THE PELAMIS WAVE ENERGY CONVERTER: IT MAY BE JOLLY GOOD IN PRACTICE, BUT WILL IT WORK IN THEORY?

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1. Background

The design of wave energy converters (WECs) has hitherto concentrated primarily on hydrodynamic efficiency, see the 1985 review by David Evans [1] (1985). The Pelamis WEC is a promising new concept which is designed instead primarily for survival in extreme seas. This is accomplished by its end-on orientation to the waves, which enables the WEC negotiate breaking waves safely, and by its relatively small diameter (3.5m), which non-linearly limits power output in extreme conditions. See Figure 1 below – further details are on the website www.oceanpd.com.

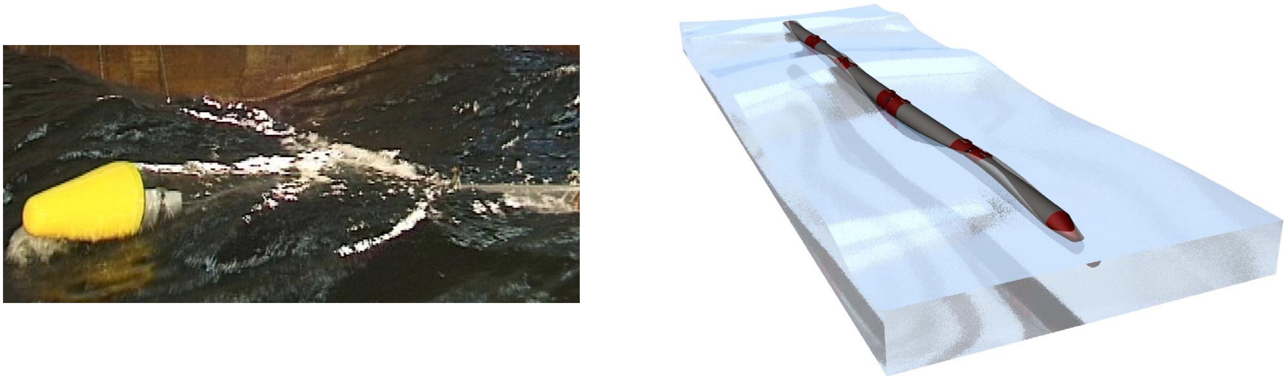

Figure 1. The Pelamis wave energy converter

The power take-off is from hydraulic jacks at the articulated joints - the hydrodynamic efficiency of the device has been optimised empirically, by extensive model testing and numerical modelling. It has been found that the efficiency is greatly improved by making the transverse motions resonant (by suitable choice of transverse stiffness at the joints), and coupling them to the vertical motions (by introducing suitable coupling terms in this stiffness). All this may be excellent in practice, but will it work in theory?

2. The waves generated by a single segment in isolation.

The theory of wave power absorption is concerned with the waves generated by the WEC. For a single segment of Pelamis in isolation, these may readily be found with a 3-D diffraction program. Figure 2 below shows a surface element mesh generated by the WS Atkins program AQWA-LINE (which gives substantially identical results to MIT's program WAMIT, see [2]). The segment is 3.5m diameter, 30m long with uniform density, and floating freely exactly half immersed in water of infinite depth (and density $\rho = 1.000$ tonne/m^3 with $g = 10.00$ m^2/s).

