Computation of Waves Generated by a Ship Using an NS Solver Based on B-Spline Solid

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ABSTRACT: A method has been discussed for obtaining the free surface turbulent flow solutions around an advancing ship, in which the pseudo-compressibility is used. The geometry grid systems are generated by the B-spline solid. The velocity and pressure are also represented by B-spline solid. The wave surface is shown for Wigley hull at Froude number $F_r = 0.348$ and Reynolds number $R_e = 3.21\times10^6$ in the paper.

1. INTRODUCTION

Some methods have been developed for wavemaking and seakeeping problem based on B-spline and NURBS surface[2][3][4][8][9][13], and some good results have been obtained. It is worth to noted that many efforts have been made in the simulation of the flow around a ship with free surface by resolving the NS equations[1][5][7][11][12]. The finite volume method and pseudo-compressibility method are adopted for solving NS equation, and the k ε , one equation and BL turbulence model are used respectively in their works. The above numerical methods are based on the difference formulation.

The objective of this work is to develop a discreting method based a B-spline solid. The pseudo-compressibility and BL turbulence model are adopted for present work, and the free surface turbulence flows around a Wigley hull is computed

2. FORMULATION

The governing equations with pseudo-compressibility form in the non-dimensional form can be written as

$$\frac{\partial u}{\partial t}+\nabla\bullet\vec{\phi}_x=0 \qquad \frac{\partial v}{\partial t}+\nabla\bullet\vec{\phi}_y=0 \qquad \frac{\partial w}{\partial t}+\nabla\bullet\vec{\phi}_z=0 \qquad \frac{\partial p}{\partial t}+\nabla\bullet\vec{\phi}_p=0 \tag{1}$$

where

$$\vec{\phi}_x=\begin{bmatrix} u^2+p-2\nu u_x \\ uv-\nu(u_y+v_x) \\ uw-\nu(u_z+w_x)\end{bmatrix} \quad \vec{\phi}_y=\begin{bmatrix} uv-\nu(v_x+u_y) \\ v^2+p-2\nu v_y \\ vw-\nu(v_z+w_y)\end{bmatrix} \quad \vec{\phi}_z=\begin{bmatrix} wu-\nu(u_z+w_x) \\ wv-\nu(v_z+w_y) \\ w^2+p-2\nu w_z\end{bmatrix} \quad \vec{\phi}_p=\Gamma\begin{bmatrix} u \\ v \\ w\end{bmatrix} \quad p=p_{real}+\frac{z}{F_r^{\,2}}$$

$$\Gamma=\beta(u^2+v^2+w^2) \qquad \nu=1/R_e+\nu_t$$

The parameter β is a positive constant , and $\beta=0.25$. F_r is the Froude number. R_e is the Reynolds number. ν_t is the eddy viscosity.

The governing equations are integrated in the control volume moving with time, and the Reynolds transport theorem and the Gauss integral theorem are adopted. The governing equation in each control volume become the following formulations

$$\frac{d}{dt}\int_{v(t)} u\, dV + \int_{s(t)} (\vec{\phi}_x - u\vec{u}) \bullet \vec{n} dS = 0 \qquad \frac{d}{dt}\int_{v(t)} v\, dV + \int_{s(t)} (\vec{\phi}_y - v\vec{u}) \bullet \vec{n} dS = 0$$

$$\frac{d}{dt}\int_{v(t)} w\, dV + \int_{s(t)} (\vec{\phi}_z - w\vec{u}) \bullet \vec{n} dS = 0 \qquad \frac{d}{dt}\int_{v(t)} p\, dV + \int_{s(t)} (\vec{\phi}_p - p\vec{u}) \bullet \vec{n} dS = 0 \tag{2}$$

where $\vec{u}$ is velocity vector,and $\vec{u} = (u,v,w)^T$. $\vec{n} = (n_x,n_y,n_z)^T$ is a unit outward normal of the control volume.The geometry shape can be represented by B-spline solid. The expression formulation is

