Drift force on an array of vertical cylinders

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Introduction

As is theoretically indicated by Maniar & Newman (1997), waves incident to a long array of equally-spaced vertical cylinders can be trapped among the cylinders and extreme water-surface elevations among the cylinders or very large hydrodynamic forces on the cylinders can be induced. However, actual water-surface elevations or hydordynamic forces caused by the waves trapped among the cylinders turned out to be much smaller than or even qualitatively different from the theoretical predictions based on the linear potential theory (Kagemoto, Muari & Saito (1999)). It has been indicated by Kagemoto et al. (1999), or in more refined way by Molin (1999), that these apparent contradictions observed between the linear potential theory and the actual phenomena can be attributed to the tiny additional damping forces that may be induced by the viscosity.
Supposing that the velocity potential Φ representing the flow field around the cylinders in waves of circular frequency ω is written as,

$$\Phi(x,y,z,t) = Re\left\{\phi(x,y,z)e^{-i\omega t}\right\} \tag{1}$$

Molin (1999) proposed to impose the following boundary condition on the vertical body-surfaces of the cylinders.

$$\frac{\partial \phi}{\partial r} = -i\varepsilon\frac{\phi}{a} \tag{2}$$

where the lefthand-side represents the normal flow velocity on the vertical surface of the corresponding cylinder and ε is a certain real positive value. When $\varepsilon = 0$, the usual 'no-penetration' body-boundary condition is reproduced. With a finite value of ε, on the other hand, the above equation physically implies that the corresponding cylinder surface is slightly porous. The body-boundary condition (2) can be easily implemented in the existing computer code based on the linear potential theory, while the the additional damping force that may be induced due to the energy dissipation in the viscous boundary layers along the cylinder surfaces can be accounted for in an approximate way. With appropriate choice of ε value, the theoretical predictions of such responses as the water-surface elevations among the cylinders turned out to agree quite well with the corresponding experimental results (Kagemoto, Murai, Saito, Molin and Malenica (2002)). This fact could have very important practical implications for the design of floating structures supported on a large number of cylindrical columns, which are actually extensively studied in Japan as possible alternatives to conventional land-based airports, in that, other than the water-surface elevations, various practically important responses of such column-supported structures may also be very different from the linear-theory predictions.

Among the possible important responses that may be affected by the additional damping force, the present work focuses on the drift force that will act on an array of vertical cylindrical columns, which is a quite important quantity for the design of actual structures.

Drift-force calculation

Prior to the actual computation of the drift force on an array of cylinders, the applicability of the conventional calculation methods of the drift force to a porous body was examined. For this purpose, a bottom-mounted vertical cylinder of raidus a in a regular wave train (wave amplitude ζ_a, circular frequency ω) progressing in x direction was considered. Here the rectangular coordinate system (x, y, z) is used, in which the x, y axes lie on the undisturbed free surface while the z axis stretches vertically upward. The water depth and thus the column length is assumed to be infinite for the sake of simplicity, although the results shown later are practically applicable for a cylinder of finite but large draft.

The velocity potential that satisfies the body-boundary condition (2) can be obtained in exactly the same manner as that shown by McCamy & Fuchs (1954) and the drift force $\overline{F}_x$ acting on the cylinder in the x direction can be written in an explicit form as follows.

$$\overline{F}_x/\rho g\pi a\zeta_a^2 = \frac{1}{2}\sum_{n=0}^{\infty} Im(\alpha_{n+1}\alpha_n^*)\left\{1 - \frac{n(n+1)}{(k_0a)^2} - \frac{\varepsilon^2}{(k_0a)^2}\right\} \tag{3}$$

with

$$\alpha_n \equiv J_n(k_0a) - \frac{J_n'(k_0a) + \frac{i\varepsilon}{k_0a}J_n(k_0a)}{H_n'(k_0a) + \frac{i\varepsilon}{k_0a}H_n(k_0a)}H_n(k_0a) \tag{4}$$

Here J_n, H_n represent the n-th order Bessel function of the first kind and the n-th order Hankel function of the first kind respectively. k_0 denotes the wavenumber of the incident wave. The prime indicates the derivative with respect to the argument and the asterisk denotes the complex conjugate.

