Verification of Fourier-Kochin representation of waves

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The purpose of this study is to present a verification of the Fourier-Kochin representation of waves given in [1,2]. This representation expresses the waves generated by a given flow at a boundary surface in terms of single Fourier integrals and spectrum functions that are defined by distributions of elementary waves over the boundary surface. The Fourier-Kochin representation of waves is given in [1,2] for three classes of free-surface flows: (i) diffraction-radiation of time-harmonic waves without forward speed, (ii) steady ship waves, and (iii) time-harmonic ship waves (diffraction-radiation with forward speed).

The Fourier-Kochin representation of waves is considered here for steady flows associated with the linearized free-surface boundary condition $w + F^2\partial u/\partial x = 0$ where $F = \mathcal{U}/\sqrt{gL}$ is the Froude number, and $(u\,,v\,,w) = \vec{u} = \vec{U}/\mathcal{U} = \nabla\phi$ is the disturbance-flow velocity; here, $\phi = \Phi/(\mathcal{U}L)$ is the velocity potential associated with the velocity $\vec{u}$. The Fourier-Kochin representation of waves defines the potential ϕ^W and the velocity $\vec{u}^W$ associated with the waves that are generated by a given velocity distribution $\vec{u}$ at a boundary surface Σ, which may intersect the mean free-surface plane $z=0$ along the boundary curve Γ. The boundary surface $\Sigma\cup\Gamma$ is divided into patches, i.e. $\Sigma\cup\Gamma = \sum_{p=1}^{p=N}\Sigma_p\cup\Gamma_p$, associated with reference points $(x_p\,,y_p\,,z_p)$, with $\vec{x} = \vec{X}/L$, located near the centroids of the patches.

The wave potential ϕ^W and velocity $\vec{u}^W$ at a field point $(\xi\,,\eta\,,\zeta)$ of the flow domain outside a boundary surface $\Sigma\cup\Gamma$ are given by the single Fourier integrals

$$4\pi\begin{Bmatrix}\phi^W\\u^W\\v^W\\w^W\end{Bmatrix} = \Re\mathrm{e}\int_{-\infty}^{\infty}\frac{d\beta\;\alpha^d}{k^d-\nu}\begin{Bmatrix}i\\\alpha^d\\\beta\\i\,k^d\end{Bmatrix}\sum_{p=1}^{p=N}\left[1+\mathrm{erf}\left(\frac{x_p-\xi}{\sigma F^2 C}\right)\right]S_p^W e^{(z_p+\zeta)k^d+i[(x_p-\xi)\alpha^d+(y_p-\eta)\beta]}$$

where $\Re\mathrm{e}$ stands for the real part. The functions $\alpha^d(\beta)$ and $k^d(\beta)$ are defined as

$$\alpha^d = \sqrt{k^d}/F \qquad k^d = \nu + \sqrt{\nu^2+\beta^2} \qquad \text{with} \quad \nu = 1/(2F^2)$$

Here, $k^d(\beta)$ stands for the value of the wavenumber k at the dispersion curves $\alpha = \pm\alpha^d(\beta)$, with $-\infty \le \beta \le \infty$, associated with the dispersion relation $F^2\alpha^2 - k = 0$. The function C in the error function erf is related to the curvature of the dispersion curves and is given by

$$C = 1 + |\,3/(F^2k^d) - 2\,|\,/\,(\,4\,F^2k^d - 3\,)^{3/2}$$

We have $C=2$ for $\beta=0$, where $\alpha^d = k^d = 1/F^2$, $C\to 1$ as $\beta\to\pm\infty$, and $C=1$ at the inflexion points defined by $F^2k^d = 3/2$ and $F^2\beta = \pm\sqrt{3}/2$. The positive real constant σ may be chosen as in [2].

The contribution S_p^W of patch p to the wave-spectrum function $S^W(\beta)$ is given by

$$S_p^W = S_p^\Sigma + F^2 S_p^\Gamma \qquad \text{with}$$

$$S_p^\Sigma = \int_{\Sigma_p} d\mathcal{A}\,\left[\,\vec{u}\cdot\vec{n} + i\,\frac{\alpha^d}{k^d}\,(\vec{u}\times\vec{n})^y - i\,\frac{\beta}{k^d}\,(\vec{u}\times\vec{n})^x\,\right]e^{k^d(z+z_p)+i[\alpha^d(x-x_p)+\beta(y-y_p)]}$$

$$S_p^\Gamma = \int_{\Gamma_p} d\mathcal{L}\,\left[\,(\,t^xt^y + \frac{\alpha^d\beta}{(k^d)^2}\,)\,\vec{u}\cdot\vec{t} - (t^y)^2\,\vec{u}\cdot\vec{\nu}\,\right]e^{i[\alpha^d(x-x_p)+\beta(y-y_p)]}$$

