Free surface deformation due to an impulsively moved plate

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The topic of free surface deformation due to the interaction between a free surface and a moving solid boundary has been extensively studied e. g.Peregrine (1972), Cointe (1987,89), Howison *et al* (1991). In 1994, King and Needham found the complete solution for the case of a vertical plate accelerated into a strip of stationary inviscid fluid. This analysis was performed in the small time limit, where all boundary value problems could be solved analytically, and expressions for the height the fluid rises up the plate and the free surface gradient were given. The utility of these results in an age when sophisticated computer codes can attempt to solve this problem is both of checking the accuracy of the code and giving insight into the structure of the flow field. In the case of an accelerating plate the flow near the intersection of the free surface and the plate is dominated by a region, of dimensions, $0\left(-t^2 lnt\right)$, in which the main feature is uniform vertical translation of a block of fluid to form the root of any subsequent jet. Corrections to this main primary flow feature, determine that the gradient of the free surface where it intersects the plate is $0\left(\frac{1}{lnt}\right)$. Some support for these conclusions can be found in the experimental study of Yong & Chwang (1992).

In this work we will extend our previous analysis to the case of a plate made to move into a stationary strip of fluid with velocity U. This flow is rather more violent than the previous case, and consequently more difficult to analyse. Firstly we find that the impulsive boundary motion produces a pressure impulse in the fluid. This is singular at the intersection of the plate and free surface. To resolve this singularity it is necessary to consider regions of size $0\left(-tlnt\right)$ and then $0(t)$ about this point in order to find a boundary value problem which captures the dynamics of the interaction between plate and free surface in such a manner that the solution is bounded. The final boundary value problem is non-linear and parameter free and takes the form

$$\nabla^2\Phi_1 = 0 \quad \text{in} \;\; 1 \le x < \infty, \;\; -\infty < y \le \eta_1 \tag{1}$$

$$\Phi_{1x} = 1 \quad \text{at} \;\; x = 1 \tag{2}$$

$$-\frac{2}{\pi}\eta_1 + \Phi_1 - x\Phi_{1x} - \eta_1\Phi_{1y} + \frac{2}{\pi}\Phi_{1y} + \frac{1}{2}\left(\Phi_{1x}^2 + \Phi_{1y}^2\right) = 0 \text{ at } y = \eta_1 \tag{3}$$

$$-\Phi_{1y} - \frac{2}{\pi} + \eta_1 - x\eta_{1x} + \Phi_{1x}\eta_{1x} \text{ at } y = \eta_1 \tag{4}$$

$$\begin{aligned}\Phi_1 \sim \Phi_\infty = &-\frac{2}{\pi}r\sin\theta\ln r + \frac{2r}{\pi}\sin\theta\left(1 + \ln\frac{4}{\pi}\right) - \frac{2}{\pi}r\theta\cos\theta \\ &- \frac{2}{\pi^2}\left(\ln\hat{r} - \ln\frac{4}{\pi}\right)^2 + \frac{2}{\pi^2}\theta^2 + \frac{4}{\pi}\theta \quad \text{as} \;\; r \to \infty\end{aligned} \tag{5}$$

$$\eta_1 \sim \eta_\infty = \frac{2}{\pi}\left(-\ln r + \ln\frac{4}{\pi} + \frac{1}{r}\right) \quad \text{as} \;\; r \to \infty. \tag{6}$$

where ϕ_1 is the velocity potential and η_1 is the free surface elevation. The conditions at infinity arise from matching to the previous region.

Due to its nature, the solution of this boundary value problem only requires a single computation, which is carried out by the boundary integral method, to find the flow field and free surface near to the tip of this nascent jet. Results of this computation will be presented and some limitations and further applications of this type of analysis will also be discussed.

References

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Discussion Sheet

Abstract Title :	Free surface derformation due to an impulsively moved plate		
(Or) Proceedings Paper No. :	22	***Page :***	085
First Author :	King, Cooper and Needham		
Discusser :	J. Nick Newman		

Questions / Comments :

There were several papers dealing with this topic in the 1980's. In particular, W.-M. Lin developed a discretized boundary-integral method where a double-collocation point was imposed at the intersections of the free surface and vertical boundaries, using both boundary conditions; this produced stable and convergent results (globally), which are summarized in the following paper:

"Nonlinear forced motions of floating bodies," by W.-M. Lin, J.N. Newman and D.K. Yue, Fifteenth Symposium on Naval Hydrodynamics (1984), pp 33-47.

Author's Reply :
(If Available)

Author did not respond.

Questions from the floor included; Marshall Tulin & Guo Xiong Wu.