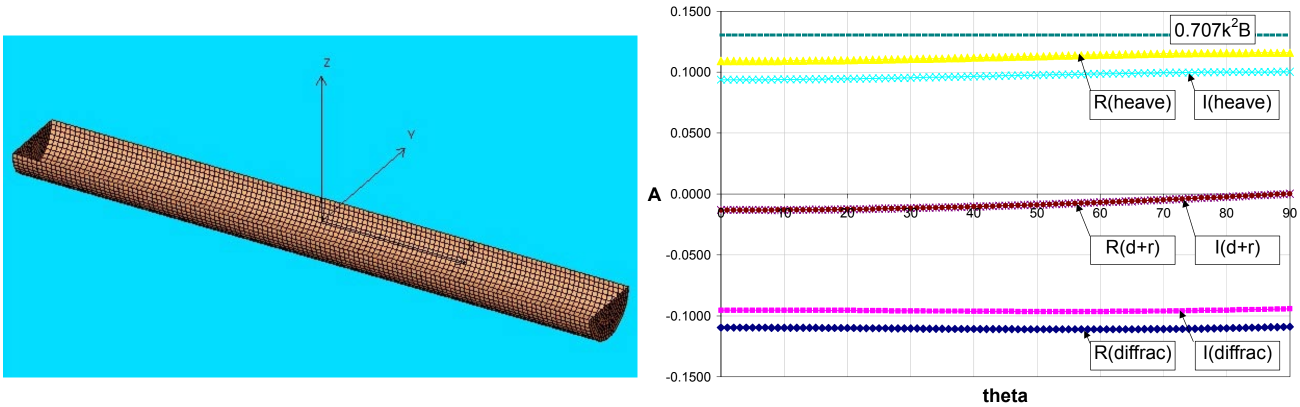

Figure 2. Analysis of a single segment with the diffraction program AQWA-LINE.

Figure 2 also shows the amplitude of the various waves produced by the segment. In this paper we characterise any waves propagating away to infinity by the complex function $A(\theta)$, from which the elevation of the waves at great distance is defined as:

$$\Re\left\{\frac{A(\theta)ae^{i(kR-\omega t)}}{\sqrt{2\pi kR}}\right\} \tag{1}$$

This is because in the polar coordinates (R, θ) these waves will ultimately decay inversely as the square root the distance R from the centre of the device, by energy conservation. The incident waves are assumed to travel in the direction $\theta = 0$ in the sense of increasing R, and to have angular frequency ω and wave number k. Finally a is the complex amplitude of the motion responsible for the waves of (1). Thus when we consider free-floating behaviour in incident waves, $\Re\{ae^{-i\omega t}\}$ is the elevation of the incident waves at the centre of the segment. And when we consider heave motion in still water $\Re\{ae^{-i\omega t}\}$ is the heave motion (positive upwards).

Figure 2 gives results (diffracted and diffracted+radiated waves) with the segment in incident waves 150m long (i.e. as long as all five segments of Pelamis combined) and also for heave motion in still water, at the same frequency. In the first case the diffracted+radiated waves are much smaller than the diffracted wave alone, so the segment is following the incident waves like a small raft, so as to be almost transparent to them. For the heave motion the waves are almost equal and opposite to the diffracted waves. Since the heave motion in waves is almost equal to the incident wave elevation (94% of its amplitude according to AQWA-LINE, and within 0.02 degrees of its phase), we may deduce that in the former case it is the waves radiated by the heave motion that are cancelling the diffracted waves.

Theoretically ([3] eqn 24b), $A(\theta)a$ for heave motion is equal to $-i\omega H(\theta)e^{i\pi/4}/g$, where $H(\theta)$ is the 'Kochin function' ([3] eqn 17). In our case of long waves $H(\theta)$ may be approximated without difficulty as $-k(-i\omega a)B$, where B is the waterplane area of the segment. This leads to a value of $A(\theta) = k^2Be^{i\pi/4}$, which is shown in Figure 2, and agrees correctly with the computations. See also [4] eqn 10 - the more sophisticated slender-body approximation there will in fact recover the θ-dependence seen in Figure 2. For present purposes it is sufficient to observe that this dependence is small, and that the amplitude of the waves radiated by heave motion is proportional to the waterplane area B.

3. Energy flux in the far field: single segment

Hitherto the segment has been floating freely; we are now in a position to assess the effect of incorporating a power take-off so as to extract some wave energy. We will depart from the earlier literature cited above by considering the energy flux in the far field directly. Since the freely-floating segment is practically transparent to the waves, and the only significant wave radiation is from heave motion, we will take the far field waves produced by the segment (the 'produced waves') as being simply those due to the *additional heave motion compared with the freely-floating segment.* Thus in (1) we will take $\Re\{ae^{-i\omega t}\}$ is this additional heave motion (positive upwards), and denote the incident wave elevation at the centre of the segment as $\Re\{be^{-i\omega t}\}$.

