Fast Multipole Method for Hydrodynamic Analysis of Very Large Floating Structures *

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1 Introduction

For linear hydrodynamic analysis of floating structures, boundary element methods (BEM) employing the free-surface Green function are frequently used as basic design tools. However, when the methods are applied for the analysis of Very Large Floating Structures (VLFS), the number of unknowns $(= N)$ reaches the order of 10^4–10^5 and thus large storage requirement $(O(N^2))$ and the excessive computation time ($O(N^3)$ for factorization solvers or $O(N^2)$ for iterative solvers) make the application of conventional BEM impractical. The precorrected-FFT method [1, 2] has been successfully applied to such a large scale analysis, but not yet for a VLFS in shallow water. We hereby present an alternative approach using fast multipole methods [3, 4, 5], which have been more commonly used in many fields that would require excessive computation resources.

It is known that a fast multipole method for Helmholtz' equation in 2D is possible with the help of Graf's addition theorem for the Hankel function (see Fukui & Katsumoto [6], for example). Because the free surface Green function in shallow water is represented by series of Bessel and Hankel functions, the fast multipole method for linear wave diffraction/radiation problems can be formulated as a straight forward extention of the method applied to 2D Helmholtz' equation. Because the Green function in the series form converges rapidly when horizontal distance between source and field points relative to the water depth is large, the method will be most efficient when the horizontal dimensions of the analyzed area are large compared with the water depth. This is just the case for a VLFS in shallow water. We have implemented the multipole acceleration algorithm to our higher-order boundary element program [7] (which is based on the integral equation proposed in [8]), and examined its efficiency by benchmark calculations including VLFS response analysis in variable water depth environment of a real sea.

2 Formulations

The Green function in finite depth water of h can be represented by

$$G = \sum_{m=0}^{\infty} \frac{2K_0(k_m R)}{N_m} \cos k_m(z+h) \cos k_m(\zeta + h) \tag{1}$$

where $N_m = \frac{h}{2}(1 + \sin 2k_m h / 2k_m h)$, $k_m \tan k_m h = -\omega^2/g$, k_m $(m \geq 1)$ is positive real and $k_0 = \mathrm{i}k$, k: the wave number, g: gravitational acceleration, and time variance of $\mathrm{e}^{\mathrm{i}\omega t}$ is assumed for all first order quantities, R denotes the horizontal distance between the reference and the source points, and z and ζ are their vertical coordinates, respectively.

Graf's addition theorem for Bessel functions yields:

$$K_0(k_m R) = \sum_{n=-\infty}^{\infty} K_n(k_m r) \mathrm{e}^{\mathrm{i}n\theta} I_n(k_m \rho) \mathrm{e}^{-\mathrm{i}n\Phi} \tag{2}$$

*This study was supported by the Program for Promoting Fundamental Transport Technology Research from the Corporation for Advanced Transport & Technology (CATT).

where $(r,\ \theta)$ and $(\rho,\ \Phi)$ represent the horizontal coordinates of the reference and the source points, respectively, measured from the origin O which can be arbitrarily (but $r > \rho$) chosen. Substitution of Eq.(2) into Eq.(1) yields:

$$G = \sum_{m=0}^{\infty} \frac{2}{N_m} \sum_{n=-\infty}^{\infty} M_{mn} K_n(k_m r) \mathrm{e}^{\mathrm{i}n\theta} \cos k_m(z+h) \tag{3}$$

$$M_{mn} = I_n(k_m \rho) \mathrm{e}^{-\mathrm{i}n\Phi} \cos k_m(\zeta + h) \tag{4}$$

Note that the Green function is now represented in the form of multipole expansion around O. Therefore, we are now ready to apply the fast multipole algorithm. The normal derivative at the source point, $\partial G/\partial n$, can also be represented by the same form, with just replacing M_{mn} by $\partial M_{mn}/\partial n$. The origin of the multipole expansion can be moved arbitrarily (under $r > \rho$), and the coefficient $\tilde{M}_{m\nu}$ for the new origin O' can be calculated from M_{mn} at O by

$$\tilde{M}_{m\nu} = \sum_{n=-\infty}^{\infty} M_{mn} I_{\nu-n}(k_m \xi) \mathrm{e}^{-\mathrm{i}(\nu-n)\psi} \tag{5}$$

where $(\xi,\ \psi)$ is the polar coordinate of O measured from O'. This can also be obtained from Graf's addition theorem.

