"ESCAPE" OF PARTICLE TRAJECTORIES IN LINEAR IRREGULAR WAVES: A NEW EXPLANATION FOR WAVE BREAKING AND MODEL OF BREAKING WAVES

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SUMMARY

A curious feature of the ordinary linear theory of irregular waves is that the particle trajectories "escape" from time to time. This is easily shown by time-history simulation (e.g. with MATHCAD), and is to be expected, since the forward velocity of a particle will occasionally exceed the phase velocity. Were the wave steady, the streamlines in a frame moving with the wave would then diverge from a stagnation-point.

The phenomenon is a simple explanation for wave breaking in deep water. If the water surface is taken as a sheet of particles (i.e. the kinematic free surface condition is applied exactly, rather than in the usual approximate way), the breaking waves appear as local eruptions of the free surface. These can be cut off when the surface at the front of the wave becomes vertical, to give a remarkably realistic-looking model of breaking waves, based entirely on the velocity potentials from linear theory.

The most striking feature of the model is that the frequency of wave breaking depends not just on the average steepness (i.e. significant wave height ÷ wavelength at spectral peak) but on the *bandwidth*. Waves which are, on average, not at all steep still break if the bandwidth is large enough. This is consistent with observation, and is in contrast to the conventional view of wave breaking as an instability developing from a regular wave train. That is the narrow-bandwidth case, where the waves are, on average, steep.

1. ENGINEERING BACKGROUND

Breaking waves are one of the main hazards to floating bodies. For small vessels, like fishing boats, the risk is capsize – the loss of the trawler *Helland Hansen,* for example, is well-established as being the result of capsize in a breaking wave [1]. For large vessels like tankers and bulk carriers, the risk is of impact damage to hull plating. Bow damage is reported not infrequently, see the case of the tanker *Wilstar*, for example, where the bow damage was very severe [2]. A recent case is the oil production ship *Schiehallion*, where the bow plating was ruptured over a small area well above the water line, by a pressure which must have been about 75 tonnes/m^2. See Figure 1 below.

Fig 1. Bow damage on the *Schiehallion* FPSO

To predict the likelihood of such damage it is necessary to know the probability of encountering a wavefront which is sufficiently steep. According to the ordinary linear theory of irregular waves, the water surface elevation is described for example by the Pierson-Moskowitz spectrum ([3] p.315):

$$S(\omega)=\frac{s^2\pi g^2}{\omega^5}e^{-(w_m/\omega)^4/\pi} \qquad (1)$$

where w_m is the mean frequency (1.408 times the peak frequency w_p in this case), and s is the average steepness based on it (i.e. significant wave height ÷ length of wave of frequency w_m), taken as 0.05 by definition of the P-M spectrum. In deep water the transfer-function between water surface elevation and water surface slope is $i\omega^2/g$ so the spectrum of water surface slope is:

$$\left|\frac{i\omega^2}{g}\right|^2\frac{s^2\pi g^2}{\omega^5}e^{-(w_m/\omega)^4/\pi}=\frac{s^2\pi}{\omega}e^{-(w_m/\omega)^4/\pi} \qquad (2)$$

Since this spectrum behaves as ω^{-1} as $\omega\to\infty$, its integral is infinite, and so the significant water slope is infinite too. Thus no useful predictions of wavefront steepness can be made with conventional sea spectra, all of which share this property.

There is an engineering literature of "freak waves" which addresses the problem of predicting extreme waves [4] - but it includes the problem of exceptionally high waves, which are a hazard to the decks of fixed offshore structures, without breaking. The problem of wave breaking in deep water has also engaged the attention of a number of mathematicians – this literature is reviewed in a most attractive manner by Peregrine [5].

2. "ESCAPE" OF PARTICLE TRAJECTORIES IN LINEAR IRREGULAR WAVES.

