EARLY STAGE OF FLOATING PLATE IMPACT

A. A. Korobkin, Lavrentyev Institute of Hydrodynamics, Novosibirsk, Russia
A. Iafrati, INSEAN (Italian Ship Model Basin),Italy, E-mail: a.iafrati@insean.it

SUMMARY
The liquid flow and the free surface shape at the initial stage of impulsive vertical motion of a floating rigid plate are investigated with the help of a small time expansion procedure. It is shown that the initial flow close to the plate edges is approximately self-similar and is governed by non-linear boundary-value problem with unknown shape of the free surface. Outside of small vicinities of the plate edges the flow is given by the pressure-impulse theory. The non-linear self-similar inner solution is matched to the outer solution and obtained numerically by the boundary-element method. Agreement of the computed free surface shape with experimental one is fairly good.

1. INTRODUCTION

Plane unsteady flow caused by a sudden vertical motion of a floating rigid plate is considered. The flow at the initial stage, when the penetration depth of the plate is much smaller than the plate dimension, is usually described with the help of the pressure-impulse concept. The pressure-impulse approach provides the flow near the impacting body after a short acoustic stage [1]. However this approach cannot be used close to the plate edges, where both unbounded velocity of the flow and unbounded free surface elevation are predicted. As a result, the pressure-impulse solution of the floating plate impact problem cannot be used to derive the initial conditions for starting accurate numerical calculations of the developed flow. Uniformly valid initial asymptotics of the flow initiated by the plate impact is helpful to get clear insight into the flow pattern and to develop adequate numerical algorithms for simulating unsteady flows in the presence of a body with corner points.

The problem of plate impact is relevant to that of floating wedge impact intensively studied earlier [2]. The main difference between them is connected to the fact that in the latter case the free surface elevation is restricted from above by the wedge surface. As a result, the jets caused by the floating wedge impact follow the wedge surface and are of infinite length within the incompressible liquid model, which means there are no intersection points in the small time solution. In the case of floating plate impact the plate edges are treated as the separation points and the jets are detached from the surface of the body. Therefore, the flow caused by a floating plate impact and unsteady flows with separation have some common features. Accurate analysis of the problem under consideration can be helpful to get ideas about flow patterns around separation points.

2. FORMULATION OF THE PROBLEM

Initially the liquid is at rest and occupies a lower half-plane ($y' < 0$), the plate of finite breadth $2L$ floats on the free surface of the liquid. The midpoint of the plate is taken as the origin of the Cartesian coordinate system $x'Oy'$ with axis Oy' being normal to the plate and directed out of the liquid. Dimensional variables are denoted by a prime. At some instant of time, taken as initial one ($t' = 0$), the plate suddenly starts to penetrate the liquid vertically. The velocity of the plate V is assumed much smaller than the sound velocity in the resting liquid. The liquid is assumed ideal and incompressible. The liquid flow is assumed potential, two-dimensional and symmetric with respect to the axis Oy'. Surface tension and gravity force are neglected. We shall determine the liquid flow and the geometry of flow region at the initial stage, when the plate displacement Vt' is much smaller than the plate dimension $2L$. The ratio Vt'/L plays a role of small parameter of the problem.

The half-width of the plate L is taken as the length scale, the ratio L/V as the time scale and the product VL as the scale of the velocity potential. In the non-dimensional variables the liquid flow is described by the velocity potential $\Phi(x, y, t)$, the initial boundary-value problem for which has the form

$$\Phi_{xx} + \Phi_{yy} = 0 \quad \text{in} \quad \Omega(t), \tag{1}$$

$$\Phi_y = -1 \quad (| x |< 1, y = -t), \tag{2}$$

$$\zeta_t + \Phi_x \zeta_x = \Phi_y, \tag{3}$$

$$2\Phi_t + \Phi_x^2 + \Phi_y^2 = 0 \quad (| x |> 1, y = \zeta(x, t)), \tag{4}$$

$$\zeta(\pm 1, t) = -t, \quad \zeta_x(\pm 1, t) = 0, \tag{5}$$

$$\zeta(x, 0) = 0, \quad \Phi(x, 0, 0) = 0 \tag{6}$$

$$\Phi \to \infty \quad (x^2 + y^2 \to \infty). \tag{7}$$

Equation $y = \zeta(x, t)$ describes the elevation of the free surface.