In case of $\varepsilon = 0$, the above expression is further reduced to the following very simple form.

$$\overline{F}_x/\rho g\pi a\zeta_a^2 = \frac{4}{(\pi k_0 a)^3}\sum_{n=0}^{\infty}\left\{1 - \frac{n(n+1)}{(k_0a)^2}\right\}^2 \frac{1}{\mid H_n'(k_0a)\mid^2 \mid H_{n+1}'(k_0a)\mid^2} \tag{5}$$

It is known that drift forces acting in horizontal direction can also be calculated from the momentum change in the fluid volume surrounding the corresponding body. The detailed mathematical manipulation can now be found in many textbooks (e.g. Mei (1983)) and the final formula are shown here.

Supposing that the scattered velocity potential Φ_s is expressed at large r as,

$$\Phi_s = -Re\left\{\frac{ig\zeta_a}{\omega}A(\theta)\left(\frac{2}{\pi k_0 r}\right)^{1/2} e^{i(k_0r-\omega t-\pi/4)}e^{k_0z}\right\} \tag{6}$$

where (r, θ, z) is the cylindrical coordinate system fixed to the center of the water-surface cross section of the cylinder and $A(\theta)$ represents a certain function of θ, then the drift force $\overline{F}_x$ can be calculated as follows.

$$\overline{F}_x/\rho g\pi a\zeta_a^2 = -\frac{1}{2(\pi k_0 a)}\left\{\frac{1}{\pi}\int_0^{2\pi}\cos\theta \mid A(\theta)\mid^2 d\theta + 2Re\left(A(0)\right)\right\} \tag{7}$$

In the present case, however, since the body-surface is assumed to be slightly porous, the following additional term $\delta\overline{F}_x$ is needed, which vanishes if the corresponding body is watertight.

$$\delta\overline{F}_x = -\overline{\int\int_{S_B} \rho \frac{\partial\phi}{\partial x}\frac{\partial\phi}{\partial n} dS} \qquad (8)$$

where the overline indicates that the time average is taken and $\partial/\partial n$ denotes the derivative in the normal direction (outward from the flow region) at the body surface S_B.
Table 1 compares the drift force $\overline{F}_x/\rho g\pi a\zeta_a^2$ calculated by the equations (3),(7),(7)+(8). It can be known that the body-surface-integraion part $\delta\overline{F}_x$ can take appreciable portion of the total drift force as ε increases.

Table 1 Comparisons of the drift force calculated by diffrent formula

	$2ka$	eq.(3)	eq.(7)	eq.(7)+eq.(8)
$\varepsilon = 0.000$	1.00	0.9103204E-01	0.9103186E-01	0.9103186E-01
	2.00	0.2116524E+00	0.2116520E+00	0.2116520E+00
	3.00	0.1911160E+00	0.1911156E+00	0.1911156E+00
$\varepsilon = 0.003$	1.00	0.9105114E-01	0.9402962E-01	0.9105095E-01
	2.00	0.2108890E+00	0.2121831E+00	0.2108886E+00
	3.00	0.1905345E+00	0.1914794E+00	0.1905341E+00
$\varepsilon = 0.030$	1.00	0.9097961E-01	0.1200670E+00	0.9097935E-01
	2.00	0.2040098E+00	0.2168095E+00	0.2040093E+00
	3.00	0.1853504E+00	0.1946938E+00	0.1853500E+00
$\varepsilon = 0.300$	1.00	0.6732617E-01	0.2985854E+00	0.6732558E-01
	2.00	0.1378781E+00	0.2515847E+00	0.1378775E+00
	3.00	0.1379171E+00	0.2208587E+00	0.1379166E+00

Drift force on an array of vertical cylinders

As shown in the previous section, since, if $\varepsilon \neq 0$, the horizontal drift force can not be calculated from the far field only, the drift forces shown in this section were calculated by the pressure integration over the body surface of the cylinders. The example results are shown in Fig.1. In the figure, the present calculations on the total horizontal drift force acting on an array of 16×4 vertical truncated cylinders are compared with the experimental ones published by Kashiwagi (2000). Although, unlike the dynamic responses such as water-surface displacements or wave-induced body motions, the drift force on a fixed body may not be very sensitive to damping forces, its frequency response characteristics shown in the figure are somewhat smoothed with the inclusion of positive ε and become closer to the experimental results.
Further investigation will be conducted on the drift force acting on each cylinder and the implications for the design of a floating structure supported on a large number of periodically-arrayed columns will be investigated.