Given the horizontal and vertical water velocity in deep-water linear irregular waves, i.e.

$$\sum \omega_j \sqrt{2S(\omega_j)\delta\omega}\cos(k_j x - \omega_j t + \Phi_j)e^{k_j z} \quad (3)$$

$$\sum \omega_j \sqrt{2S(\omega_j)\delta\omega}\sin(k_j x - \omega_j t + \Phi_j)e^{k_j z} \quad (4)$$

in the usual notation (see e.g. [3] p.312), the trajectory of a water particle can be found simply by numerical time-history integration. Figure 2 below shows the trajectory of a particle obtained in this way with MATHCAD, using the P-M spectrum (1) and taking 300 discrete frequencies in (3) and (4), equally spaced from 0 to $3w_m$. The particle is from the still-water free surface $z=0$ and is started from its linear-theory position.

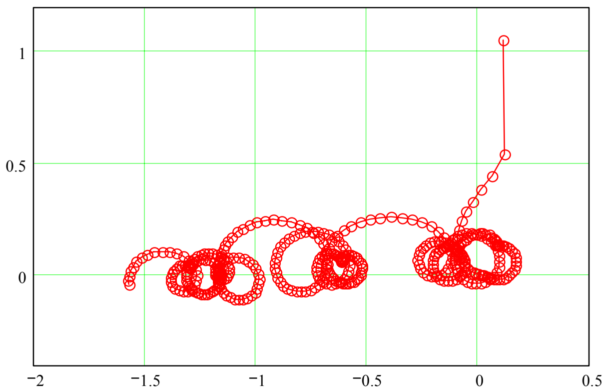

Fig 2. Particle trajectory in linear irregular waves

The Stokes drift ([3] p.252) of the particle is very evident, as is its eventual "escape". The latter occurs when the forward velocity of the particle exceeds the phase velocity of the wave, which occurs from time to time due to the magnifying effect in wave crests of the exponential term in (3). That this will produce an "escape" can be seen from Figure 3 below, which shows the streamlines in the linear-theory model of a regular wave, viewed in a frame moving with the velocity V of the wave, so that the flow is steady.

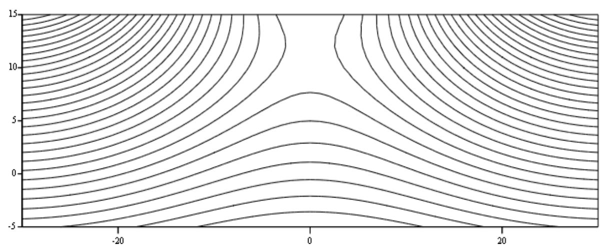

Fig 3. A linear-theory regular wave seen as a steady flow, in a frame moving at the wave speed.

If the horizontal and vertical velocity components in a stationary frame were $\{ve^{kz}coskx,\ ve^{kz}sinkx\}$, say, then they become $\{ve^{kz}coskx\text{-}V,\ ve^{kz}sinkx\}$ in the moving frame, so there will be a stagnation-point as shown, in line with the crest position. At this stagnation-point the vertical velocity is nil and the wave velocity V equals the horizontal particle velocity $ve^{kz}coskx$ in a stationary frame. Above this the flow in the moving frame reverses, producing the divergence shown. Particles above a certain dividing streamline thus "escape".

3. THE FREE SURFACE AS A SHEET OF PARTICLES.

A natural extension from this single particle is a sheet of such particles. This moving surface satisfies the kinematic free-surface condition exactly, by definition. Moreover, the pressure felt by a particle as it orbits is constant (i.e. it satisfies the dynamic free-surface condition), except for slow second-order variations, which seem unlikely to affect wave breaking.

This can be seen by considering the rates-of-change seen by the particle in the hydrostatic, transient and Bernoulli pressure components. The hydrostatic pressure $-\rho gz$ only changes convectively, i.e. as a result of the particle's motion. It rate-of-change is $-\rho g dz/dt = -\rho g\partial\varphi/\partial z$, where φ is the velocity potential. The transient pressure $-\rho\partial\varphi/\partial t$ changes non-convectively: if the jth component of the velocity potential is φ_j , the non-convective rate-of-change of the jth component of transient pressure is:

$$\frac{\partial}{\partial t}(-\rho\frac{\partial\phi_j}{\partial t}) = \frac{\rho\omega_j^2}{k_j}\frac{\partial\phi_j}{\partial z} = \rho g\frac{\partial\phi_j}{\partial z} \qquad (5)$$

