Second order internal loads on floating bodies

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Introduction

This note is devoted to the evaluation of the second order internal loads on the floating bodies oscillating about their mean position under the action of an monochromatic incident wave field. The internal loads are defined, in the usual manner, as a difference between the hydrodynamic forces and inertial forces at a given section.

As far as the hydrodynamics is concerned, the calculations are usually performed for a linear case. Due to the assumptions of the linear theory (hydrodynamic model limited by the mean water level $z = 0$ and mean wetted surface of the body, simplified free surface condition, ...) some unnatural results occur, notably the fact that there is no difference between the sagging and hogging moments. This lack of the linear theory can be supressed only by introducing the different kinds of nonlinearities in the model. In principle, any kind of nonlinear aspects will produce the difference between the sagging and hogging moments, so that some authors introduce just the hydrostatic nonlinearities associated with the integration of the hydrostatic pressure under the exact linear wetted surface of the body (the most easy term to calculate). This, of course, is not correct way to treat the problem because the nonlinearities introduced by other effects (nonlinearities of the free surface, dynamic pressure under the wave profile, quadratic term in Bernoulli equation, ...) can influence the results and sometimes even cancel the effects of the hydrostatic nonlinearities. Thus, the coherent way to treat the problem, should be the evaluation of all nonlinear effects at the following order of approximation, linear theory being considered as a first order one. This leads us to the formulation of the second order hydrodynamic problem which has already been used successfully in the studies of global behaviour (slow drift oscillations, springing excitation, ...) of some offshore structures (semi-submersibles, TLP, ...).

The present work was started with the idea that only when we are able to evaluate the importance of different effects we can be (hopefully) in position to eliminate some of them. Otherwise it will be difficult to conclude something serious.

General

The second order theory for global behaviour of the floating body oscillating in waves, is a well established topic nowadays. Thus we will not enter into much details here and we just recall the final expressions for the motion equations in frequency domain at first two orders:

$$\left(-\omega^2([\,\mathbf{M}\,]+[\,\mathbf{A}(\omega)\,])-i\omega[\,\mathbf{B}(\omega)\,]+[\,\mathbf{C}\,]\right)\{\boldsymbol{\xi}^{(1)}\} = \iint_{S_B} p_{DI}^{(1)}\boldsymbol{N}dS \tag{1}$$

$$\begin{aligned}\left(-4\omega^2([\,\mathbf{M}\,]+[\,\mathbf{A}(2\omega)\,])-2i\omega[\,\mathbf{B}(2\omega)\,]+[\,\mathbf{C}\,]\right)\{\boldsymbol{\xi}^{(2)}\} &= \\ \iint_{S_B}[(p_{DI}^{(22)}+p^{(21)})\boldsymbol{N}+p^{(1)}\boldsymbol{N}^{(1)}-\varrho g(Z^{(21)}\boldsymbol{N}+Z\boldsymbol{N}^{(21)}+Z^{(1)}\boldsymbol{N}^{(1)})]dS& \\ +\frac{1}{4}\varrho g\int_{C_B}(\eta^{(1)}-Z^{(1)})^2\frac{\boldsymbol{N}}{\cos\gamma}dC+\left\{\begin{matrix}0\\ \omega^2\boldsymbol{\Omega}^{(1)}\wedge([\,\boldsymbol{I}\,]\boldsymbol{\Omega}^{(1)})\end{matrix}\right\}&\end{aligned} \tag{2}$$

where the detailed definitions of different terms may be found in [2].

Once the first order potential $\varphi^{(1)}$ calculated almost all terms in the above equations may be evaluated quite straightforwardly, except the part related to the second order diffraction potential $\varphi_D^{(2)}$. The usual way of calculating this part is by using the Haskind relations which allows the evaluation of the forces without explicit calculation of the potential. Since the modified form of these relations will be used in this work, we briefly recall the basics of this method. An assisting radiation potential is defined by the following boundary value problem (BVP):

$$\left.\begin{array}{ll} \Delta\psi_i = 0 & \boldsymbol{r} \in \forall \\ -4\nu\psi_i + \dfrac{\partial \psi_i}{\partial z} = 0 & z = 0 \\ \dfrac{\partial \psi_i}{\partial n} = N_i & \boldsymbol{r} \in S_B \\ \lim\Big[\sqrt{\kappa_0 r}\Big(\dfrac{\partial \psi_i}{\partial r} - i\kappa_0\psi_i\Big)\Big] = 0 & r = \sqrt{x^2+y^2} \to \infty \end{array}\right\} \tag{3}$$

where $\kappa_0 \tanh \kappa_0 H = 4\nu$.

