# Theoretical and experimental analysis of the velocity profile under crest of extreme water waves

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Enhanced evidence and description of the kinematics during large wave events at sea are requested by the offshore and ocean engineering industry. The velocities in steep waves are required for subsequent analysis of loads on, e.g., ships, offshore platforms, tension legs of the platforms and risers. Despite numerous studies, proper knowledge of kinematics of steep ocean waves above mean sea level is still lacking. This provides the motivation of the present investigation. Our focus is the kinematics of large wave events which may occur on the surface of the ocean. We combine theoretical and numerical predictions using a recent fully nonlinear wave model with high-resolution Particle Image Velocimetry (PIV) experiments in a laboratory wave tank.

Our main objective is to identify the velocity profile under crest of what which generally can be characterized by a large wave event. The wave event is a result of a process on the surface of the fluid where the wave field has developed due to nonlinearity and dispersion from an initial state. Deep water waves are considered.

With the experiments in mind, we identify theoretical references of the wavenumber, wave slope and fluid velocity. For this purpose we employ third-order Stokes wave theory that is valid for moderately steep periodic wave motion deep water. We proceed as follows:

**1.** For the progressing waves we identify the wave period $T$ and the maximal elevation $\eta_m$. The local angular frequency is given by $\omega = 2\pi/T$.

**2.** We next compute the wavenumber $k$ and the wave slope $\epsilon$ solving numerically the (truncated) system of equations $\omega^2/gk = 1 + \epsilon^2$, $k\eta_m = \epsilon + \frac{1}{2}\epsilon^2 + \frac{1}{2}\epsilon^3$. A wave amplitude is defined by putting $ak = \epsilon$.

**3.** Nondimensional velocity profiles are obtained dividing the fluid velocity by $u_{\mathrm{ref}} \equiv \epsilon\sqrt{g/k}$.

An important point is that we use this procedure to a general large wave event in an arbitrary sea state. The wave period is, in this case, identified from the time history of the surface elevation at a fixed location and is taken as the trough-to-trough period of the event. $\eta_m$ is taken as the maximal wave elevation of the event. The outcome is an estimate of the wavenumber, wave slope and a reference velocity.

We shall see from both fully nonlinear computations and experiments that almost uniform nondimensional velocity profiles result from such a scaling. We also compare with the exponential profile $\mathrm{e}^{ky}$ and obtain a favourable agreement.

The third order analysis is complemented by fully nonlinear computations of unsteady wave fields using the rapid fully nonlinear method by Clamond & Grue (2001). Simulations of very large (freak) waves are obtained with long wave packets as initial condition. A self-evolution leads to a large wave event where the maximal wave elevation is about three times higher than the initial one. Other relatively large wave events are produced using the same procedure, with less strong initial conditions (Clamond & Grue 2002).

## Experiments

Several different wave scenarii in a total of 62 different steep wave events are investigated in the wave tank. The total number of PIV-wave experiments reported here is 122, counting all repetitions which are carried out. The large events take place in the leading unsteady part of a wave train, focusing wave fields and random wave series. The wave tank is 24.6 m long, 0.5 m wide and with water depth 0.72 m. The kinematics is obtained by employing an extended PIV system (Jensen et al. 2001).

The experiments show a strong collapse of the nondimensional velocity profiles, see the figures. Velocity measurements by Baldock et al. (1996) using Laser Doppler Anemometry (LDA), put on nondimensional form, fit excellent with our velocity profiles, both those obtained by PIV and theoretical ones. LDA measurements by Kim et al. (1992) of waves in moderately shallow water show larger velocity at the crest and smaller below mean water line, than in deep water (figure 1c).

Our fully nonlinear theoretical model is useful for the rather steep waves, inducing a fluid velocity of up to 40 % of the wave speed. In the present experiments we have been able to push the wave slope measuring a fluid velocity up to 75 % of an estimated wave speed. The estimated wave slope is then 0.46 and the value of $k\eta_m$ is 0.62.

A surprising result is that the exponential profile $e^{ky}$ compares rather well with all measurements of the waves above mean water line (put on nondimensional form). The good comparison suggests that the exponential profile is quite useful for obtaining estimates of the kinematics of steep waves (see the figures).

We remark that the Wheeler stretching method is widely used in engineering practice. The vertical coordinate $y$ is then replaced by $y' = y - \eta$ (infinite water depth). Our results do not support the Wheeler stretching method. They are closer a velocity profile that is linearly extrapolated above mean water line, i.e. $e^{ky} = 1 + ky$ (Longridge et al. 1996). A linear extrapolation underestimates the kinematics of large wave events, however.