which exactly cancels the jth component of the rate-of-change of hydrostatic pressure. The convective rate-of-change in transient pressure is:

$$\mathbf{v}.\nabla(-\rho\frac{\partial\phi}{\partial t}) = -\rho\mathbf{v}.\frac{\partial}{\partial t}(\nabla\phi)$$
$$= -\rho\mathbf{v}.\frac{\partial\mathbf{v}}{\partial t} = \frac{\partial}{\partial t}(-\frac{1}{2}\rho\mathbf{v}.\mathbf{v}) \qquad (6)$$

where $\mathbf{v}$ is the total particle velocity. This is clearly second order, so, to second order, it is sufficient to evaluate (6) at the mean position of the particle. It can readily be shown to have difference-frequency components only, which is not surprising because in regular waves $\mathbf{v}.\mathbf{v} = |\mathbf{v}|^2$ is constant at any fixed point. (6) is also equal to the non-convective rate-of-change of the Bernoulli pressure $-½\rho|\mathbf{v}|^2$ – which it is likewise sufficient to evaluate it at a fixed point, to second order. And by the same token, its convective rate-of-change is zero, to second order.

Thus if all the particles in the sheet are at the same pressure initially, they will remain approximately so, and thus be a good model of the free surface. And the approximation could readily be improved, by including the second-order potential from second-order irregular wave theory, which cancels the slow pressure variations just mentioned. To prevent the simulation from starting when the wave is breaking, the particles can be held on the zero-pressure surface until it crosses z = 0, and then released. As the particles drift downwave by Stokes drift, new particles can continuously be introduced in this way, and deleted when they have drifted too far.

Without the refinement of the second-order potential, the linear-theory surface-particle position can be used as an approximation to a zero-pressure surface. In regular waves this potential is zero anyway, and the same pressure error is applied to each particle (because of the release strategy just described), thus giving the correct second-order wave shape.

4. BREAKING WAVES FROM ESCAPING PARTICLES

The interest in such a simulation is of course in what happens when the particles "escape" in irregular waves. Figure 4 below shows a typical case – in fact the same case as Figure 2, at the final instant there.

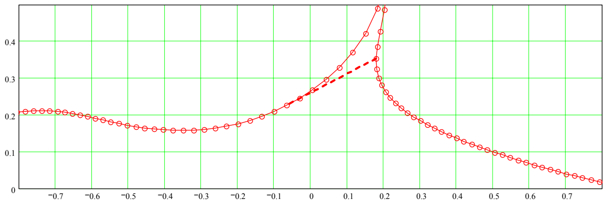

Fig 4. Eruption of particle-sheet wave (travelling from left to right) when particles "escape"

As can be seen, the water surface simply erupts, with the front face of the eruption looking remarkably like the front face of a breaking wave. If the eruption is cut off at the point where the front face becomes vertical, and the surface is sloped down behind that as a straight line, as shown dotted in Figure 4, the resemblance to a breaking wave is very strong indeed.

With MATHCAD the particles which have been cut off can simply be relocated to this straight line (and the particle distribution along the whole surface can be made uniform, for good measure, by interpolation), and the simulation continued. The resulting waves can be viewed with the animation facility in MATHCAD, and look remarkably realistic. Figure 5 below shows a typical breaking wave from such an animation (the next wave in the same case as Figure 4), compared with an exactly-computed deep water breaking-wave profile, from [5].

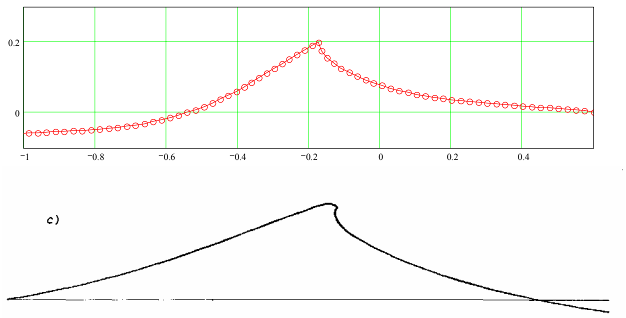

Fig 5. Comparison with Peregrine's exactly-computed deep-water breaking wave ([5] fig 8c).

