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SOME DYNAMICAL ASPECTS OF FREAK WAVES

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1 Introduction

The wave environment determines important loads on marine structures that operate at sea, either in transport, kept in position or being stationary. The description of the ocean waves is an important part of the determination of the loads acting on the structures. A proper design wave may in many cases fulfil engineering needs. However, a number of observations suggest the existence of certain giant waves - sometimes called freak waves or rogue waves. They are significantly higher and steeper than what is expected by current knowledge of wave statistics, under the given weather conditions. Visual observations of very large and steep waves are documented, see ISSC-Report (2000). As a first step towards a description of interaction between extreme waves and marine structures we here analyse what can be a realistic wave input. We model large wave events using a fully nonlinear solver and compare the results with simulations using simplified Schrödinger-like equations.

2 Numerical experiment

We investigate the evolution of a localized long wave packet. First, we compute an exact steady Stokes wave, with wavenumber k_0 and half-height a. The surface elevation and the tangential velocity at the surface are then multiplied by the 'bell' function: $\operatorname{sech}\left[\epsilon\sqrt{2}ak_0^2(x-x_0)\right]$, where the parameter ϵ determines the length of the packet. The case $\epsilon = 1$ corresponds to an exact NLS soliton. This problem has been studied analytically by Satsuma and Yajima (1974) using NLS, and numerically by Lo and Mei (1983) using Dysthe's equation. We complete their works in comparing simulations using the fully nonlinear equations (Clamond & Grue 2001) with the (fully dispersive) extended Dysthe's equation of Trulsen *et al.* (2000).

We consider here a wave packet with $ak_0 = 0.091$ and $\epsilon = 0.263$. The computational domain involves 128 wavelengths, and the carrier wave is discretized over 32 nodes per wavelength. This means that all harmonics up to the 15th are resolved, and that 128 Fourier modes are included in the spectral band $[k_0-\frac{1}{2}k_0; k_0+\frac{1}{2}k_0]$. (Runs with several resolutions have been made, for check.)

According to predictions by NLS, three solitons should be formed, and, in addition, some dispersive tails. This is indeed demonstrated by all the models. The initially long group splits, after some time, into solitons which become interacting. Large waves up to three times the initial maximum elevation are formed. These freak waves appear for a while. After a period, the 'freaking' stops, also for a while. Then it starts again, and stop, and restart, etc. To understand this recurrent phenomenon, we consider the wave envelope of the fundamental wavelength.

With the fully nonlinear simulations, two solitons appear in the front of the train (Fig. 1). The flow is highly unsteady. A rapid exchange of energy takes places. Large and frequent waves are thus formed. More surprisingly, the small soliton can "pass ahead" the larger one (Fig. 1-b). This is an unexpected behaviour, according to simplified models. Such a behaviour has neither been observed experimentally. This is most possibly due to the spatial limitation of physical wave tanks.

With the extended Dysthe equation, the two-soliton appear in the back of the train. Once formed, the evolution of the solitons is almost steady. A large steady soliton run ahead. Little energy is exchanged between the solitons. Weak interaction indicates a gentle variation of the height of the two last solitons. As a consequence, no very big waves are formed (Fig. 3).

For short times, however, the maximum elevation predicted by the two models match. In particular, the first large wave event is rather well predict in both time and amplitude (Fig. 1-a). This is logical since Dysthe's equation is valid while $(ak_0)^3\omega_0 t \leq O(1)$.

When the wave packet propagates without creating large waves, the energy is concentrated in a narrow-banded region of the spectrum (Fig. 2-a-c). During the 'freaking' events, an important quantity of energy is transferred between the Fourier modes. The large waves being steep, high wavenumbers contains a significant portion of the energy (Fig. 2-b-d). In other words, important energy is transferred from low to high wavenumbers. Figure 2 illustrates a growth in the form of side-band instability. This instability is not a Benjamin–Feir one. Indeed, in this experiment, all the Fourier modes are locked in phase. Shorter packet (i.e. when only one soliton can be formed) have spectra that are more broad-banded spectra. These spectra also include the unstable modes in the Benjamin–Feir sens. In such cases, no large waves are formed, however.