The mean energy flux in the far field is the product of the wave pressure and the wave velocity, integrated over a fixed cylindrical control surface where R has some large value R_c, say. The product of wave velocity and hydrostatic pressure does not count because it is purely oscillatory, and the flux of kinetic energy does not count because it is of higher order in wave height. The mean value of the product of any two complex oscillatory variables a and b is $\frac{1}{2}\Re(\bar{a}b)$, so if the incident and produced complex wave pressures are p_I and p_P, and their complex velocities into the control surface are v_I and v_P, the mean energy flux into the control surface is:

$$\int_{-\pi}^{\pi} \tfrac{1}{2}\Re\left\{\overline{(p_I + p_P)}(v_I + v_P)\right\} R_c \frac{d\theta}{2k} \tag{2}$$

Here the variables are taken at the still-water position, the depth integration having yielded the factor $1/(2k)$. We now observe that:

- The p_Iv_I term must integrate to zero because there is zero mean energy flux from the incident waves alone
- The p_Pv_P term must always give an energy loss because the produced waves are leaving the system
- The cross terms must therefore be the source of any energy input

Dealing with energy loss term first, and taking $A(\theta) = k^2Be^{i\pi/4}$ as above, this is readily integrated as:

$$\int_{-\pi}^{\pi}\frac{1}{2}\Re\left\{\overline{\{(\rho g\frac{k^2Bae^{i(kR_c+\pi/4)}}{\sqrt{2\pi kR_c}}}\text{)}(-\omega\frac{k^2Bae^{i(kR_c+\pi/4)}}{\sqrt{2\pi kR_c}})\}\right\}R_c\frac{d\theta}{2k}=-\int_{-\pi}^{\pi}\frac{\rho g\omega k^2B^2|a|^2}{8\pi}d\theta=-\frac{\rho g\omega k^2B^2|a|^2}{4} \tag{3}$$

The energy input form the cross terms is:

$$\int_{-\pi}^{\pi}\frac{1}{2}\Re\left\{\overline{(\rho gbe^{ikR_c\cos\theta}}\text{)}(-\omega\frac{k^2Bae^{i(kR_c+\pi/4)}}{\sqrt{2\pi kR_c}})+(\rho g\frac{k^2Bae^{i(kR_c+\pi/4)}}{\sqrt{2\pi kR_c}})\overline{(-\omega be^{ikR_c\cos\theta}\cos\theta)}\right\}R_c\frac{d\theta}{2k}$$

$$=-\frac{\rho g\omega B}{2}\Re\left\{a\bar{b}\frac{\sqrt{kR_c/2}}{2\sqrt{\pi}}\int_{-\pi}^{\pi}(1+\cos\theta)e^{i(kR_c[1-\cos\theta]+\pi/4)}d\theta\right\} \tag{4}$$

The integrand in (4) is plotted in Figure 3 below for kR_c=30.

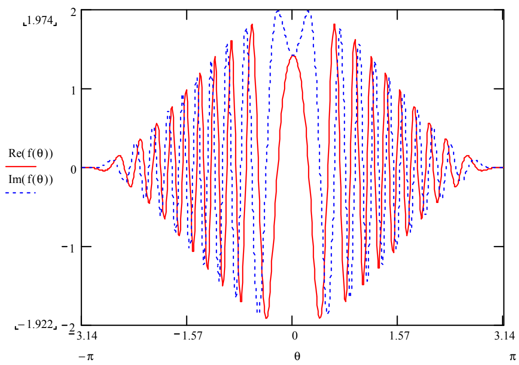

Figure 3. Angular dependence of input energy flux

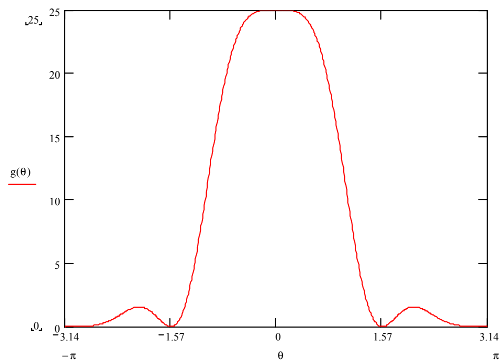

Figure 4. Angular dependence of energy loss, 5 segments

The mean energy flux evidently fluctuates rapidly around the control surface, with the significant peak being at θ = 0, i.e. downwave of the segment. This is because at that point the produced waves are travelling in the same direction as the incident waves, so that the total energy flux is proportional to the square of their combined amplitude. Depending on their relative phase, this is either greater or less than the sum of their individual amplitudes squared, so requiring a substantial cross term. At θ = ±π, on the other hand, the integrand is zero because the waves are travelling in opposite directions, and their total energy flux is thus equal to the sum of their individual fluxes (as is well-known, and can be seen from (2) given the opposite directions of the velocities under wave crests). The required integral can be found exactly by the method of stationary phase ([5] art 241). Only the energy flux around θ = 0 is significant to the integral, and (4) becomes:

$$-\frac{\rho g\omega B}{2}\Re\{a\bar{b}e^{i(\pi/4+\pi/4)}\}=\frac{\rho g\omega B}{2}\Re\{i\bar{b}a\} \tag{5}$$

