The Coupled Finite Element and Boundary Element Analysis of Nonlinear Interactions Between Waves and Bodies

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1. INTRODUCTION

The fully nonlinear wave/body interaction problem is usually solved through a time stepping technique based on the potential flow theory. At each time step, the potential is commonly found by using the boundary element method (BEM) or the finite element method (FEM). The BEM divides only the boundary of the fluid domain into small panels. Early examples of the BEM for two dimensional (2D) flow include the work by Longuet-Higgins & Cokelet (1976), Faltinsen (1977), Vinje & Brevig (1981) and Lin et al (1984). More recent applications of the BEM to three dimensional (3D) flow include those published by Ferrant (1994) and Celebi et al (1998). The FEM, on the other hand, divides the entire fluid domain into small elements. Typical applications for 2D flow include those by Wu & Eatock Taylor (1994, 1995), Clauss & Steinhagen (1999), Robertson & Sherwin (1999); and for 3D flow include Wu et al (1998) and Ma et al (2001a, 2001b).

As argued by Wu & Eatock Taylor (1995, 1996) and Ma et al (2001a, 2001b), although the BEM has far fewer unknowns when applied to the wave/body interaction problem, it usually requires considerably more memory for "large" meshes because it leads to fully populated matrices. (An exception is when a multi-subdomain BEM approach is used, as for example by Wang et al 1995; and there may be other ways of improving the efficiency of the BEM). By contrast, direct application of the FEM needs significantly less memory and it is computationally more efficient. A drawback of the FEM, however, is the mesh generation. For a body having a complicated geometry, a sophisticated mesh generator is usually required to follow the motion of the body and the wave. Greaves et al (1997), for example, adopted a quadtree-based mesh generation scheme for the 2D problem. The scheme was found to be efficient when the horizontal and vertical dimensions of the fluid domain are comparable. For an extremely long or thin domain, the CPU requirement for the mesh generator increases rapidly. As remeshing is needed at every time step or after every few time steps, excessive CPU consumed by the mesh generator at each time step will make the overall computation very inefficient.

The present work therefore explores the use of a coupled BEM and FEM approach. Near the body, the BEM is used, as a boundary element mesh is easier to create in that region. Also, when the BEM is confined to a small domain, its memory requirement is limited. Away from the body, the fluid domain will be regular if the wave does not overturn or break. This allows some simple mesh generator to be used, which can deal efficiently with a large (including extremely long or thin) fluid domain. The adopted BEM and FEM are based on the approach described in Wu & Eatock Taylor (1995). The additional work required is to ensure that the potential and velocity are continuous at the interface of the BEM and FEM domain. This is achieved through iteration, in a similar manner to the approach used in that paper where we implemented the domain decomposition method for the FEM.

2. COUPLED FINITE AND BOUNDARY ELEMENT METHOD

We consider the interaction of a wave generated by a piston-like wavemaker with a two dimensional body. A Cartesian coordinate system Oxy is defined in which y coincides with the initial position of the wavemaker and points vertically upwards, and the origin of the system is on the mean free surface. All the physical parameters are nondimensionalized by the density of the fluid ρ, a typical dimension of the body L and the time $\sqrt{L/g}$, where g is the acceleration due to the gravity. The fluid is assumed to be incompressible and inviscid, and the flow is assumed to be irrotational. A velocity potential ϕ can then be introduced, which satisfies the Laplace equation and the usual non-linear boundary conditions on the body surface, free surface and wavemaker .

As shown in Figure 1, the fluid domain is divided into three regions. R_1 and R_3 are away from the body, where the finite element method can be adopted as the mesh can be generated easily. R_2 encloses the body, where the boundary element method can be used. The continuity of the potential and velocity is enforced on the two interfaces Γ_{12} and Γ_{23}.