If any reader is interested in reproducing these results, which are all based on linear wave theory (and are accurate to second order when the waves are regular, see section 3), I would be pleased to e-mail them a copy of my complete MATHCAD spreadsheet.

5. FREQUENCY OF WAVE BREAKING: IMPORTANCE OF BANDWIDTH

To return to the problem posed in section 1, a first application for this simulation is to investigate the frequency of wave breaking. Since wave breaking occurs when the particle velocity in a crest exceeds the phase velocity, the pertinent parameter will be the significant value of the former. The spectrum of particle velocity at a crest elevation equal to the significant wave height h, say, is obtained from the transfer function for horizontal velocity at this elevation, which is ωe^{kh}. For the P-M spectrum (1), this gives the velocity spectrum as:

$$\left|\omega e^{(\omega^2/g)0.05(2\pi g/w_m{}^2)}\right|^2 \frac{s^2\pi g^2}{\omega^5} e^{-(w_m/\omega)^4/\pi} \qquad (7)$$

which is plotted in Figure 6 below.

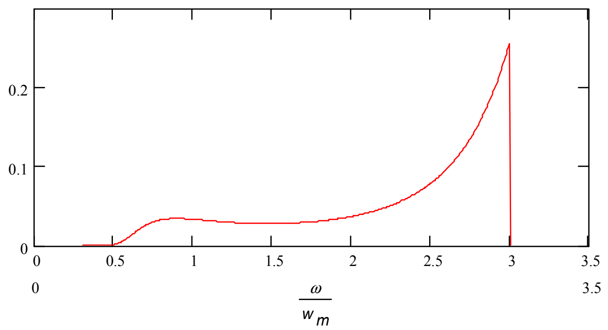

Fig 6. Spectrum of particle velocity in crest

As can be seen, the spectrum diverges rapidly at large ω – it is limited in Figure 6 only by the upper frequency limit of $3w_m$ arbitrarily chosen after equations (3) and (4).

Thus we reach the important conclusion that the frequency of wave breaking will be very sensitive to the *bandwidth* of the spectrum, especially the extent of its high-frequency "tail". The other, practical, conclusion is that no useful results can be obtained using any of the conventional sea spectra, because the high-frequency "tails" are too dominant - the problem is effectively a more severe version of that mentioned after equation (2).

We therefore switch to a simple sine-squared spectrum:

$$S(\omega) = A\cos^2\left\{\frac{\omega - w_p}{w_b}\right\} \quad \text{if } \{\ \} < \pi/2$$

$$= 0 \ \text{ otherwise} \qquad (8)$$

where the peak frequency is w_p and the bandwidth is πw_b. This clearly avoids all problems at large ω, because it is zero there. Of course, this is only the "underlying" spectrum of the linear waves - the as-seen spectrum (i.e. the spectrum of the water surface elevation time-history generated by our particle-sheet waves) will contain contributions from all the non-linearities. The breaking waves, in particular, will give contributions which only decay approximately as ω^{-2} as $\omega \to \infty$ (because the Fourier transform of a "step" discontinuity decays as ω^{-1} as $\omega \to \infty$, see [6] table 1, and thus its contribution to the spectrum decays as ω^{-2} as $\omega \to \infty$).

When A in (8) is chosen to give the same average steepness as the P-M spectrum (1) (i.e. $4\sqrt{(\tfrac{1}{2}A\pi w_b)} = 0.025(2\pi g/w_p^2)$, since for a P-M spectrum the average steepness based on w_p is 0.025), and to have a similar-looking peak (i.e. $\pi w_b = w_p$, say), the MATHCAD simulation gives no breaking at all, in simulations of hundreds of waves. This is in contrast to the earlier simulations of Figures 2 and 4, with the P-M spectrum. To confirm that this is due to the omission of the high-frequency "tail" of the spectrum, we can add another small sine-squared spectrum of 3 times the frequency, while keeping the overall significant waveheight the same. If these additional waves have the same steepness as the original ones, say, they will only have 1/9 of the significant waveheight and 1/243 of the spectral ordinate, but will substantially increase the significant crest velocity considered above. And the MATHCAD simulation now gives a breaking wave every ten waves or so, confirming the importance of the bandwidth.