In the evaluation of the integral $\int GV\mathrm{d}S$ (same procedure can be applied to $\int G_n \phi \mathrm{d}S$ in the followings), the influences from near panels are evaluated directly in a conventional manner; however, the influences from far panels can be evaluated using Eq.(3) where the M_{mn} is replaced by

$$M_{mn}^B = \int_{S_{far}} M_{mn} V \mathrm{d}S \tag{6}$$

where S_{far} represents the surface at the far distance from the collocation point. Firstly, we calculate M_{mn}^B for each panel locating the multipole expansion point at the horizontal coordinates of the panel center, then gather them at the center of 'leaf-cell' as a group of several panels, where Eq.(5) can be utilized. Further, a group of four cells form an upper level cell, and M_{mn}^B is also calculated at the center of the upper cell. Note that at this stage, we need not specify the reference (or collocation) points. We may define the level 0 cell as a square cell including all panels, and then level 1 cell as a quater portion of the level 0 cell, level n cell as a quarter of level $n-1$ cell, *etc.* [5]. After setting up the multipole coefficients M_{mn}^B for all cells at each level, we calculate $\int GV\mathrm{d}S$ utilizing Eqs.(3) and (6) for each collocation point, where larger cells are selected as far as possible. This hierarchical algorithm is known to be $O(N \log N)$ for the computation time [4]. Although the $O(N)$ algorithm [3, 5] is also possible as has been made by Fukui & Katsumoto [6] we hereby implemented the $O(N \log N)$ algorithm because of its simplicity and easiness for developing parallelized program. Because the integrals can be evaluated very fast using these algorithms, we can solve the integral equation with an iterative solver without holding large part of coeffieient matrices. The requirement of only $O(N)$ storage may be the most attractive feature of the method.

When a factorization solver is used, we usually use modal expansion approach to VLFS hydroelastic analysis, where a number of radiation problems are solved separately for each mode, and then generalized added-mass and radiation damping are calculated for modal coordinates [7]. However, it is not the case when an iterative solver is used; we would like to have the final solution from only one iterative procedure without solving a number of separate radiation problems. This can be done by solving the structural problem in each step of the iterative procedure and finding the relationships between ϕ_n and ϕ on the wetted-surface of the structure. If the modal method is applied, this may be represented schematically for a flat-bottom VLFS by

$$\{\phi_n\} = \rho\omega^2 [f][K - \omega^2 M]^{-1}[L]\{\phi\} \tag{7}$$

where $[f]$, $[K - \omega^2 M]$, and $[L]$ are $8N_E \times P$, $P \times P$, and $P \times N$ matrices, respectively, where N_E and N are the numbers of panels and nodes on the bottom surface of the VLFS, and P is the number of deflection modes as an elastic plate. It should be noted that a large part of memory allocation is consumed for storing these matrices for the Fast Multipole Method in Table 1; the memory allocations only for hydrodynamic part are less than half of the indicated values.

A box-like VLFS, either in constant depth sea $h = 8$m or in variable depth sea (Fig.1), has been analyzed for the oblique wave of $\beta = \pi/4$ ($\beta = 0$ corresponds to the head wave from positive x direction and $\beta = \pi/2$ to the beam wave from positive y direction). The specifications of the VLFS are: the length $L = 1,500$m, the beam $B = 150$m, the draft $d = 1$m, the rigidity as an elastic plate $D = 3.88 \times 10^7$ kNm, and the Poisson's ratio $\nu = 0.3$. Number of modal functions employed are 160 (20 in longitudinal & 8 in beam). Results are shown in Tables 1 and 2, and Figs.2–5.

Table 1: Performance of the fast multipole method and the direct method using LU factorization on an EWS node (IBM RS/6000SP; POWER3 375MHz). The residual tolerance in GMRES $\epsilon = 10^{-4}$, $L/\lambda = 9.57$ ($T = 18$sec), $h = 8$m (constant). Values in parentheses are estimates.

Model	Typical panel size, Δ	Number of nodes	Fast Multipole Method				Direct Method	
			Number of iter.	CPU time per iter.	CPU time	Memory allocation	CPU time	Memory allocation
A	25m	1,609	31	3 sec	1.80 min	27 MB	1.55 min	54 MB
B	12.5m	5,377	32	15 sec	10.4 min	89 MB	28.5 min	489 MB
C	6.25m	19,393	32	86 sec	77.4 min	315 MB	(907 min)	(6 GB)
D	3.125m	73,345	32	570 sec	775 min	1.15 GB	(708 hr)	(85 GB)

Table 2: Numbers of iterations of GMRES and computation times for various L/λ. $h = 8$m (constant).

Model	L/λ	Wave period	Number of iterations		CPU time	
			$\epsilon = 10^{-3}$	$\epsilon = 10^{-4}$	$\epsilon = 10^{-3}$	$\epsilon = 10^{-4}$
C	9.57	18 sec	25	32	1.15 hr	1.29 hr
C	17.9	10 sec	112	148	3.46 hr	4.40 hr
C	33.2	6 sec	252	497	8.23 hr	15.7 hr
D	62.0	4 sec	287	—	63.8 hr	—
D	62.0	4 sec	287	—	14.5 hr*	—

*Parallel computation using 5 CPUs.

