = 0.42$.

2) I believe so. There is a lengthy account of this casualty on the website of the American Bureau of Shipping (www.eagle.org), but it should be borne in mind that there is currently a lawsuit in New York between ABS and the Spanish Government.

Question by : D.H. Peregrine

Your approximation only works for short periods of time, does it not?

Author’s reply:

Absolutely. I am ignoring the effect of higher-order pressure errors on the free surface. The associated changes to the surface take time to build up, because the fluid takes time to respond to a force acting on it. So my approximation should be at its best if the waves quickly come to a focus and break.

If you are talking about phenomena that take a long time to build up – like the Benjamin-Fier instability – then my approach is completely irrelevant (indeed, I predict that this instability should not occur).