The use of the Green theorem for ψ_i and $(\varphi_I^{(2)} + \varphi_D^{(2)})$ allows the derivation of the following expression for the part of the forces due to $p_{DI}^{(22)}$:

$$F_{DIi}^{(22)} = 2i\omega\varrho \iint_{S_B} (\varphi_I^{(2)} N_i + \psi_i Q_B) dS + 2i\omega\varrho \iint_{S_F} \psi_i Q_F dS \tag{4}$$

where the forcing functions Q_F and Q_B are the right hand side terms of the second order boundary conditions on the body and the freee surface respectively:

$$\begin{aligned} Q_B &= -\frac{\partial \varphi_I^{(2)}}{\partial n} - \frac{i\omega}{2}\boldsymbol{\Omega}^{(1)} \wedge (\boldsymbol{\Omega}^{(1)} \wedge \boldsymbol{R}) - \frac{1}{2}\{[(\boldsymbol{\xi}^{(1)} - \boldsymbol{\Omega}^{(1)} \wedge \boldsymbol{R})\nabla]\nabla\varphi^{(1)}\}\boldsymbol{n} \\ &\quad + \frac{1}{2}(\boldsymbol{v}^{(1)} - \nabla\varphi^{(1)})(\boldsymbol{\Omega}^{(1)} \wedge \boldsymbol{n}) \end{aligned} \tag{5}$$

$$\begin{aligned} Q_F &= \frac{i\omega}{g}\{(\nabla\varphi_B^{(1)}\nabla\varphi_B^{(1)} + 2\nabla\varphi_I^{(1)}\nabla\varphi_B^{(1)}) \\ &\quad - \frac{1}{2}[(\varphi_B^{(1)} + \varphi_I^{(1)})(\frac{\partial^2 \varphi_B^{(1)}}{\partial z^2} - \nu^2\varphi_B^{(1)}) + \varphi_B^{(1)}(\frac{\partial^2 \varphi_I^{(1)}}{\partial z^2} - \nu^2\varphi_I^{(1)})]\} \end{aligned} \tag{6}$$

The most difficult part to calculate is the part associated with the slowly convergent free surface integral. However, the efficient methods exist today for evaluation of this integral with high accuracy and acceptable CPU time.

Internal loads

Till now we were concerned with the evaluation of the global forces on the floating objects but we stated in the introduction that we are mainly interrested in the internal loads so that further clarifications are necessary. By definition, the internal loads are the difference between the external forces and inertial forces at considered section k of the body (fig. 1).

We consider first and second order cases separately.

First order

In the linear case, the above operation can be performed relatively easily because we solve directly the BVP for the potential so that the total pressure is explicitely known at each body

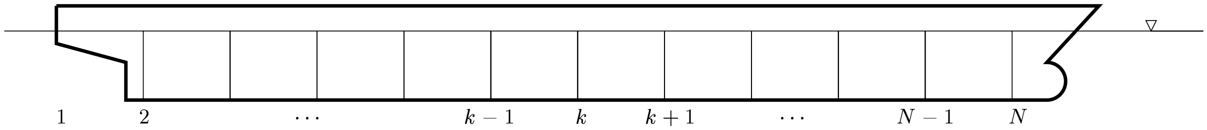

Figure 1: *Subdivision of the mean wetted part of the body.*

panel. According to the figure 1 we can write for the first order internal loads, at section k, following expression :

$$\boldsymbol{f}^{(1)k} = \boldsymbol{F}^{(1)k} + \omega^2 [\ \mathbf{M}\ ]^k \{\boldsymbol{\xi}^{(1)}\} \tag{7}$$

where $\boldsymbol{F}^k$ means the external pressure force obtained by integrating pressure over the "cutted" part (from the section k to the end of the body) only and $[\ \mathbf{M}\ ]^k$ is the corresponding inertia matrix.

Second order

The second order problem is more complicated because we don't know the distribution of the second order potential over the body surface, since we used Haskind relations for the calculation of $F_i^{(22)}$. However, the Haskind relations can be used again, in the modified form, for the calculation of this part of the forces at each section. In fact, in order to calculate the internal loads due to the second order potential, we need to calculate the following quantity :

$$F_{DIi}^{(22)k} = 2i\omega\varrho \iint_{S_k} (\varphi_I^{(2)} + \varphi_D^{(2)}) N_i dS \tag{8}$$

where the surface S_k is the mean wetted surface contained between the section k and the end of the body.
As for the global forces, we introduce the assisting radiation potential ψ_i^k associated with the surface S_k. The BVP for this potential is the same as that for ψ_i (3) except for the condition on the body which is now :

$$\frac{\partial \psi_i^k}{\partial n} = \begin{cases} N_i & \boldsymbol{R} \in S_k \\ 0 & \boldsymbol{R} \neq S_k \end{cases} \tag{9}$$