$$\vec{x}(\xi,\eta,\zeta) = \sum_{i=0}^{L_G}\sum_{j=0}^{M_G}\sum_{k=0}^{N_G} \vec{X}_{ijk} N_{i,l}(\xi) N_{j,m}(\eta) N_{k,n}(\zeta) \tag{3}$$

where $\vec{x}$ is the locate vector, and $\vec{x} = (x,y,z)^T$. $\vec{X}_{ijk} = (X_{ijk},Y_{ijk},Z_{ijk})^T$ are the B-spline control nets representing the geometry shape. $N_{i,l},N_{j,m}$ and $N_{k,n}$ are the B-spline basis function of order l,m and n , defined by the Cox-de Boor recursive expressions.
The velocity and pressure can also be expressed by B-spline solid in follow formulation.

$$\vec{u}(\xi,\eta,\zeta) = \sum_{i=0}^{L_S}\sum_{j=0}^{M_S}\sum_{k=0}^{N_S} \vec{U}_{ijk} N_{i,p}(\xi) N_{j,q}(\eta) N_{k,r}(\zeta) \tag{4}$$

$$p(\xi,\eta,\zeta) = \sum_{i=0}^{L_S}\sum_{j=0}^{M_S}\sum_{k=0}^{N_S} P_{ijk} N_{i,p}(\xi) N_{j,q}(\eta) N_{k,r}(\zeta) \tag{5}$$

where $\vec{U}_{ijk} = (U_{ijk},V_{ijk},W_{ijk})^T$ and P_{ijk} are the B-spline control nets representing the velocity and pressure. The integral equation(2) transformed to the parameter coordinate system can be written as

$$\frac{d}{dt}\int_{v(t)} uJd\xi d\eta d\zeta + \int_{s1+s2} (\vec{\phi}_x - u\vec{u}) \bullet \vec{n}_1 H_1 d\eta d\zeta + \int_{s3+s4} (\vec{\phi}_x - u\vec{u}) \bullet \vec{n}_2 H_2 d\xi d\zeta + \int_{s5+s6} (\vec{\phi}_x - u\vec{u}) \bullet \vec{n}_3 H_3 d\xi d\eta = 0 \tag{6}$$

$$\frac{d}{dt}\int_{v(t)} vJd\xi d\eta d\zeta + \int_{s1+s2} (\vec{\phi}_y - v\vec{u}) \bullet \vec{n}_1 H_1 d\eta d\zeta + \int_{s3+s4} (\vec{\phi}_y - v\vec{u}) \bullet \vec{n}_2 H_2 d\xi d\zeta + \int_{s5+s6} (\vec{\phi}_y - v\vec{u}) \bullet \vec{n}_3 H_3 d\xi d\eta = 0 \tag{7}$$

$$\frac{d}{dt}\int_{v(t)} wJd\xi d\eta d\zeta + \int_{s1+s2} (\vec{\phi}_z - w\vec{u}) \bullet \vec{n}_1 H_1 d\eta d\zeta + \int_{s3+s4} (\vec{\phi}_z - w\vec{u}) \bullet \vec{n}_2 H_2 d\xi d\zeta + \int_{s5+s6} (\vec{\phi}_z - w\vec{u}) \bullet \vec{n}_3 H_3 d\xi d\eta = 0 \tag{8}$$

$$\frac{d}{dt}\int_{v(t)} pJd\xi d\eta d\zeta + \int_{s1+s2} (\vec{\phi}_p - p\vec{u}) \bullet \vec{n}_1 H_1 d\eta d\zeta + \int_{s3+s4} (\vec{\phi}_p - p\vec{u}) \bullet \vec{n}_2 H_2 d\xi d\zeta + \int_{s5+s6} (\vec{\phi}_p - p\vec{u}) \bullet \vec{n}_3 H_3 d\xi d\eta = 0 \tag{9}$$