Here, the unit vector $\vec{n} = (n^x, n^y, n^z)$ is normal to the boundary surface Σ and points into the flow region outside Σ, and the unit vectors $\vec{t} = (t^x, t^y, 0)$ and $\vec{\nu} = (-t^y, t^x, 0)$ are tangent and normal to the boundary curve Γ in the mean free-surface plane $z=0$. The normal vector $\vec{\nu}$ points into the flow region outside Γ, like the normal vector $\vec{n}$, and the tangent vector $\vec{t}$ is oriented clockwise (looking down). The spectrum functions $S^\Sigma(\beta)$ and $S^\Gamma(\beta)$ are defined by distributions of elementary waves over the boundary surface Σ and the boundary curve Γ, respectively, with amplitudes given by the normal components $\vec{u}\cdot\vec{n}$, $\vec{u}\cdot\vec{\nu}$ and the tangential components $\vec{u}\times\vec{n}$, $\vec{u}\cdot\vec{t}$ of the velocity $\vec{u}$ at Σ and Γ.

Thus, the Fourier-Kochin wave representation defines the wave potential $\phi^W(\vec{\xi})$ and velocity $\vec{u}^W(\vec{\xi})$ at a field point $\vec{\xi}$ of the flow region outside a boundary surface $\Sigma \cup \Gamma$ in terms of the velocity distribution $\vec{u}(\vec{x})$ at the boundary surface Σ and the boundary curve Γ. This representation of the waves generated by a flow at a boundary surface only involves the boundary velocity $\vec{u}(\vec{x})$; i.e. the Fourier-Kochin wave representation does not involve the potential $\phi(\vec{x})$ at the boundary surface $\Sigma \cup \Gamma$, unlike the classical boundary-integral representation that defines the potential in a potential-flow region in terms of boundary-values of the potential ϕ and its normal derivative $\partial\phi/\partial n = \vec{u} \cdot \vec{n}$. The Fourier-Kochin wave representation is based on several recent new fundamental results obtained within the framework of the Fourier-Kochin theory [3,2] : (i) the boundary-integral representation, called velocity representation, given in [1,2], (ii) the representation of the generic super Green function defined in [4,5,2], and (iii) the transformations of spectrum functions given in [3,1,2]. The flow generated by a given flow at a boundary surface can be expressed as

$$\phi = \phi^W + \phi^L \qquad \vec{u} = \vec{u}^W + \vec{u}^L$$

where ϕ^W, $\vec{u}^W$ is the wave component defined by the Fourier-Kochin wave representation, and ϕ^L, $\vec{u}^L$ is a local-flow component. The Rankine and Fourier-Kochin nearfield flow representation given in [6] expresses the local component ϕ^L, $\vec{u}^L$ in terms of distributions of elementary Rankine singularities and Fourier-Kochin distributions of elementary waves over the boundary surface Σ and the boundary curve Γ. The local component ϕ^L, $\vec{u}^L$ is not considered here.

For the purpose of verifying the foregoing Fourier-Kochin wave representation, the flow due to a source-sink pair is considered here. Fig.1 shows the disturbance velocity (u, v, w) generated by a point source and a point sink, of strength $q = Q/(\mathcal{U}L^2) = 0.001$, located at $(x, y, z) = (\pm 0.5, 0, -0.02)$ over the lower half $z \leq 0$ of the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2$ with $(a, b, c) = (0.55, 0.05, 0.1)$. The velocity distribution (u, v, w) generated by the point source-sink pair is evaluated, for a Froude number $F = 0.316$, using integral representations of the Green function given in [7]. The upper half of Fig.2 depicts the free-surface elevation, computed using integral representations of the Green function, due to the source-sink pair. The lower half of Fig.2 depicts the free-surface elevation obtained using the Fourier-Kochin wave representation and the velocity distribution generated by the source-sink pair at the ellipsoidal boundary surface depicted in Fig.1. The free-surface elevations computed using expressions for the Green function (upper half) and reconstructed using the Fourier-Kochin wave representation (lower half) are not identical in the vicinity of the ellipsoidal boundary surface because the local-flow component u^L is ignored in the Fourier-Kochin wave representation. The wave elevations shown in Fig.3 along the four longitudinal cuts $y = 0$, $y = 0.06$, $y = 0.1$, $y = 0.5$ show that the local component u^L in fact is only significant in the vicinity of the elliptical boundary curve.

The results depicted in Figs 1-3 provide a verification of the Fourier-Kochin representation of waves. Furthermore, Fig.3 shows that the wave component is dominant even in the nearfield. Illustrative practical applications of the Fourier-Kochin representation of waves are given in [8,9]. Specifically, the Fourier-Kochin representation of steady ship waves is coupled with nearfield calculations based on the Euler equations in [8] and is applied to the design of a wave cancellation multihull ship in [9].

