DYAMICS OF A BODY FALLING IN WAVES

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Summary. At the 17-th IWWWFB (2002) Y.Kim, Y. Liu & D. Yue have presented a study of the dynamics of a body falling through calm water. It has been indicated by the authors that even a small disturbance induced by currents and surface waves may strongly influence the body's trajectory. Here we present some preliminary results of a similar problem where a symmetric body of revolution is falling in a time-dependent ambient velocity field $\vec{U}$. If the body is small compared to the wavelength of a surface wave, the wave velocity field may be also envisaged as an unsteady non-uniform current.

The dynamics of a 3-D slender body falling in a weakly non-uniform unsteady current is investigated. Analytic expressions for the hydrodynamic forces and moments acting on a body moving in a non-uniform current are presented. Numerical simulations of the nonlinear equations of motion of the body are performed. The kinematic parameters of motion and body trajectories are also calculated.

1 EQUATIONS OF MOTION

The equations of motion of a rigid body are represented in the body fixed coordinate system as:

$$\frac{d\vec{Q}}{dt} + \vec{\omega} \times \vec{Q} = \vec{F}_B + \vec{F} \tag{1}$$

$$\frac{d\vec{K}}{dt} + \vec{\omega} \times \vec{K} + \vec{V} \times \vec{Q} = \vec{M}_B + \vec{M} \tag{2}$$

where $\vec{Q}$ is the linear momentum of the body, $\vec{K} = \mathbf{I}\vec{\omega}$ is the moment of momentum, $\vec{\omega}$ is the body angular velocity, $\mathbf{I}$ is the inertia tensor of the body and $\vec{V}$ is the velocity vector of the origin of the attached coordinate system. The r.h.s of these equations represent the total vector of forces and moments which are decomposed into a sum of body forces and moments F_B, M_B and body surface stresses terms, F, M, correspondingly. Once the vectors of linear and angular velocities are known, the location and orientation of the body can be thus calculated.

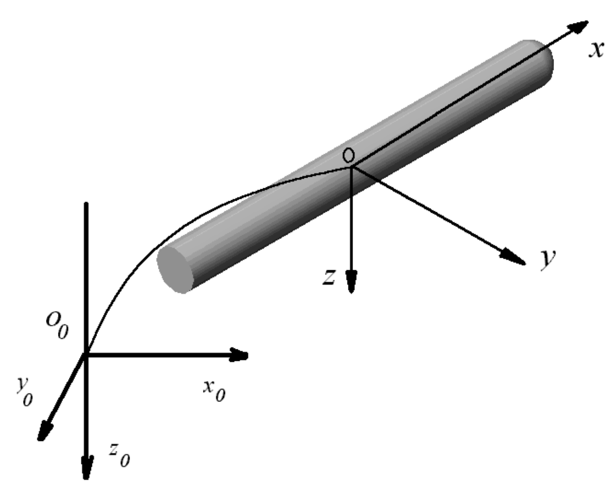


Figure 1: Coordinate system

2 HYDRODYNAMIC FORCES

Assuming that the fluid is perfect, the inertial hydrodynamic force and moment are represented as (Haskind 1977, Miloh 2003):

$$\vec{F} = -\mathbf{T}\frac{d\vec{V}}{dt} + (\mathbf{T} + \rho B\mathbf{1})\frac{D\vec{U}}{Dt} - \vec{\omega}\times\mathbf{T}(\vec{V}-\vec{U}_0) + \left[\mathbf{T},\mathbf{E}\right](\vec{V}-\vec{U}_0) \tag{3}$$

$$\vec{M} = -\mathbf{R}\frac{d\vec{\omega}}{dt} - \vec{\omega}\times\mathbf{R}\vec{\omega} + (\vec{V}-\vec{U}_0)\times\mathbf{T}(\vec{V}-\vec{U}_0) + \vec{d}\times\vec{\omega} - \frac{1}{2}\mathbf{J}\frac{\partial\mathbf{E}}{\partial t} + \varepsilon_{ijk}\frac{\partial E_{kl}}{\partial t}\int_B x_j x_l dB \tag{4}$$