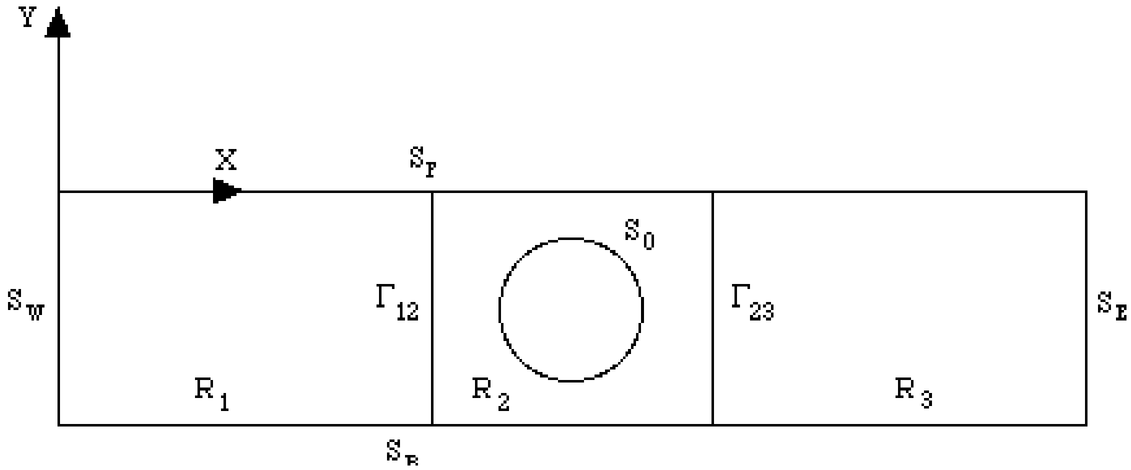

Figure 1 The coupled FEM and BEM computational domain

Based on the finite element method, the potential in R_1 can be written as

$$\phi^{(1)} = \sum_{j=1}^{n_1} \phi_j^{(1)} N_j(x, y) \tag{1}$$

where ϕ_j are the nodal values of the potential, n_1 is the number of the nodes in R_1 and $N_j(x, y)$ are the shape functions, which have been chosen to vary linearly over triangular elements in the present analysis. Application of the Galerkin method within R_1 leads to

$$\int_{R_1} \nabla N_i \sum_{j=1}^{n_1} \phi_j^{(1)} \nabla N_j dR |_{j \notin S_{F_1}} = -\int_{R_1} \nabla N_i \sum_{j=1}^{n_1} \phi_j^{(1)} \nabla N_j dR |_{j \in S_{F_1}} - U_0 \int_{S_W} N_i dS - \int_{\Gamma_{12}} \frac{\partial \phi^{(2)}}{\partial n} N_i dS \, . \tag{2}$$

It should be noted that the direction of the normal of Γ_{12} changes sign from R_1 to R_2.

Within R_2, the complex potential $\beta = \phi^{(2)} + i\psi^{(2)}$ is defined, where $\psi^{(2)}$ is the stream function. Along the boundary we can write

$$\beta = \sum_{j=1}^{n_2} \beta_j N_j(z) \, . \tag{3}$$

β_j are the nodal values of the complex potential and the interpolation function is chosen as

$$N_j(z) = \begin{cases} (z - z_{j+1})/(z_j - z_{j+1}) & z \in (z_j, z_{j+1}) \\ (z - z_{j-1}))/(z_j - z_{j-1}) & z \in (z_{j-1}, z_j) \\ 0 & z \notin (z_{j-1}, z_{j+1}) \end{cases} \, . \tag{4}$$

Application of Cauchy's theorem gives

$$\sum_{j=1}^{n_2} A_{kj}\phi_j^{(2)}|_{j\in S_0+S_{B_2}} + i\sum_{j=1}^{n_2} A_{kj}\psi_j^{(2)}|_{j\in S_{F_2}+\Gamma_{12}+\Gamma_{23}} = -\sum_{j=1}^{n_2} A_{kj}\phi_j^{(2)}|_{j\in S_{F_2}} - \sum_{j=1}^{n_2} A_{kj}\phi_j^{(2)}|_{j\in \Gamma_{12}} - \sum_{j=1}^{n_2} A_{kj}\phi_j^{(2)}|_{j\in \Gamma_{23}} - i\sum_{j=1}^{n_2} A_{kj}\psi_j^{(2)}|_{j\in S_0+S_{B_2}}, \tag{5}$$

where

$$A_{kj} = \frac{z_k - z_{j-1}}{z_j - z_{j-1}}\ln\frac{z_j - z_k}{z_{j-1} - z_k} + \frac{z_k - z_{j+1}}{z_j - z_{j+1}}\ln\frac{z_{j+1} - z_k}{z_j - z_k}. \tag{6}$$