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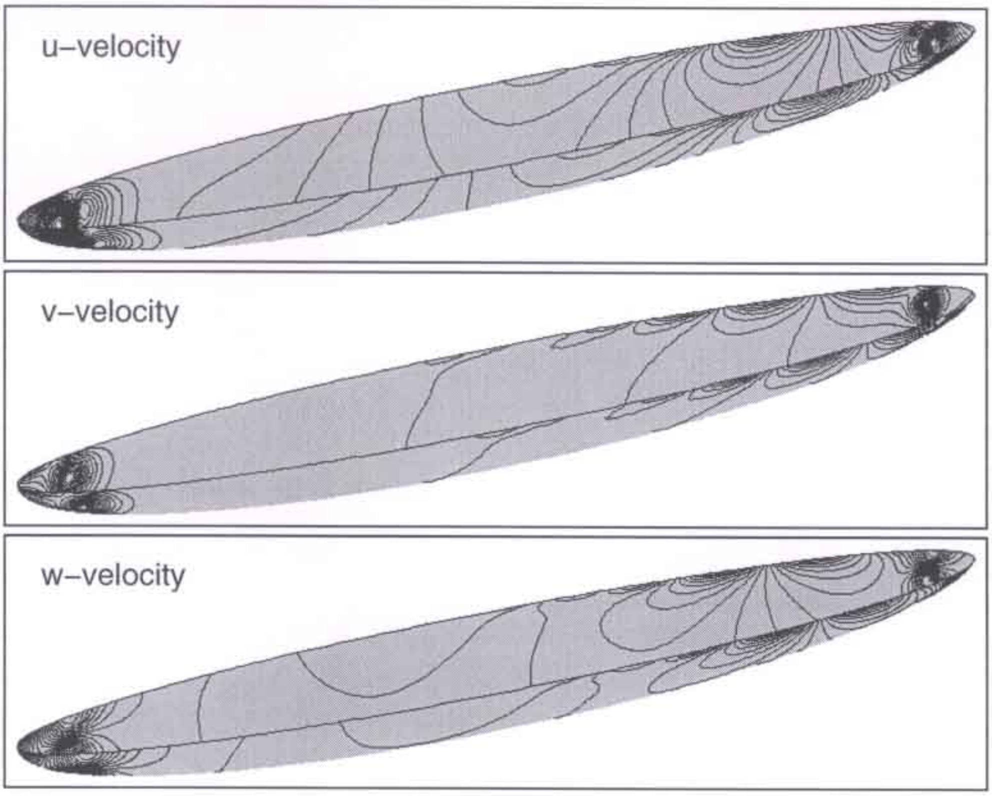