where

$$J = \left|\frac{\partial(x,y,z)}{\partial(\xi,\eta,\zeta)}\right| \quad \vec{n}_1 = \frac{\vec{x}_\eta \times \vec{x}_\zeta}{|\vec{x}_\eta \times \vec{x}_\zeta|} \quad \vec{n}_2 = \frac{\vec{x}_\zeta \times \vec{x}_\xi}{|\vec{x}_\zeta \times \vec{x}_\xi|} \quad \vec{n}_3 = \frac{\vec{x}_\xi \times \vec{x}_\eta}{|\vec{x}_\xi \times \vec{x}_\eta|} \quad H_i = \sqrt{E_i G_i - F_i^2}, i = 1,2,3$$

$$E_1 = \vec{x}_\eta \bullet \vec{x}_\eta \qquad G_1 = \vec{x}_\zeta \bullet \vec{x}_\zeta \qquad F_1 = \vec{x}_\eta \bullet \vec{x}_\zeta$$

The similar formulation can be obtained for E_2,F_2,G_2 and E_3,F_3,G_3 . $\vec{x}_\xi$, $\vec{u}_\xi$ can be calculated by following formulation

$$\vec{x}_\xi = \sum_{i=0}^{L_G}\sum_{j=0}^{M_G}\sum_{k=0}^{N_G}\vec{X}_{ijk}\frac{dN_{i,l}(\xi)}{d\xi}N_{j,m}(\eta)N_{k,n}(\zeta) \tag{10}$$

$$\vec{u}_\xi = \sum_{i=0}^{L_S}\sum_{j=0}^{M_S}\sum_{k=0}^{N_S}\vec{U}_{ijk}\frac{\partial N_{i,p}(\xi)}{\partial\xi}N_{j,q}(\eta)N_{k,r}(\zeta) \tag{11}$$

The similar formulation can be obtained for $\vec{x}_\eta, \vec{x}_\zeta$ and $\vec{u}_\eta, \vec{u}_\zeta$. $\vec{u}_x, \vec{u}_y, \vec{u}_z$ can be represented in following

$$\begin{bmatrix}\vec{u}_x\\ \vec{u}_y\\ \vec{u}_z\end{bmatrix} = \frac{1}{J}\begin{bmatrix} y_\eta z_\zeta - z_\eta y_\zeta & y_\zeta z_\xi - z_\zeta y_\xi & y_\xi z_\eta - z_\xi y_\eta \\ z_\eta x_\zeta - z_\zeta x_\eta & z_\zeta x_\xi - z_\xi x_\zeta & z_\xi x_\eta - z_\eta x_\xi \\ x_\eta y_\zeta - x_\zeta y_\eta & x_\zeta y_\xi - x_\xi y_\zeta & x_\xi y_\eta - x_\eta y_\xi \end{bmatrix}\begin{bmatrix}\vec{u}_\xi\\ \vec{u}_\eta\\ \vec{u}_\zeta\end{bmatrix} \tag{12}$$

The formulation (6) is calculated by adopting the iteration formulaton(13).

$$\begin{aligned}&\sum_{i=0}^{L_S}\sum_{j=0}^{M_S}\sum_{k=0}^{N_S}U_{ijk}^{n+1}\left(\int_{v(t)}N_{i,p}(\xi)N_{j,q}(\eta)N_{k,r}(\zeta)Jd\xi d\eta d\zeta\right)^n = \left(\int_{v(t)}uJd\xi d\eta d\zeta\right)^n \\ &-\Delta t\left(\int_{s1+s2}(\vec{\phi}_x - u\vec{u})\bullet\vec{n}_1 H_1 d\eta d\zeta + \int_{s3+s4}(\vec{\phi}_x - u\vec{u})\bullet\vec{n}_2 H_2 d\xi d\zeta + \int_{s5+s6}(\vec{\phi}_x - u\vec{u})\bullet\vec{n}_3 H_3 d\xi d\eta\right)^n\end{aligned} \tag{13}$$