This latter result could be readily tested in a wave flume (or in a numerical wave flume) by allowing a group of longer-period waves to overtake a group of much smaller shorter-period waves, and seeing if breaking is initiated.

That waves do sometimes break even if they are not very steep is certainly observed at sea. Figure 6 below is a striking example from [4], with eye-witness account by Capt. G.A.Chase of the Maine Maritime Academy.

Fig 7. Large wave breaking at sea

Taken aboard the SS Spray (ex-Gulf Spray) in about February of 1986 (best recollection), in the Gulf Stream, off of Charleston. Circumstances: A substantial gale was moving across Long Island, sending a very long swell down our way, meeting the Gulf Stream. We saw several rogue waves during the late morning on the horizon, but thought they were whales jumping. It was actually a nice day with light breezes and no significant sea. Only the very long swell, of about 15 feet high and probably 600 to 1000 feet long. This one hit us at the change of the watch at about noon. The photographer was an engineer (name forgotten), and this was the last photo on his roll of film. We were on the wing of the bridge, with a height of eye of 56 feet, and this wave broke over our heads. This shot was taken as we were diving down off the face of the second of a set of three waves, so the ship just kept falling into the trough, which just kept opening up under us. It bent the foremast (shown) back about 20 degrees, tore the foreword firefighting station (also shown) off the deck (rails, monitor, platform and all) and threw it against the face of the house. It also bent all the catwalks back severely. Later that night, about 1930, another wave hit the after house, hitting the stack and sending solid water down into the engine room through the forced draft blower intakes.

Note the observation that there was "only a very long swell, of about 15 ft high and probably 600-1000 ft long".

6. CONTRAST WITH VIEW THAT WAVES BREAK DUE TO INSTABILITY

The view that waves break due to instability is described at some length (but without DHP's enthusiastic endorsement!) in the admirable review already cited [5]. The origin of this view lies in the famous Benjamin-Feir instability [7], by which a regular wavetrain becomes increasingly unstable as it steepens. From the viewpoint of wave breaking as particle "escape" under linear theory, that is a very special case, where the bandwidth is vanishingly narrow. If the bandwidth is larger, the waves break sooner.

7. CONTRAST WITH VIEW THAT LINEAR WAVE KINEMATICS NEED "STRETCHING".

A more prosaic contrast is with the widely-held view in the oil industry (e.g. [8] para 2.3.1.c(2)) that in irregular waves linear wave theory over-predicts water velocities in wave crests, and requires "Wheeler stretching" to reduce them. The contrary view is expressed in this present paper - that those "anomalous" crest velocities are realistic, and explain wave breaking.

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Discussion Sheet

Abstract Title :	"Escape" of particle trajectories in linear irregular waves: A new explanation for wave breaking and model of breaking waves.		
(Or) Proceedings Paper No. :	40	***Page :***	155
First Author :	Rainey, R.C.T.		
Discusser :	Alexander H. Day		

Questions / Comments :

The author presents some interesting and thought-provoking results. However I am a little concerned about the consistency of the approach presented. In particular I am worried about the use of linear superposition (as in equations 3 & 4) in conjunction with a second order particle velocity (implied by the use of instantaneous position rather than mean position). I'm also worried about the consistency of the free surface boundary conditions, since it seems that the kinematic condition is satisfied to a different order of approximation than the dynamic condition. I wonder if these inconsistencies contribute to the particle "escape"?

Author's Reply :
(If Available)

There is nothing inconsistent in my approach. There is no single "consistent" approach to water waves, any more than there is, for example, to the problem of a simple pendulum. There, we can either write a differential equation for the angular motion, and obtain it as as $\theta\sin(\omega t)$, or one for the translational motion, and obtain it as $X\sin(\omega t)$. These solutions have different higher-order errors, but they are both correct to first order.

Likewise with water waves, the classical approach of applying a boundary condition on z=0, is not the only approach. We can alternatively observe that the solution to Laplace's equation must of the form (3) & (4), by separation of variables. Then, we can apply boundary conditions on a sheet of particles, and observe that the pressure cancellation (5) applies, provided $\omega^2=kg$. And that we can achieve pressure cancellation to second order, if we add a second-order potential to cancel (6).