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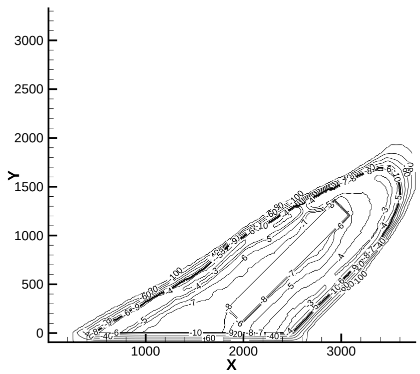


Figure 1: Contour plot of the variable depth configuration.

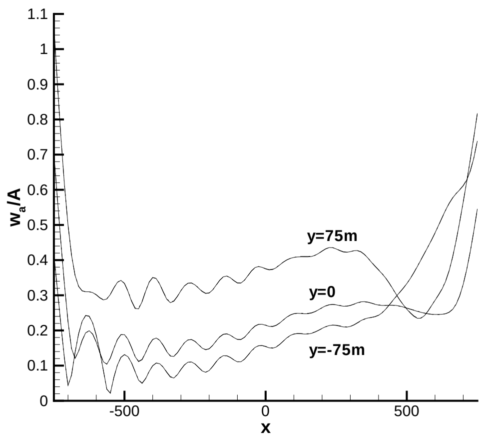


Figure 2: Deflection amplitudes of VLFS in costant depth sea (model B) at $T = 18$sec. —: direct method, - - -: fast multipole method ($\epsilon = 10^{-3}$).

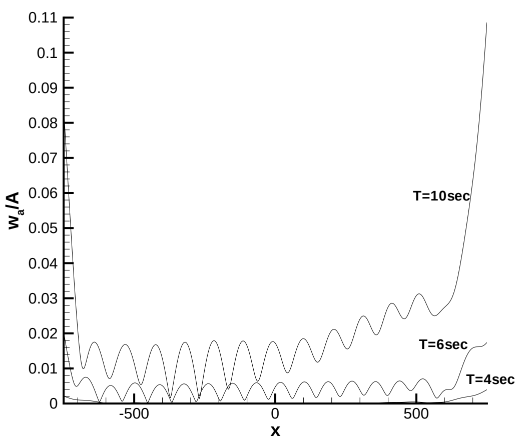
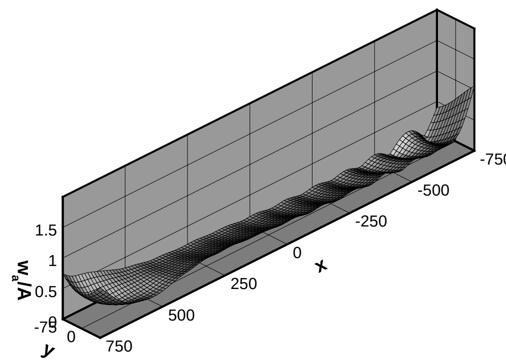
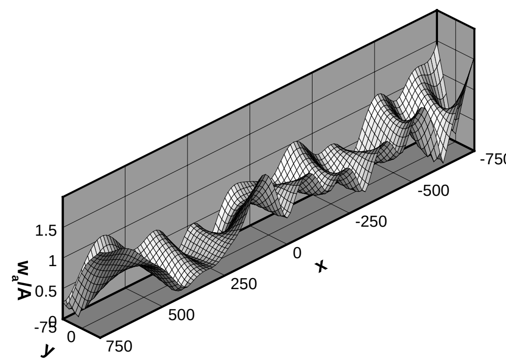


Figure 3: Deflection amplitudes of VLFS in costant depth sea for various T along $y = 0$.



(a) in constant depth sea ($h = 8$m).



(b) in variable depth sea.

Figure 4: Deflection amplitudes at $T = 18$sec.

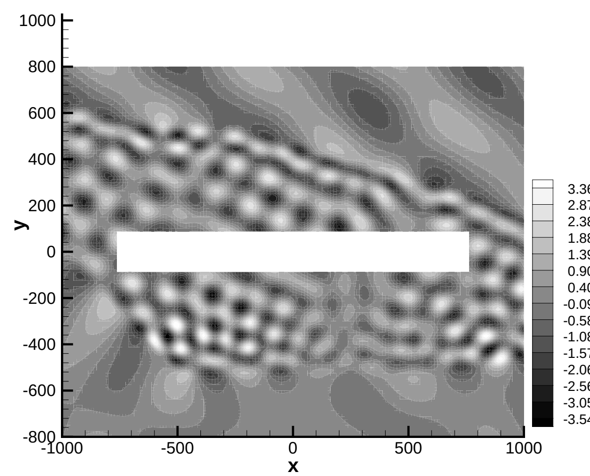


Figure 5: Snapshot of the surface elevation around the floating body in the variable depth sea. $T = 18$sec, total nodes: 67,098 (61,721 for the sea-bottom surface, 5377 for the VLFS).