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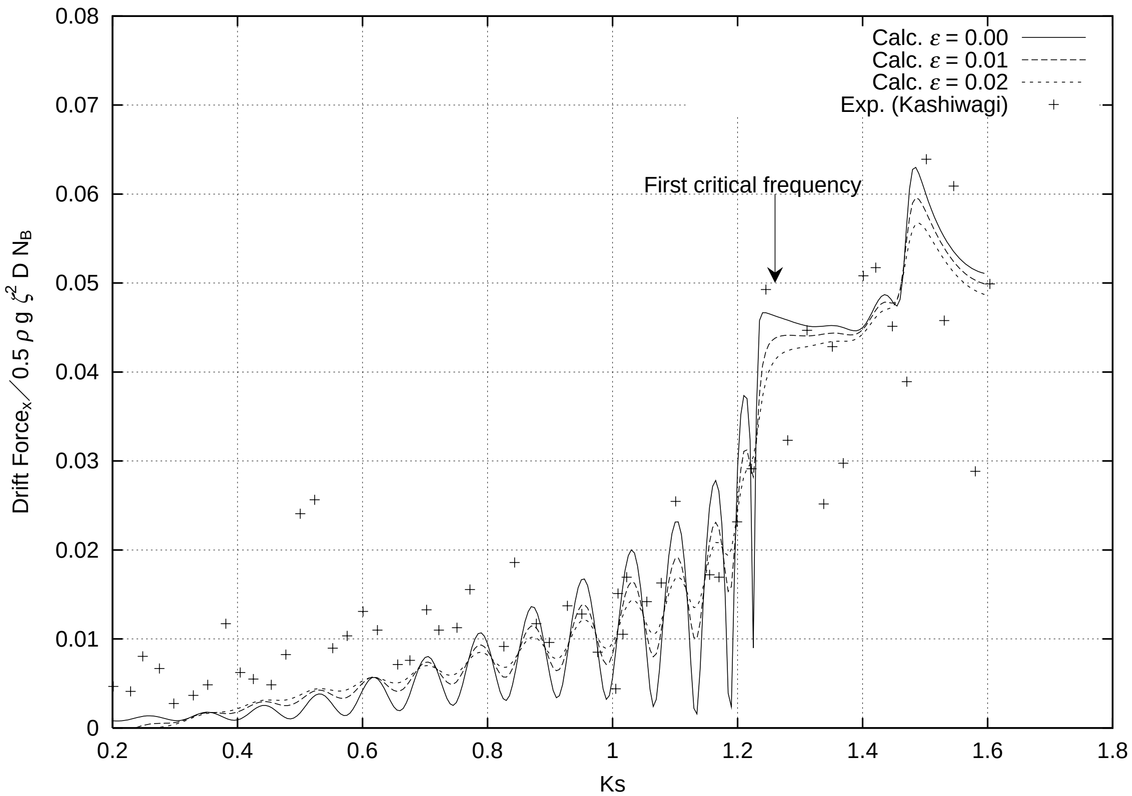

Figure 1: Drift force on an array of 16 × 4 vertical truncated cylinders (d/a=4, s/a=2) (A:incident-wave amplitude, D:cylinder diameter a:cylinder radius, d:cylinder draft, 2s:center-to-center distance, N_B:number of cylinders)

Discussion Sheet

Abstract Title :	Drift force on an array of vertical cylinders		
(Or) Proceedings Paper No. :	17	***Page :***	065
First Author :	Kagemoto, H		
Discusser :	David V. Evans		

Questions / Comments :

The interesting effect at kr = ½ may be connected to the fact that you have an array of 4 x 16 cylinders. The near-trapping at kr = 1.2 corresponds to a single array of cylinders, first shown by Maniar and Newman. But, we [Evans & Porter JFM] have found there are 4 trapped mode frequencies for 4 cylinders equally spaced in a tank and it may be that kr = ½ corresponds to one of these.

Author's Reply :
(If Available)

As is observed in http://www.ocean.jks.ynu.ac.jp/~murai/iwwwfb17-p65.htm, the numerical fact is that nothing particular happens at around Ks=0.5.

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(Or) Proceedings Paper No. :	17	***Page :***	65
First Author :	Kagemoto and Murai		
Discusser :	J. Nick Newman		

Questions / Comments :

There seems to be a singular regime in the vicinity of Ks=1/2, both in Kashiwagi's experimental data shown in Figure 1 and also in some of your own results. Do you know of any explanation for this?

Author's Reply :
(If Available)

As is observed in http://www.ocean.jks.ynu.ac.jp/~murai/iwwwfb17-p65.htm, the numerical fact is that nothing particular happens at around Ks=0.5.