In R_3, we can write

$$\int_{R_{31}} \nabla N_i \sum_{j=1}^{n_3} \phi_j^{(3)} \nabla N_j dR|_{j\notin S_{F_3}+S_E} = -\int_{R_1} \nabla N_i \sum_{j=1}^{n_3} \phi_j^{(3)} \nabla N_j dR|_{j\in S_{F_3}+S_E} - \int_{\Gamma_{23}} \frac{\partial \phi^{(2)}}{\partial n} N_i dS \tag{7}$$

similar to equation (2). Here the potential on the boundary at the far end S_E is treated as known, because it can be obtained through the solution at the previous time step using the radiation condition (Ma et al 2001a, 2001b).

Equations (2), (5) and (7) can be solved iteratively. When $\partial\phi^{(2)}/\partial n$ in (2) is assumed, the equation becomes complete and can be solved. From the solution of $\phi^{(1)}$ we replace $\phi^{(2)}$ on Γ_{12} in (5) by $\phi^{(2)} + \gamma(\phi^{(1)} - \phi^{(2)})$, where γ is the relaxation coefficient. When $\phi^{(2)}$ on Γ_{23} is assumed in (5), the equation can be solved to give $\partial\phi^{(2)}/\partial n$ on Γ_{23} through the derivative of the stream function along the boundary. Subsequently, equation (5) can be solved to give the new value of the potential on Γ_{23} through $\phi^{(2)} + \gamma(\phi^{(3)} - \phi^{(2)})$. The solution procedure then returns to R_1 and is repeated until the desired accuracy has been achieved.

3. NUMERICAL RESULTS

The results given in Figure 2 are for a cylinder of radius r_0 with submergence $1.5r_0$ in water of depth $4r_0$. The cylinder is placed at a distance equal to $65r_0$ from both the wavemaker and the far end. The length of the BEM is equal to $4r_0$ with the cylinder being in the middle. The wavemaker is stationary and the cylinder moves with horizontal velocity $U = \omega a \sin\omega t$ (a = 0.1 and ω = 1.0, using the non-dimensional parameters previously specified). Based on the linear solution in the frequency domain, the vertical force is zero because of anti-symmetry. The result in Figure 2b, which has been divided by a, is therefore due to the nonlinear effect. In particular, it has been shown by Wu (1993, 2000) that when the motion becomes periodic, the nonlinear vertical force has only components of $2n\omega$ while the horizontal force has only components of $(2n+1)\omega$, $n = 0,1,2,...$, which can be seen to be consistent with the results in Figure 2. More results and discussion will be given at the workshop.

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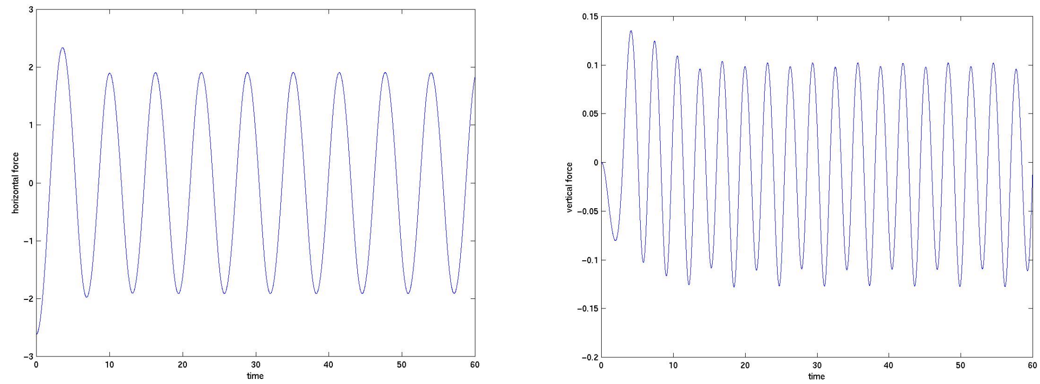

(a) horizontal force (b) vertical force

Figure 2 Time history of forces on a submerged circular cylinder in forced sinusoidal horizontal motion

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