The temporal scenario presents different interesting characteristics. Intermittent oscillations appear when the waves are large. The period of these oscillations is twice the fundamental period (see the zoom in Fig. 3). Some energy is then transfered to the lower frequencies. No equivalent phenomenon occurs in space (i.e. no double wavelength appears). This phenomenon is not at all predicited by any simplified equations. The reason is that the period doubling is automatically excluded by the assumption of narrow-banded spectra.

Period-doubling is a well-known mechanism explaining the transition to turbulence of viscous flows. In our knowledge, this phenomenon for water wave has never been described. It is certainly an important aspect for understanding the mechanisms leading to breaking.

3 Conclusion

Comparisons — between a fully nonlinear and approximate equations — of some freak wave events, show both quantitatively and qualitatively very different behaviours. Some apparently very important phenomenae, like intermittence, are not at all predicted by simplified equations of Schrödinger-type. The cause is more due to their narrow-banded spectra assumption than due to their weakly nonlinearities.

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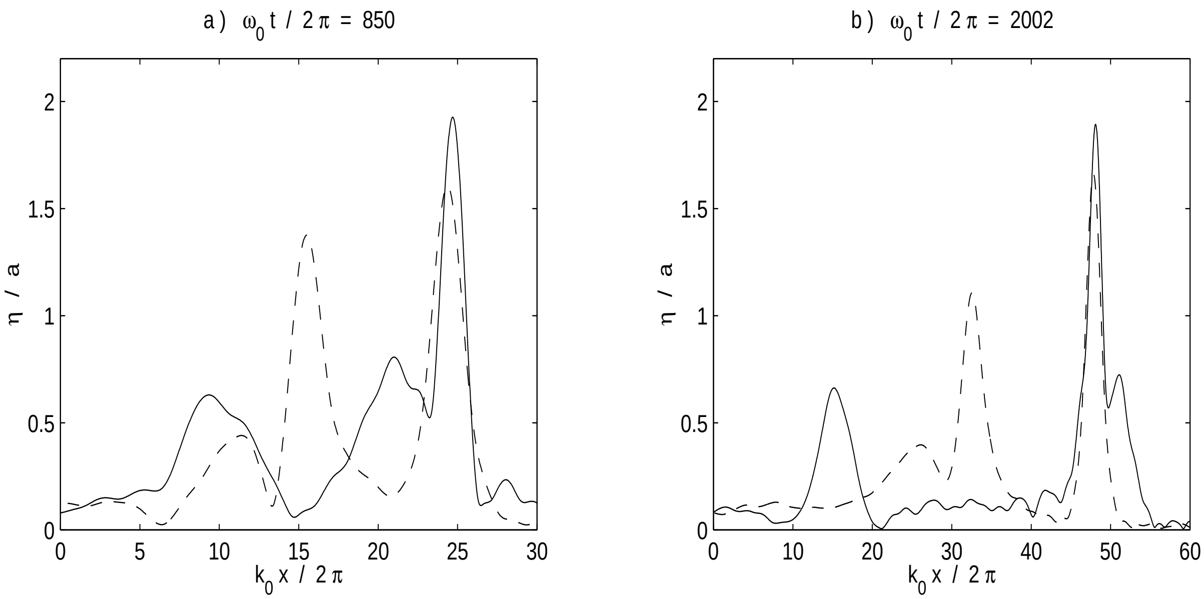

Figure 1: *Surface's envelope at two different times.*
— Fully nonlinear, – – Extended Dysthe's equation.