Within the small time expansion procedure the solution of the original problem (1) - (7) is sought in the forms

$$\Phi(x, y, t) = \Phi_0(x, y) + t\Phi_1(x, y) + t^2\Phi_2(x, y) + ..., \tag{8}$$

$$\zeta(x, t) = t\zeta_0(x) + t^2\zeta_1(x) + t^3\zeta_2(x) + ... \tag{9}$$

By substituting (8) and (9) into equations (1) - (7) and collecting terms of the same order with respect to the non-dimensional time t, we obtain recurrent sequence of boundary-value problems for the coefficients in expansions (8) and (9). The boundary-value problem for

$\Phi_0(x,y)$ is that provided by the pressure-impulse theory. The solution of this problem is well known. It gives, in particular,

$$\Phi_0(x,0) = \sqrt{1-x^2} \qquad (|x|<1), \tag{10}$$

$$\zeta_0(x) = |x|/\sqrt{1-x^2} - 1 \qquad (|x|>1). \tag{11}$$

This solution does not satisfy condition (5) at the plate edges and has to be properly corrected close to the edge points.

Yakimov [3] was the first, who studied the local flow close to the edge of a plate entering water. He used physical arguments to show that the local flow is self-similar and introduced the stretched variables $(x-1)t^{-\frac{2}{3}}$ and $yt^{-\frac{2}{3}}$. However, his main concern was the influence of the air density on the motion of the jet initiated by the plate impact, in order to explain different dimensions of cavity behind a flat-nosed body entering liquid. The boundary-value problem, which governs the local flow was derived in [4] in more general case of variable velocity of the plate entry.

3. SELF-SIMILAR LOCAL FLOW

In order to derive an inner solution valid close to the plate edges, the original problem (1) - (7) is reformulated with the help of stretched variables

$$x = 1 + But^{\frac{2}{3}}, \quad y = Bvt^{\frac{2}{3}}, \tag{12}$$

$$\Phi = \sqrt{2B}t^{\frac{1}{3}}\varphi(u,v,t), \quad B = (9/2)^{\frac{1}{3}} \tag{13}$$

introduced for a small vicinity of the right-hand side edge point. By substituting (12) and (13) into equations (1) - (7) and retaining terms of the leading order as $t \to 0$, we arrive at the boundary-value problem with respect to the first-order inner velocity potential $\varphi(u,v)$

$$\varphi_{uu} + \varphi_{vv} = 0 \quad \text{in } \omega \tag{14}$$

$$\varphi_v = 0 \quad (v=0, u<0), \tag{15}$$

$$\varphi - 2(u\varphi_u + v\varphi_v) + (\varphi_u^2 + \varphi_v^2) = 0 \ \text{ on } h=0, \tag{16}$$

$$uh_u + vh_v = \varphi_u h_u + \varphi_v h_v \quad \text{on } h=0, \tag{17}$$

$$\eta = 0\ ,\ \eta_u = 0 \quad \text{at } u=0, \tag{18}$$

$$\varphi \to -\sqrt{r}\sin\theta/2 \quad \text{as } r = \sqrt{u^2+v^2} \to \infty, \tag{19}$$

where θ is the angular coordinate, $u = r\cos\theta$ and $v = r\sin\theta$, equation $h(u,v)=0$ describes the free surface elevation in stretched variables, $h(u,v) = v - \eta(u)$ close to the plate edge and at the infinity, where the free surface can be one-to-one projected on the horizontal axis.

The boundary-value problem (14)-(19) is very complicated owing to the non-linear terms in the free surface boundary conditions (16) and (17) and by the fact that the free surface shape itself is unknown and has to be derived as a part of the solution. This boundary-value problem can be significantly simplified by following an approach similar to that used in [2] to study the self-similar solution in floating wedge impact problem. A modified velocity potential, defined as

$$S(u,v) = \varphi(u,v) - \frac{r^2}{2}, \tag{20}$$

makes it possible to rewrite the dynamic boundary condition (16) in the form

$$S_\tau = \pm\sqrt{\frac{1}{2}r^2 - S}, \tag{21}$$

which is suitable for its integration along the free surface, and reduce the kinematic boundary condition to

$$S_\nu = 0, \tag{22}$$

which, together with (15), provides that the normal derivative S_ν of the modified velocity potential is zero along the boundary of the inner flow domain. Here $\boldsymbol{\tau}$ and $\boldsymbol{\nu}$ are the tangent and normal unit vectors to the fluid boundary. In the following it is assumed that the normal vector is oriented inward the flow region while, along the free surface, the tangent vector is oriented from the plate edge toward the far field (see Fig. 1).