The similar formulation can be obtained for formulation(7),(8),(9). The integrations items in the above formulation are calculated by applying Gaussian quadrature. The linear equation systems can be constructed by satisfing the integral equation in each control volume and the boundary conditions, thus $\vec{U}_{ijk} = (U_{ijk}, V_{ijk}, W_{ijk})^T$ and P_{ijk} can be obtained by solving the linear equation systems. The velocity $\vec{u}$ and pressure p can be obtained by appling the De Boor algorithm. The stress condition is adopted for the free surface boundary condition. It can be see in ref.[7]. The other boundary conditions are listed in Table 1.

Table 1 Boundary Condition

Boundary	u, v, w	p
Upstream	$u = 1, v = 0, w = 0$	$p = 0$
Downstream	$\partial u/\partial\vec{n} = 0, \partial v/\partial\vec{n} = 0, \partial w/\partial\vec{n} = 0$	$\partial p/\partial\vec{n} = 0$
Center	$\partial u/\partial\vec{n} = 0, v = 0, \partial w/\partial\vec{n} = 0$	$\partial p/\partial\vec{n} = 0$
Free Surface	Stresses	Stresses
Body	$u = 0, v = 0, w = 0$	$\partial p/\partial\vec{n} = 0$
Bottom	$u = 0, v = 0, w = 0$	$\partial p/\partial\vec{n} = 0$
Side	$\partial u/\partial\vec{n} = 0, \partial v/\partial\vec{n} = 0, \partial w/\partial\vec{n} = 0$	$\partial p/\partial\vec{n} = 0$

The new grid systems can be reconstructed by the following formulation approximately, and

$$\vec{X}_{ijk}^{n+1} = \vec{X}_{ijk}^{n} + \Delta t \cdot \vec{u} \tag{14}$$

the new velocity and pressure are smoothed by B-spline solid for next time calculation. It will take wave filter effect. The goal is for the numerical stability.

3. RESULT OF WIGLEY HULL

The Wigley hull is selected for turbulent flow computation. The Froude number and Reynolds number are $Fr = 0.348, R_e = 3.21\times10^6$. The grid system is shown in Fig.1. The wave surface is shown in Fig.2 . The range of grid system is $-1.5 \le x \le 1, 0 \le y \le 1.5, -1.5 \le z \le wave \quad height$.

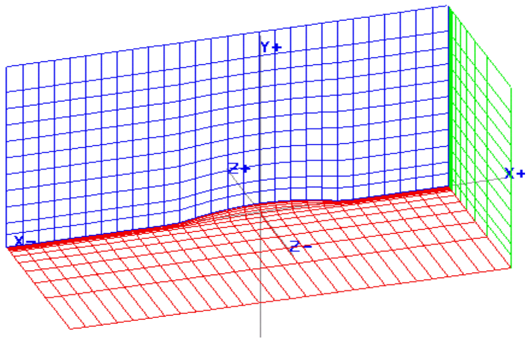

Fig.1.Grid System

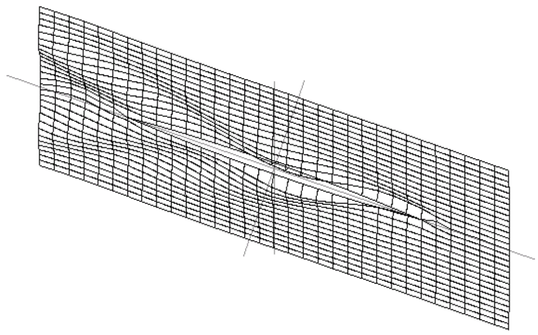
Fig.2. Wave Surface

4. CONCLUSION

Although this is a initial and incomplete work, it is feasible that the turbulence flow is calculated by adopting B-spline solid. The detail work need to do in the following several monthes.

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