Here $\vec{U}_0$ is the velocity vector of the non-uniform current evaluated at the origin of the attached coordinate system, $\mathbf{1}$ is a unitary matrix, $\mathbf{T}$ is the diagonal matrix of linear added masses, $\mathbf{R}$ is the diagonal matrix of rotational added masses, $\left[\mathbf{T},\mathbf{E}\right]$ denotes the comutator operator between the two tensors, B is the body volume, $\mathbf{E},\mathbf{J}$ and $\vec{d}$ are matrices with the following elements:

$$E_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right), \quad J_{ijl} = \int_S \phi_{i+3}\frac{\partial}{\partial n}(x_j x_k)ds, \quad d_i = J_{kli}E_{kl} + \frac{1}{2}J_{ikl}E_{kl} \tag{5}$$

where ϕ_{i+3} are the conventional rotational Kirchhoff unit potentials, and S denotes the body surface. All matrices and vectors are calculated at the origin of the body coordinate system.

The above formulae do not account for fluid viscosity. However, the dynamics of a slender falling body has much in common with the maneuvering of a slender body, where the effects of fluid viscosity play a crucial role. In our model the viscous effects are introduced on the basis of slender body theory and the cross-flow approach. For instance, the quasi-steady transverse lift force and yawing moment acting on a slender body, are represented as an integral over the body length L:

$$\begin{Bmatrix} F_y \\ M_z \end{Bmatrix} = -\int_L V_x \left\{\frac{d}{dx}\left(V_y + \omega_z x\right)\lambda_{22}(x) + (V_y + \omega_z x)\frac{d\lambda_{22}(x)}{dx}\right\}\begin{Bmatrix} 1 \\ x \end{Bmatrix} dx \tag{6}$$

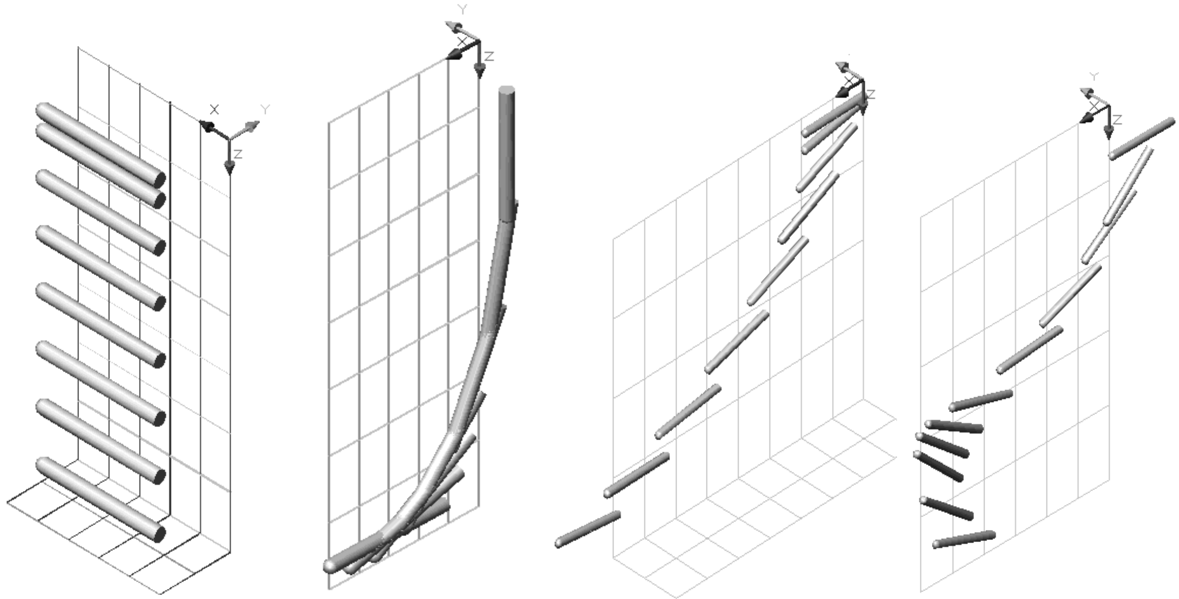
where V_y is the transverse velocity of the cross-section and ω_z is the yawing angular velocity. In order to obtain the lift, the integration of the second term of the integrand is performed only for positive values of the derivative of the transverse added mass $\lambda_{22}(x)$. In this case the yawing and pitching moments include terms of the same nature as those incorporated in (4). Thus, for consistency similar terms should be excluded from (4), or, alternatively, from (6). The cross-flow terms, which are attributed to the viscous drag of the body cross-sections, are represented as integrals over the total body length. For instance, the transverse force and yawing moment can be written as follows:

$$\begin{Bmatrix} F_y \\ M_z \end{Bmatrix} = \rho\int_L C_d(x)a(x)(V_y + \omega_z x)\left|V_y + \omega_z x\right|\begin{Bmatrix} 1 \\ x \end{Bmatrix} dx \tag{7}$$

where $a(x)$ denotes the radius of the body and C_d is the two-dimensional drag coefficient of the corresponding cross-section.

For a body moving in a non-uniform ambient flow, the hydrodynamic reactions can be calculated using the same formulae by rewriting them with respect to components of the relative velocity vector $\vec{V}-\vec{U}_0$.

3 NUMERICAL COMPUTATIONS



(a) **(b)** **(c)** **(d)**

Figure 2: Representative patterns of a body falling in calm water.

a) Initially horizontal oriented body; the center of mass coincides with the center of buoyancy.
b) Initially vertical oriented body; the motion starts with a small disturbance of the initial conditions.
c) Initially horizontal oriented body; the center of buoyancy is located at a distance $0.01L$ aft the center of mass.
d) Initially horizontal oriented body; the center of buoyancy is located at a distance $0.2L$ aft the center of mass.

The system of five differential equations of motion is solved numerically for different parameters of the body such as the body's weight and displacement, locations of its center of gravity and body's kinematic initial conditions. Some typical patterns of a falling body are represented in Fig. 2. They are in a favorable agreement with the experimental and numerical results of Kim *et al* (2002).

The dynamics of a body moving in the field of a progressive monochromatic wave is investigated for different ratios of the wavelength λ to the body length L, different ratios of the mass of the body m to its displacement ρB and for different initial conditions. Fig. 3 illustrates the motion of a 1m-length body falling in an ambient field flow, which is induced by a monochromatic surface wave with a 20 m wavelength. It is shown that such short waves do not actually influence the motion. However, for a wave with a wavelength of the order of 100-body lengths, the situation is essentially different. Fig. 4 illustrates the motion of initially horizontal oriented bodies with relatively large amplitudes of motion. The motion of an initially vertical oriented body is shown in Fig. 5. It is demonstrated that the presence of non-uniform unsteady ambient velocity fields can drastically alter the trajectories and kinematic parameters of the body.