The derived boundary-value problem in the stretched variables is rather similar to that of floating wedge impact. However, differently from the floating wedge case, the sign in the dynamic boundary condition (21) changes along the free surface. Taking into account the definition (20) and the far field behavior (19), we may conclude that the modified velocity potential has to decay when moving along the free surface toward the far field, that is $S_\tau < 0$. On the contrary, at the junction of the free surface with the plate edge, we obtain that $\varphi_u(0,0) > 0$ from physical consideration, which implies $S_\tau(0,0) > 0$ and, therefore, $S_\tau > 0$ along a part of the free surface attached to the plate edge. The point P_I, where the sign in the dynamic boundary condition (21) changes, has to be determined together with the solution.

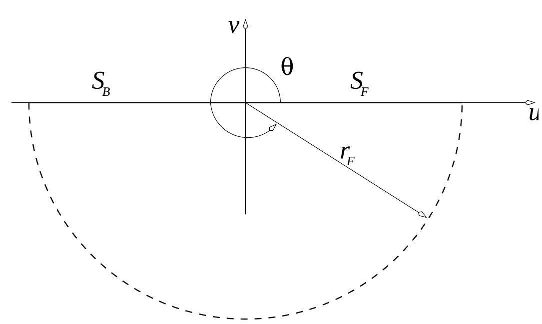

Figure 1: *Sketch of the inner flow region.*

4. NUMERICAL PROCEDURE

In order to solve the inner boundary-value problem, a pseudo time-stepping iterative procedure is developed. Owing to analogies with the inner solution for the impact of floating wedges [2], a similar approach is adopted here.

The velocity potential φ is written in the form of a boundary integral representation. A Dirichlet condition is applied along the free surface while a Neumann boundary condition is applied along the plate. The computational domain extends up to a far field boundary located at $r = r_F$ (Fig. 1). Along the far field boundary either Dirichlet or Neumann boundary conditions can be applied on the basis of the asymptotic behavior (19). The velocity potential along the body and its normal derivative along the free surface are recovered by solving the boundary integral equation, which is derived by taking the limit of the boundary integral representation as the collocation point approaches the domain boundary. By using the relation (20) and the normal derivative of the velocity potential along the free surface, the normal derivative of S along the free surface can be evaluated, thus allowing to check whether the kinematic boundary condition (22) is satisfied or not. In the latter case the free surface shape is advanced in time by using S_ν and φ_τ as normal and tangential velocity components. This choice was proved to provide good convergence properties of the iterative procedure. A flat free surface is used as a starting configuration.

The key point in the procedure concerns the determination of the velocity potential along the free surface and, in particular, the identification of the position, where the sign in the dynamic condition (21) changes. The determination of the velocity potential starts from the intersection of the free surface with the far field boundary, where the value given by the far field asymptotics (19) is used. From equation (20) the value of S is calculated at that point. This value is used to compute S_τ through the dynamic condition (21) taken with the minus sign. Then S_τ is integrated moving from the far field toward the plate edge, thus allowing us to reconstruct the distributions of S and φ along the free surface.

In order to explain the way used to identify the point, at which the sign in the dynamic condition (21) has to be changed, it is worth observing that two parts of the fluid boundary with different boundary conditions applied on them, match each other at the plate edge at angle π. In general case, behavior of the solution with mixed boundary conditions is characterized by the eigensolution, which gives local rise to the vertical velocity, $\varphi_v = O(1/\sqrt{r})$ as $r \to 0$. This behavior of the velocity close to the plate edge is not allowed by the edge conditions (18). The development of such an eigensolution in the numerical procedure is avoided by moving P_I along the free surface up to achieving a minimum of the modulus of the vertical velocity at the first free surface panel.

From the numerical standpoint, the part of the free surface lying between the plate edge and P_I is discretized with panels of uniform size. A progressively growing size is used for the panels along the body and along the portion of free surface which lies on the right hand side of the point P_I. Due to the low order of the asymptotic expansion (19) in the far field, a very large extension of the computational domain has been found necessary to correctly apply the far field boundary conditions. In the computations reported below the size of panel in the region between the plate edge and the point P_I is 0.01 and a growing factor 1.05 is used along the body and along the free surface on the right of P_I. The radius of far field boundary was chosen as $r_F = 200$.