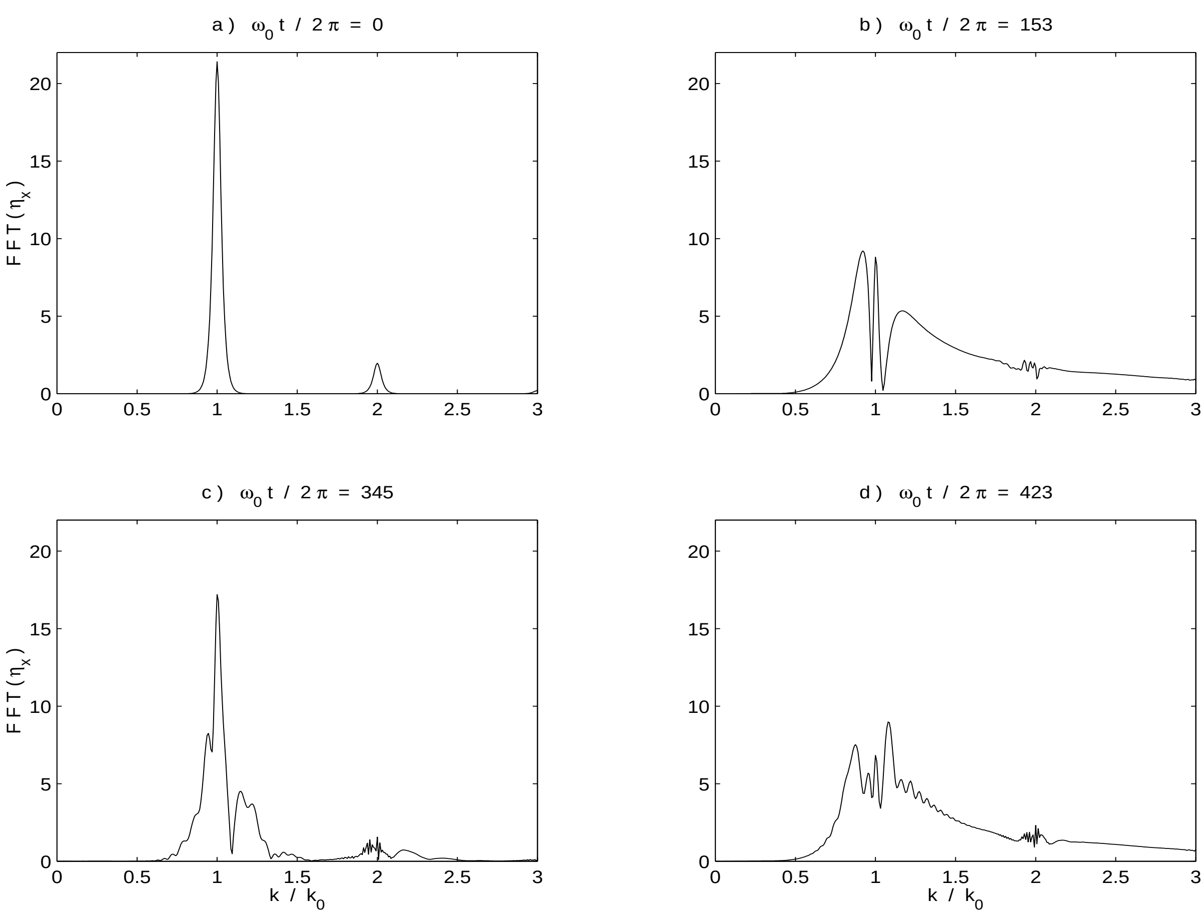

Figure 2: *Spectrum of surface's slope at two different times.*

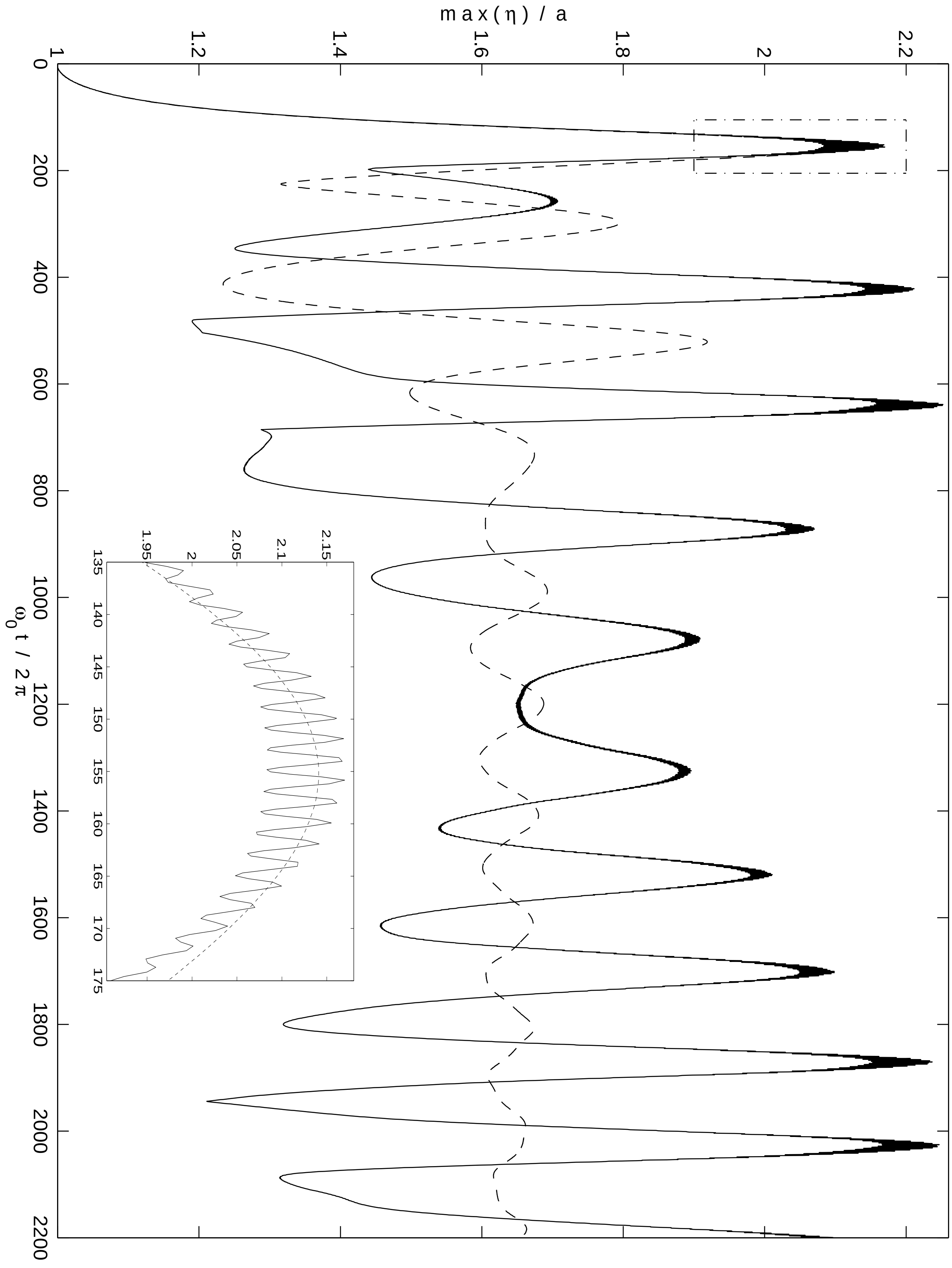

Figure 3: *Temporal evolution of the surface maximum elevation.*
— Fully nonlinear, – – Extended Dysthe's equation.

Discussion Sheet

Abstract Title :	Some dynamical aspects of freak waves		
(Or) Proceedings Paper No. :	08	***Page :***	029
First Author :	Clamond, D.		
Discusser :	D. Howell Peregrine		

Questions / Comments :

The maximum wave events are remarkably similar to those found in other studies such as the Henderson, Peregrine & Dold (Wave Motion, 29, pp341-361,1999) and indicate that for 2D waves they can represent a model of a freak wave event, since they occur from a variety of initial conditions. It is much harder to evaluate their frequency.

Author's Reply :
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Author did not respond.