The waves we obtain in this way agree exactly with the classical Stokes 1st and 2nd order waves, to 1st and 2nd order respectively. They are effectively "alternative" linear or second order waves, every bit as consistent and rigorous as the classical linear and second order waves. It is only the higher-order components which are different.

You could object that the particle "escape" that we obtain is just such a higher-order effect, and thus has no validity. But you could likewise object that the "escape" to infinity of an Euler strut, when it buckles, has no validity because it violates the small-deflection assumption used to derive it. It is suggestive, though, is it not?

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First Author :	Rainey, R.C.T.		
Discusser :	D. Howell Peregrine		

Questions / Comments :

In computations with "exact" irrotational 2D flow, for initial conditions taken as realisations of wind wave spectra, wave breaking usually occurred in less than 50 periods.
The domain was periodic and the spectrum has maximum wave number twice the minimum plus all second-order interactions.

Author's Reply :
(If Available)

That is very interesting - your initial conditions should make your velocity at first like mine (i.e. like my (3) and (4), plus 2nd order difference-frequency terms), but with a very narrow bandwidth (2:1 ratio of maximum to minimum wave number, plus difference frequencies, whereas I have at least 20:1). Perhaps my simulation would not give breaking waves very often for this case. If so, my suggestion would be that your waves are evolving over the first part of your simulation, so that by the time breaking occurs, your bandwidth (in the same sense, i.e. the range of frequencies in the "underlying" spectrum of the velocity field) is much wider.

If I am right, the time taken for the wave to break in your simulation will not follow an exponential distribution, as it would if the waves were equally likely to break at all times, but will be biased towards long times, because the waves need to evolve first.

Some dependence of breaking-frequency on bandwidth has of course long been acknowledged. Dr. R.G.Standing has drawn my attention to a branch of 1980s literature (papers by Snyder & Kennedy in vol 13 of J.Phys.Oceanography, by Srokosz in vol 16 of same, and Greenhow in vol 16 of Ocean Engng.) which seeks to correlate breaking with m4, i.e. the fourth moment of the surface elevation spectrum. This is the same as the linear-theory water surface slope or particle acceleration in g, see my equation (2) (these authors acknowledge the major difficulty I highlight there, that both are infinite with all the standard spectra). They predict quite a strong dependence of breaking-frequency on bandwidth - but not as strong as I do.

For example, in regular waves, I get breaking when ka is 0.42 (i.e. a linear-theory acceleration of 0.42g), see my reply to Tuck. But in the double sine-squared spectrum considered above figure 7, where I get breaking every ten waves or so, the linear-theory RMS acceleration is only 0.061g, so the acceleration reached every 10 waves is only 0.061(2ln10)^0.5 = 0.13g. So I am saying the breaking-frequency is not simply a matter of the linear predictions of acceleration - increasing the bandwidth is more important than that.

However, my main argument is not a quantitative one about bandwidth, but is about our theoretical understanding of the problem of breaking. Experimentally, regular waves are observed to break at about ka = 0.4, yet conventional linear theory only gives a water surface steepness of 0.4 for that case, and conventional second-order theory only gives a steepness of 0.5. Since the breaking requires infinite steepness, the conventional view is that the explanation of wave breaking must be sought in some fully-nonlinear process beyond these simple wave theories. I disagree. If the assumptions of wave theory are taken in the order I advocate (which is no less rigorous than the classical one, see my reply to Day), breaking appears with those simple theories, at ka = 0.42..

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First Author :	Rainey, R.C.T.		
Discusser :	Paul D. Sclavounos		

Questions / Comments :

In a little known section of Wehausen and Laitone (see p.740 of "Surface Waves", in Encyclopedia of Physics, vol.9, Berlin-Gotingen-Heidelberg: Springer-Verlag, 1960) there exists an exact result by Fritz John who develops an exact relation (eqn. 34.29) between the wave particle velocity and acceleration on the free surface and the Eulerian wave elevation and slope in the deterministic and stochastic case. The relation is an ordinary differential equation with time dependent coefficients. There exists a large body of literature on the stability properties of such equations which may confirm your very interesting result. I wonder if you are aware of it and if you think this is a fruitful approach to follow?