In Fig. 2 the convergence history of the free surface shape is shown and in Fig. 3 the last configuration is plotted. It can be seen that convergence is essentially achieved, although the tip of the jet is still evolving. Due to the very thin layer developing there, a finer resolution should be employed. Nevertheless, the obtained numerical results are in rather good agreement with the experimental observation by Yakimov [3].

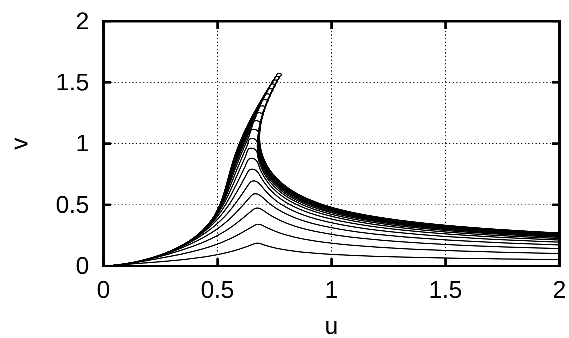

Figure 2: *Convergence history of the free surface shape: one hundred iterations are done between two successive curves.*

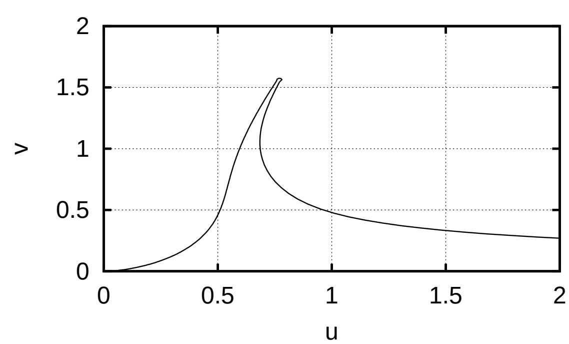

Figure 3: *Last free surface configuration: a thin layer appears solution of which requires a finer resolution.*

In Fig. 4 the behavior of the solution far from the main region is shown and a comparison with the far field asymptotics, which follows from the pressure-impulse solution, is established. It is seen that the asymptotic behavior is recovered with a reasonable accuracy. Higher order terms in the far field asymptotics would be important to significantly reduce the extension of the computational domain.

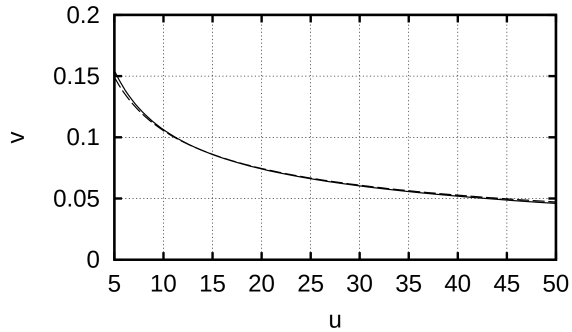

Figure 4: *Comparison between the last free surface configuration computed (*solid line*) and the free surface elevation predicted by the pressure-impulse solution (*dash line*) in the far field of the inner flow region.*

5. CONCLUDING REMARKS

The developed model describing a fine flow pattern close to the edges of the floating plate entering liquid, and the developed numerical algorithm to solve the corresponding non-linear boundary-value problem with unknown in advance position of the free surface, provide reasonable comparison with experimental results. The numerical algorithm may be of help to deal with the separation unsteady flows. The presented analysis has a potential to be improved and optimized. In particular, shallow model approximation can be used to improve description of the jet flow and obtain its characteristics in a way similar to that developed for the floating wedge impact problem. Higher-order far field asymptotics of the velocity potential is available. This asymptotics can be incorporated into the numerical algorithm, in order to reduce the CPU time. However, it is important to notice that the inner boundary-value problem contains no parameters and has to be solved accurately just once.

References

[1] A.A. Korobkin & D.H. Peregrine 'The energy distribution resulting from an impact on a floating body.', *J. Fluid Mech*, **417**, 2002, pp. 157-181.

[2] A. Iafrati & A.A. Korobkin 'Starting flow generated by a floating wedge impact.', submitted to J. Fluid Mech, 2002.

[3] Yu.L. Yakimov 'Influence of atmosphere at falling of bodies on water.' *Izv. Akad. Nauk SSSR Mekh. Zhidk. Gaza* **5**, 1973, pp. 3-6.

[4] A.A. Korobkin 'Plate impact on the free surface of an incompressible liquid.' In: *NON-LINEAR PLANE PROBLEM ON WATER IMPACT*, Chapter 3 of Final Report on the project of the Royal Society of London, 1999, pp. 50-58.