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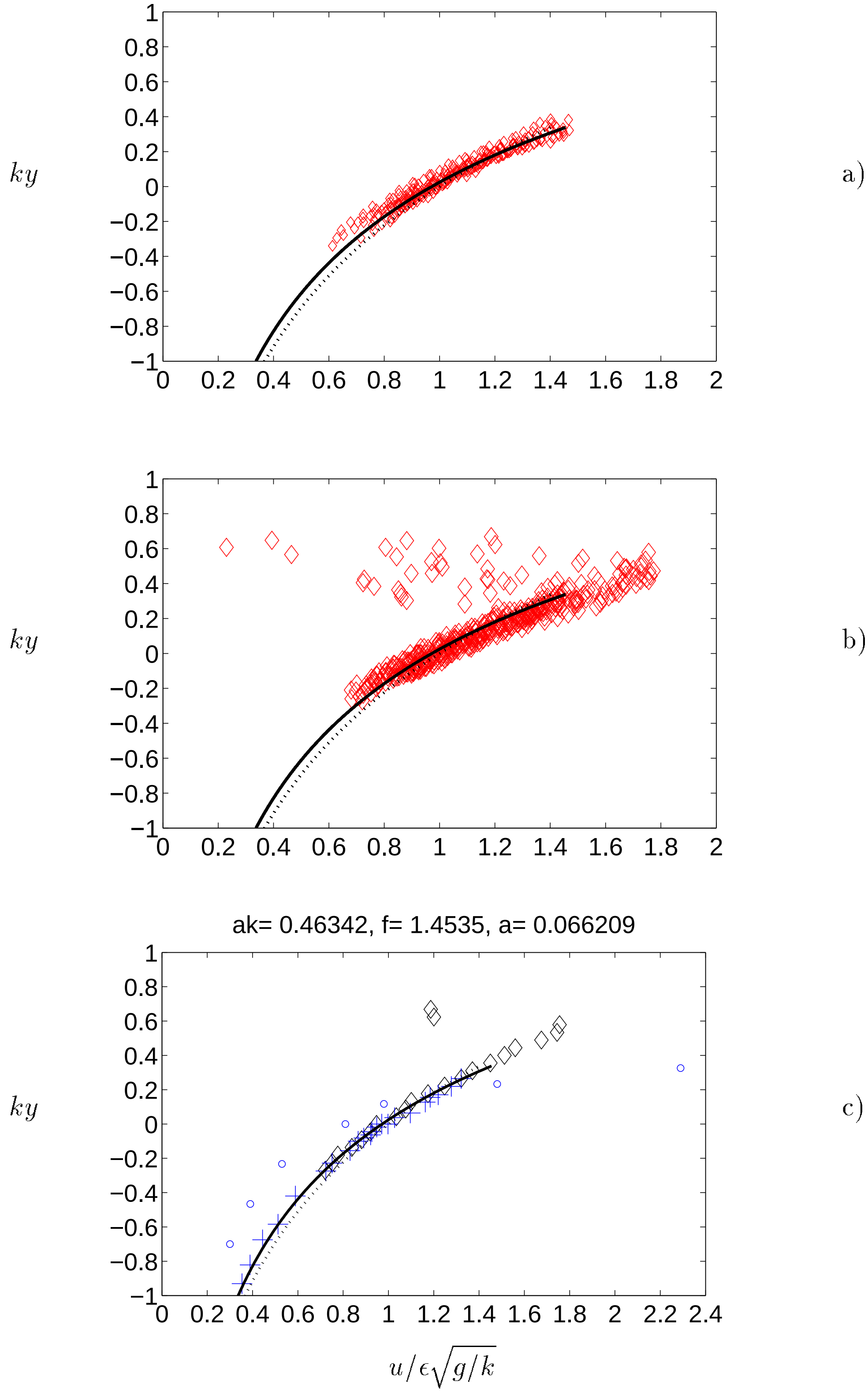


Figure 1: Horizontal velocity profiles below wave crest in deep water. a) 13 large wave events in the leading unsteady part of a wave group, with $0.213 < ak < 0.348$. b) 19 large wave events of focusing wave groups, with $0.217 < ak < 0.463$. c) Largest focusing wave event in: present measurements ($\diamond$), Baldock et al. (1996) ($+$), Kim et al. (1992) ($\circ$). Solid line: Velocity profile in fully nonlinear freak wave computation with $ak = 0.29$. Dotted line: $\mathrm{e}^{ky}$. (Note the differences in horizontal scale.)