For any given $|a|$, the energy loss term (3) will be fixed, so we are at liberty to choose the phase of a as equal to that of ib, so as to maximise the energy input (5). The net energy input will then be:

$$\frac{\rho g\omega B}{2}|b||a|-\frac{\rho g\omega k^2B^2|a|^2}{4}=\frac{\rho g\omega B}{4}(2|b||a|-k^2B|a|^2) \tag{6}$$

The energy input term is proportional to $|a|$ whereas the energy loss term is proportional to $|a|^2$, so the maximum net energy can easily be found to occur when $|a|=|b|/(k^2B)$ and to be:

$$\frac{\rho g\omega B}{4}\left(\frac{2|b|^2}{k^2B}-\frac{|b|^2}{k^2B}\right)=\left[\frac{\rho g\omega|b|^2}{4k}\right]k^{-1} \tag{7}$$

Since the term in square brackets is the incident energy per unit length of wave crest, we have recovered the well-known result (e.g. [1] eqn 4.2) that the maximum crest 'capture width' is k^{-1} for a device radiating waves equally in all directions. However, this requires $|a|/|b| = (k^2B)^{-1} = 5.4$ in our case – since $|a|$ is limited to about 1m, this limits the wave amplitude at which this 'capture width' can be achieved, to about 20cm.

4. Energy flux in the far field: complete device

So far there is nothing new in these results. However, Figure 3 and the argument below it suggest that the design objective for the complete device should be to produce waves concentrated on a *downwave* direction ($\theta = 0$). This will maintain the energy input, which only depends on the waves produced in a downwave direction, while reducing the energy loss which depends on the waves produced in all directions. For example, an effective strategy would appear to be to arrange the additional heave motions of each segment considered above (i.e. the heave motions compared with freely floating segments – the segments will be transparent to the waves produced by other segments, just as they are to incident waves) to be phase lagged between each other in the same way as the incident wave itself. The waves produced by each segment will then interfere constructively in a downwave direction, but interfere destructively in the upwave direction. Given the five 30m segments exactly spanning a 150m wavelength, with the centre of the central one at $R=0$, the complex amplitude will in fact vary with θ as:

$$e^{-i0.8\pi(1-\cos\theta)} + e^{-i0.4\pi(1-\cos\theta)} + 1 + e^{i0.4\pi(1-\cos\theta)} + e^{i0.8\pi(1-\cos\theta)}$$

$$= 1 + 2\cos(0.4\pi[1-\cos\theta]) + 2\cos(0.8\pi[1-\cos\theta]) \qquad (8)$$

As far as the energy loss integral (3) is concerned, it is the variation of (amplitude)^2 with θ which counts; this is shown in Figure 4 above. For the same energy input, numerical integration shows that the energy loss is about one third of a single segment, which will increase the 'capture width' to about half a wavelength. Given the larger waterplane area of the complete device, moreover, the limiting wave amplitude is increased from 20cm above to 35cm. In waves of 3.5m amplitude, say (i.e. 7m height, which is quite steep for the given 150m wavelength), the capture width will reduce to about a tenth of a wavelength. This is consistent with the measured performance of the Pelamis WEC. The required large heave motions are achieved in practice by the coupling to a resonant sway mode, as described at the start of this paper. According to AQWA-LINE the sway damping force is less than 1% of the heave damping force (both per unit motion in still water), so the additional waves produced by this sway motion will be negligible.

In the earlier literature cited above, by contrast, the design aim of WECs appears to be to concentrate radiated waves (as opposed to the 'produced' waves considered above which are radiated+diffracted) in the *upwave* direction. See in particular [4] eqn (2), which applies to a device like the Pelamis WEC. The diffracted waves from the Pelamis WEC will clearly be in a downwave direction (by the argument of (8) above, in fact, which applies equally to the diffracted waves) - it is not clear how combing these with radiated waves in an upwave direction will make the Pelamis WEC produce waves downwave overall, as appears to be necessary from the arguments of this paper.

Indeed, it appears that the emphasis on radiating waves upwave has encouraged the development of WECs which are wave reflectors in the quiescent condition, like the Salter "duck". Here such a radiated wave is clearly required, to cancel the wave reflection, without introducing losses behind the WEC. From an engineering point of view, however, it is very difficult to make such a device which is also capable of surviving extreme storms.

Acknowledgement
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