Fig. 1. Velocity distribution generated by souce-sink pair at boundary surface

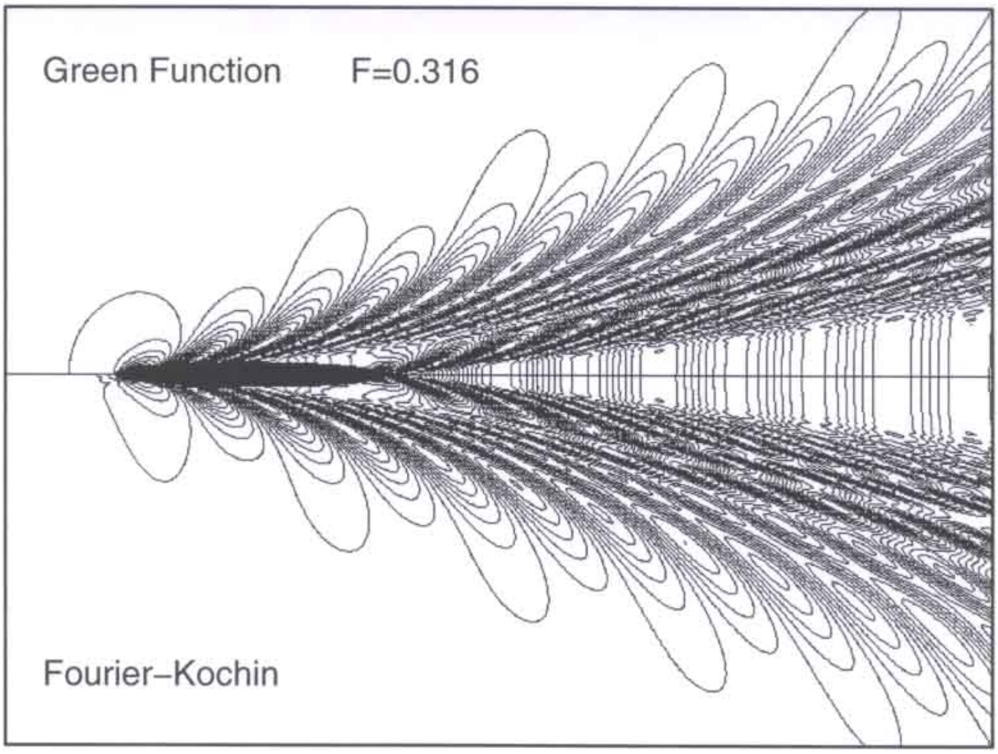

Fig. 2. Wave patterns due to souce-sink pair
top: wave pattern computed using Green function
bottom: wave pattern reconstructed using Fourier-Kochin wave representation

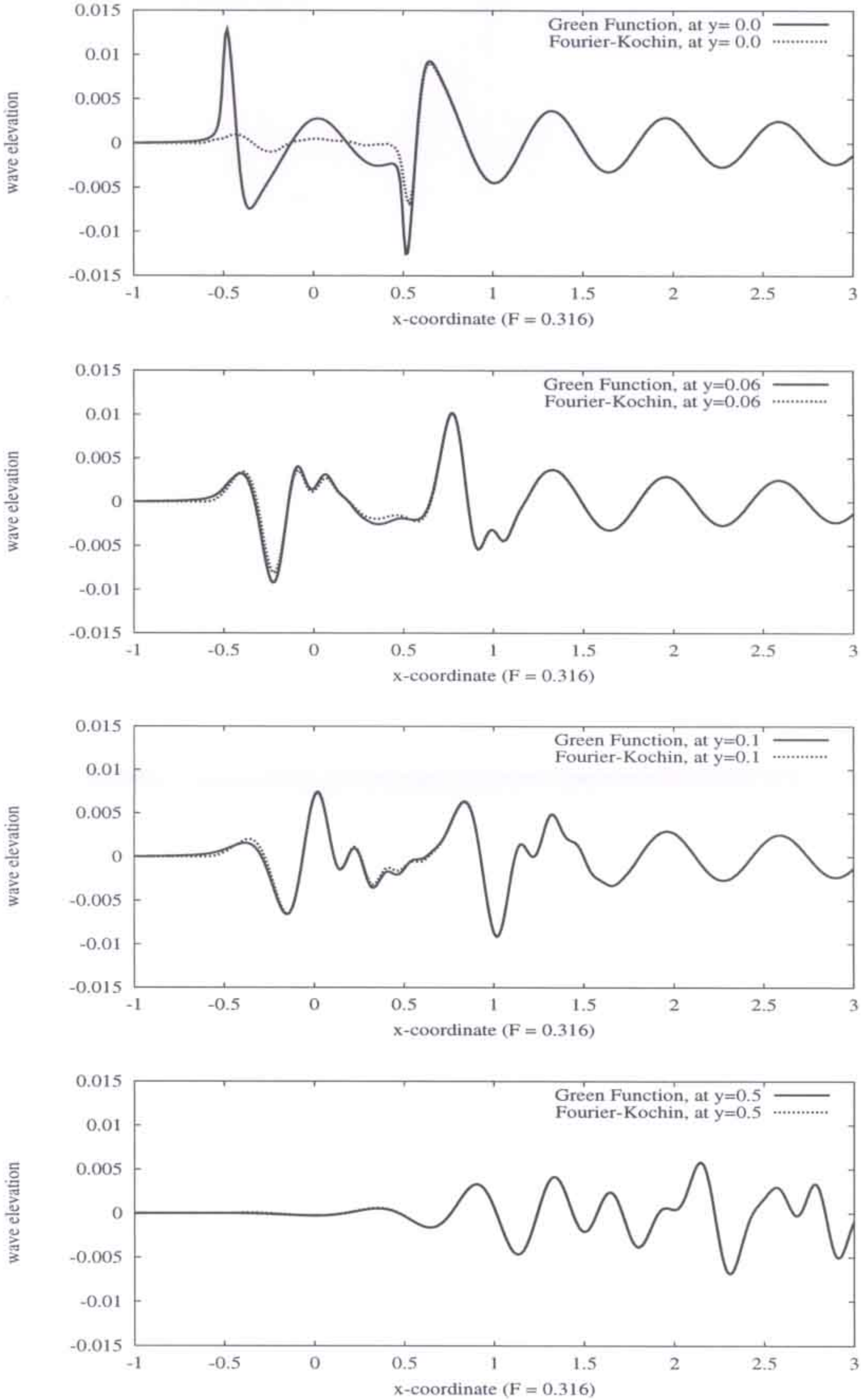

Fig. 3. Wave elevations along four cuts at y=0, 0.06, 0.1, 0.5 for F=0.316