We apply now the Haskind theorem to the potential ψ_i^k and the second order diffraction potential $\varphi_D^{(2)}$ and we obtain the following expression for $F_{DIi}^{(22)k}$:

$$F_{DIi}^{(22)k} = 2i\omega\varrho \iint_{S_k} \varphi_I^{(2)} N_i + 2i\omega\varrho \iint_{S_B} \psi_i^k Q_B dS + 2i\omega\varrho \iint_{S_F} \psi_i^k Q_F dS \tag{10}$$

Due to the same form of the BVP's, the potential ψ_i^k will have the same characteristics as ψ_i, so that the same method can be used for the evaluation of the troublesome free surface integral. The remaining part of the second order force $\boldsymbol{F}^{(2)k}$ is easily calculated by integration of known quantities over S_k, so that we can now write the final expression for the second order internal loads at section k:

$$\boldsymbol{f}^{(2)k} = \boldsymbol{F}^{(2)k} + \begin{Bmatrix} 0 \\ \omega^2 \boldsymbol{\Omega}^{(1)} \wedge ([\ \boldsymbol{I}\ ]^k \boldsymbol{\Omega}^{(1)}) \end{Bmatrix} + 4\omega^2 [\ \mathbf{M}\ ]^k \{\boldsymbol{\xi}^{(2)}\} \tag{11}$$

The last things we should note is that the internal loads, as calculated above, are expressed with respect to the center of gravity and care should be taken when transfering them to the reference point on the section k. The correct way to do is:

$$\boldsymbol{f}_P^k = \boldsymbol{f}^k + \left\{ \begin{matrix} 0 \\ (\boldsymbol{R}_G - \boldsymbol{R}_P) \wedge \boldsymbol{f}^k \end{matrix} \right\} \tag{12}$$

where $\boldsymbol{R}_P$ is the position vector of the reference point and, of course, the vector product involves only the force component of $\boldsymbol{f}^k$. The above (12) is true both for the first and second order.

Numerical results and discussions

On the figure 2 we present some preliminary results for the container ship in oblique wave conditions. The comparison between the numerical calculations and the different experimental results is presented. More results will be presented at the Workshop.