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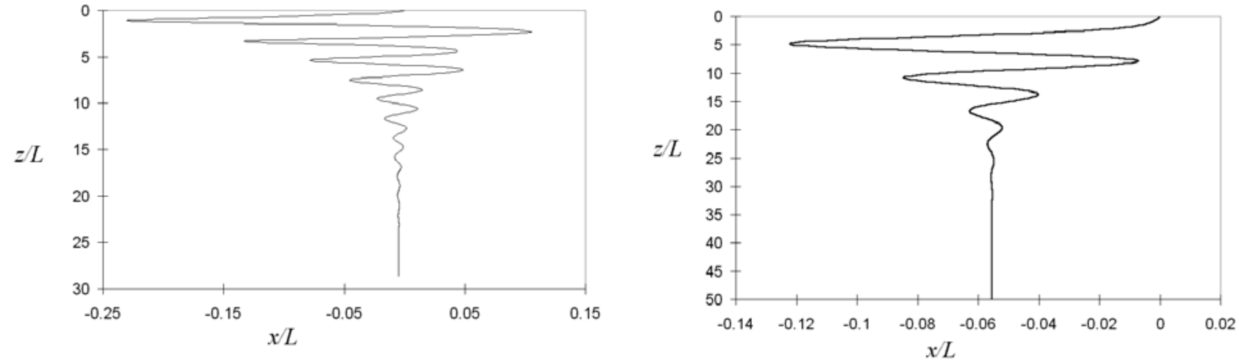
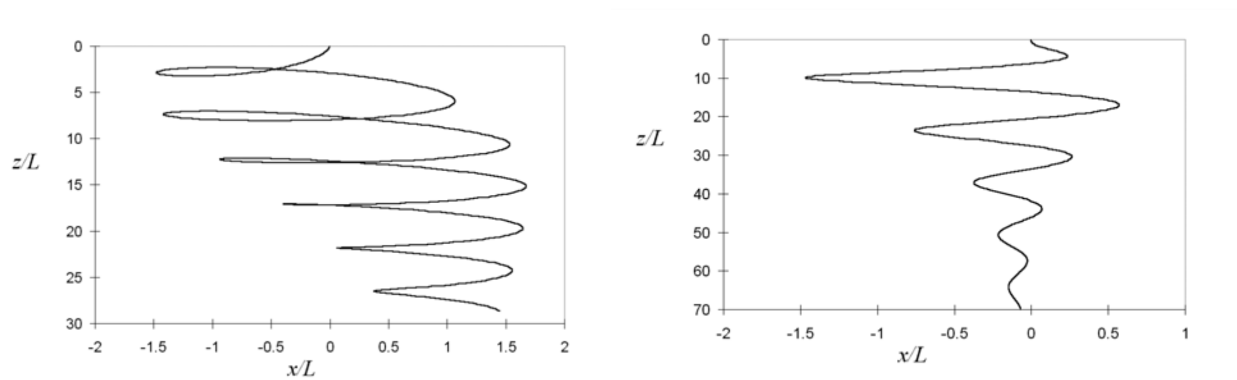


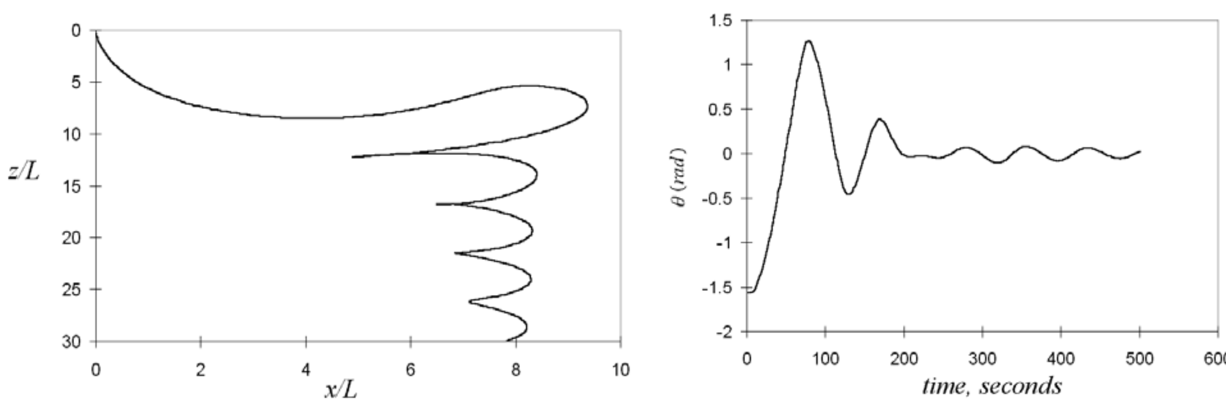
(a) $m/\rho B = 1.15$ **(b)** $m/\rho B = 2.3$

Figure 3: Trajectories of the center of mass in the vertical plane ($\lambda = 20L$ **)**



(a) $m/\rho B = 1.15$ **(b)** $m/\rho B = 2.3$

Figure 4: Trajectories of the center of mass in the vertical plane ($\lambda = 100L$ **)**



(a) the center mass trajectory **(b) pitch angle as a function of time**

Figure 5: Kinematic parameters of the body in the vertical plane ($\lambda = 100L$, $m/\rho B = 1.15$ **)**