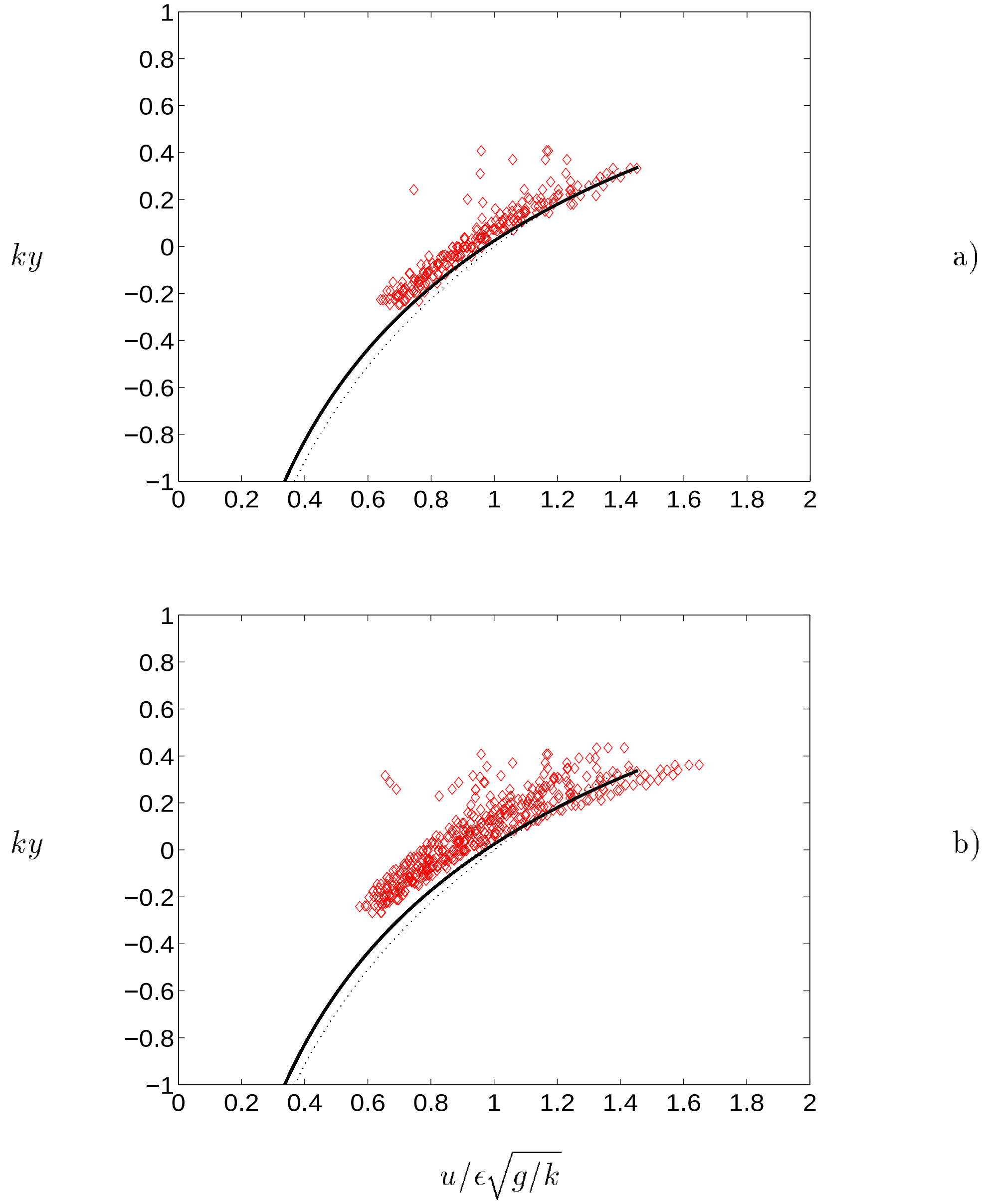


Figure 2: Horizontal velocity profiles below wave crest. a) 5 events of random wave series ($\diamond$). b) All measurements of large events in the random waves, with $ak > 0.3$. Solid line: Velocity profile in fully nonlinear freak wave computation with $ak = 0.29$. Dotted line: $\mathrm{e}^{ky}$.

**Question by** : M. Tulin

1. Since Mirhauq (Trondheim) has shown that extreme waves in the North Sea are not symmetric (steep front faces), how can you claim to model them as Stokes Waves?
2. Isn't it true that the maximum orbital velocity (and therefore the design condition) is simply the wave phase speed augmented by a factor depending on the breaking jet strength?
3. You say: "Despite numerous studies, proper knowledge of kinematics of steep ocean waves above mean level is still lacking". How can this be true when there exist several published studies of fully nonlinear wave propagation, including wave grouping and breaking, and these allow detailed knowledge of kinematical and velocity fields?

**Author's reply**:

We are very thankful for these valuable comments and questions.

1. Regarding question 1 we do not claim that steady Stokes waves is a proper model for irregular waves at sea. Our angle of attack is this: with the purpose of having a theoretical reference of a slowly varying wave field, that is weakly nonlinear and narrow banded, we identify a local wave period and a local wave length. This can be justified mathematically. Moreover, accurate laboratory measurements confirm the weakly nonlinear dispersion relation, both for small, moderate and even large waves. The weakly nonlinear reference velocity, $e\sqrt{g/k}$ ,see the paper, is then used in all the measurements. The collapse of the velocity recordings, put on nondimensional form, is a surprising result of the investigation. They are useful for simple predictions of wave induced fluid velocities.
2. It may be true that the maximal velocity in ocean waves at sea is the wave speed. In our laboratories we have been able to measure fluid velocities up to 75% of the estimated wave speed. LDA experiments by Baldoch, Swan & Taylor showed velocities up to 40% of the wave speed, and by Kim et al. up to 64% of C. References are given in the paper.
3. You are right in referring to the many publications on fully nonlinear wave computations, with lots of documentation of the wave induced velocities, from a theoretical point of view. Our main contribution precisely consists in 122 different PIV measurements of large wave events giving an experimental confirmation and supplement to the existing knowledge. Such a documentation has repeatedly been suggested by experts in the ocean engineering community.

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