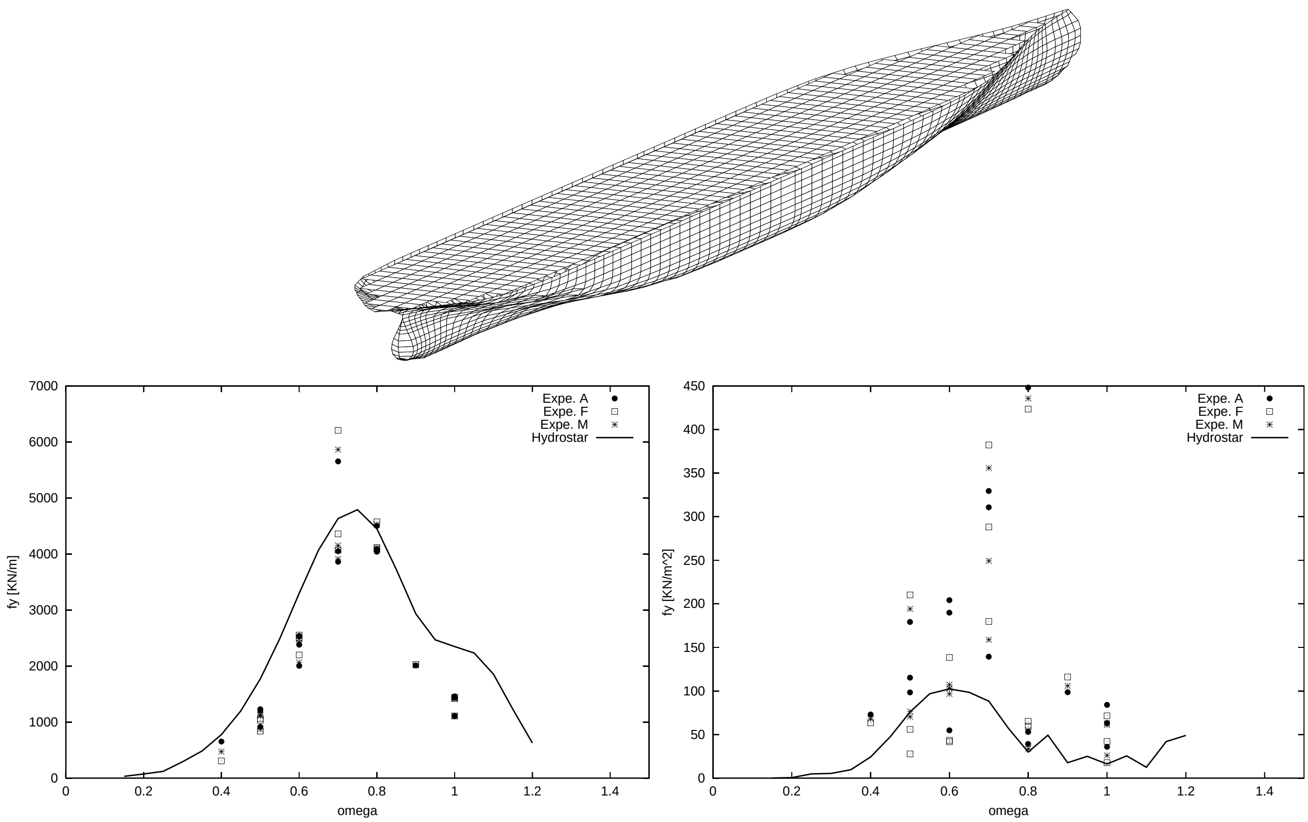

Figure 2: *First and second order horizontal shear force F_y at midship section of an container vessel.*

References

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Discussion Sheet

Abstract Title :	Second order internal loads on floating bodies		
(Or) Proceedings Paper No. :	30	***Page :***	115
First Author :	Malenica and Mravak		
Discusser :	J. Nick Newman		

Questions / Comments :

How important is the nonlinear component of hogging and sagging in practical problems? It seems that naval architects may overemphasize this relatively small effect.

The indirect (Haskind) method is awkward to apply here since the auxiliary potential must be evaluated separately for each structural load point. We have found that the computational cost of the integrated force and moment using the direct solution for the second-order potential is practically the same as using the Haskind method, as explained in the following reference:

"The computation of second-order wave loads," by C.-H. Lee, J.N. Newman, M.-H. Kim and D.K.P. Yue, 1991 OMAE Conference, Volume 1-A, pp 113-123.

Author's Reply :
(If Available)

The difference between sagging and hogging moments may sometimes be very large, (up to 30 percent for some ships), and correct evaluation of sagging and hogging moments seems to be quite an important issue in shipbuilding. (e.g. see. 14th ISSC Report, Ch. Extreme hull girder loading). After short bibliogaphial research we realized that these non-linearities are usually taken into account using very rough approximations, so that we decided to compare more consistent theories like a second order one. That was the main purpose of this paper. The second order loads are certainly not the only non-linear part contributing to the global loads but, what we want to show here is that, all non-linearities should be taken into account properly. If this is true at second order it should probably be true for higher order effects too.

Thanks for the second comment.
When we started to work on this problem, we had in hands the original BV-HYDROSTAR code where the Haskind relations were already implemented, so that it was much easier for us to continue to use this method. At that time, we didn't really look into the details of the CPU issues. However after your comments we investigated this point in more detail and we realized that you should be right, and CPU time should be equivalent for the two methods.

Discussion Sheet

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(Or) Proceedings Paper No. :	30	***Page :***	115

First Author :	Malenica and Mravak
Discusser :	Rod C.T. Rainey

Questions / Comments :

This is really a comment on the comment from J.N. Newman. I agree that naval architects appear to have over-emphasised nonlinear hogging and sagging - I believe the reason may be that they base their calculations on the nonlinear surface profile of an undisturbed wave, apparently in ignorance of the fact that there is no corresponding 2nd order pressure (in deep water). See for example Ch.6 in Rawson & Tupper's "Basic Ship Theory" (vol.1, 4th Ed. 1994).
But there are other internal loads on ships where the 2nd order contribution is important - the longitudinal force near the bow, for example.

Author's Reply :
(If Available)

Part of the answer is in the answer to Prof. Newman question. As we already said, our understanding is that the methods commonly used in the calculations of the sagging-hogging differences are very simplified and not consistent. As you mention, some of them just use the incident wave profile and account for some hydrostatic effects, which is not correct, even if the non-linear theories are used for incident wave definitions. There are many other nonlinear effects (second or higher order) missing in these models, and these non-linear effects may affect the forces and moments in any direction.

Questions from the floor included; Odd Faltinsen, Masashi Kashiwagi & Paul Sclavounos.