Author's Reply :
(If Available)

I was certainly unaware of John's relation 34.29. If I understand it correctly, it gives the surface particle velocity, starting from a water surface shape $\eta(x,t)$ which already satisfies both the dynamic and kinematic free-surface conditions exactly.

My "escape" phenomenon, however, is associated with water surface shapes which do not satisfy the dynamic free-surface condition exactly. With the exact waves envisaged by John, the manfestation of breaking is not an "escape" of the surface particles, but points where the surface elevation $\eta(x,t)$ becomes mutivalued. It is not clear to me that his relation 34.29 is well-defined at these points - but even if it is, the relation would be taking the fact of breaking as its starting-point.

From my point of view, that is tantamount to assuming what I am seeking to explain.

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First Author :	Rainey, R.C.T.
Discusser :	Ernest O. Tuck

Questions / Comments :

An equivalent of Fig.3 appears in a 1965 JFM paper by me. I actually showed in my paper a somewhat greater domain, and the streamlines at greater heights get even more bizarre. I have an artistic needle-pointed version of this flow held displayed as a hanging decoration on my office wall. When some people see this picture, they say to me - "surely this describes a breaking wave"? Up till now, I have always said, "No, it has nothing to do with breaking". My purpose was to show how "unrealistic" the linearized streamlines can be. However, maybe now I must change my answer from "No", to at least "Maybe"!

Author's Reply :
(If Available)

The figure from your 1965 paper (JFM 22:401-414) is reproduced by J.V. Wehausen on p.215 of his 1973 review paper "The Wave Resistance of Ships" (Adv. Appl. Mech. 13:93-245), where it appears as shown below. Like you, Wehausen believed that the "escape" of the particles had no physical significance, remarking "Whereas streamlines computed according to the linearised theory would have given physically reasonable, athough approximate, streamlines, the 'exact' streamlines are physically nonsensical".

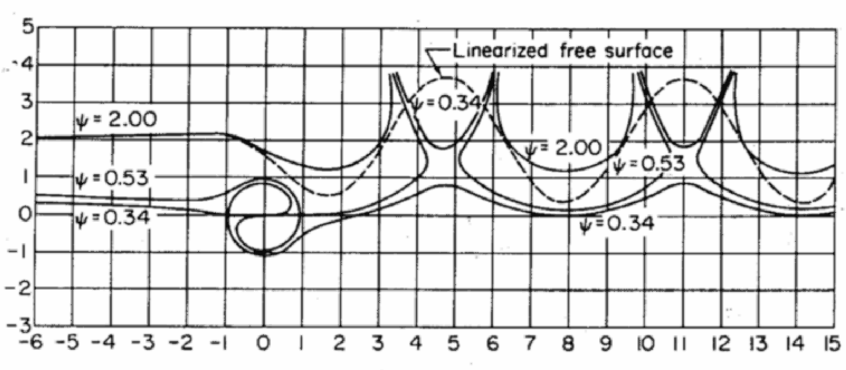

FIG. 38. Streamlines associated with the linearized potential for two-dimensional flow past a submerged dipole. [From Tuck, 1965 (Fig. 1, p. 405).]

However, the waves in the figure are over half a wavelength high, which is, physically, well beyond breaking. Thus my alternative interpretation of the streamlines, which is that the waves have already broken, is physically plausible.

It also raises the question of the threshold steepness of regular waves at which the particles begin to "escape", and thus (according to me) the waves break. If my MATHCAD simulation is run with a linear wave potential of steepness ka, and started with the particles in their linear-theory positions (both horizontal and vertical motions) in a wave trough, then when $ka = 0.48$, the waves just begin to break (i.e. their front face becomes vertical, and the simulation relocates the particles as shown in my Fig.4) as the first crest passes that trough position.

If the particles are started in their second-order positions in a wave trough (which merely requires them to be raised by $0.5ka^2$, the second-order potential being nil in regular waves, of course), the corresponding value of ka is 0.42.

These are also physically plausible values.