A Panel-Free Method for the Time-Domain Radiation Problem

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1. INTRODUCTION

Based on the desingularized Green's formula of Landweber & Macagno (1969), a method has been developed to solve the radiation problem of a floating body in the time domain. In this method, the singularity in the Rankine source of the Green function is removed. The body surface is mathematically represented by Non-Uniform Rational B-Spline (NURBS) surfaces. Thus, the integral equation can be globally discretized over the body surface by Gaussian quadrature. We call this method as the panel-free method (PFM). The accuracy of PFM based on the NURBS surface representation was demonstrated by its application to a classical problem of uniform flow past a sphere. Computed impulse response function, added-mass and damping coefficients of a hemisphere at zero speed are compared with other published results.

2. MATHEMATICAL FORMULATION

For the radiation problem of a floating body with forward speed U_0 in the time domain, the potential function, $\phi(P;t)$ can be represented as a source distribution as follows:

$$\phi(P;t) = \int_0^t d\tau \int_{\bar{S}_b} G(P,Q;t-\tau)\sigma(Q;\tau)dS + \frac{U_0^2}{g}\int_0^t d\tau \oint_\Gamma n_1 G(P,Q;t-\tau)\sigma(Q;\tau)dl \tag{1}$$

where $\bar{S}_b$ is the mean wetted surface, Γ denotes the waterline, n_1 is the x-component of the inner unit normal vector $\mathbf{n}$ which points into the body surface, and the time-dependent Green function is given by (Wehausen & Laitone 1960, Eq. (13.49)):

$$G(P,Q;t-\tau) = -\frac{1}{4\pi}(\frac{1}{r} - \frac{1}{r_1})\delta(t-\tau) + F(P,Q;t-\tau)H(t-\tau) \tag{2}$$

with

$$F(P,Q;t-\tau) = -\frac{1}{2\pi}\int_0^\infty \sqrt{gk}\sin[\sqrt{gk}(t-\tau)]e^{k(z+z')}J_0(kR)dk \tag{3}$$

where r and r_1 are distances between $P(x,y,z)$ and $Q(x',y',z')$, and P and the image point Q' of Q, $R = \sqrt{(x-x')^2+(y-y')^2}$, J_0 is the Bessel function of the zeroth order, and $\delta(t-\tau)$ and $H(t-\tau)$ are the delta function and the Heaviside step function, respectively. The source strength, σ, can be solved from

$$V_n(P;t) = -\frac{1}{2}\sigma(P;t) + \int_0^t d\tau \int_{\bar{S}_b} \frac{\partial G(P,Q;t-\tau)}{\partial n(P)}\sigma(Q;\tau)dS + \frac{U_0^2}{g}\int_0^t d\tau \oint_\Gamma n_1 \frac{\partial G(P,Q;t-\tau)}{\partial n(P)}\sigma(Q;\tau)dl \tag{4}$$

For a floating body, if the waterline integral is omitted, Eq.(4) can be desingularized as follows:

$$\begin{aligned} V_n(P;t) = -\sigma(P;t) &+ \int_{\bar{S}_b}\left[\sigma(Q;t)\frac{\partial G_1(P,Q)}{\partial n(P)} - \sigma(P;t)\frac{\partial G_1(P,Q)}{\partial n(Q)}\right]dS \\ &+2\int_{\bar{S}_b}\sigma(Q;t)\frac{\partial G_2(P,Q)}{\partial n(P)}dS + \int_0^t d\tau\int_{\bar{S}_b}\frac{\partial F(P,Q;t-\tau)}{\partial n(P)}\sigma(Q;\tau)dS \end{aligned} \tag{5}$$

The LHS of Eq.(5) is known from the body boundary condition, and $G_1(P,Q) = -1/(4\pi)(1/r + 1/r_1)$ and $G_2(P,Q) = 1/(4\pi r_1)$. The source strength can be obtained by solving Eq.(5). Based on the work by Landweber and Macagno, the non-singular representation for the velocity potential can be derived as follows:

$$\phi(P;t) = \int_{\bar{S}_b} G_1(P,Q)\left[\sigma(Q;t) - \gamma(Q)\frac{\sigma(P;t)}{\gamma(P)}\right]dS + 2\int_{\bar{S}_b}\sigma(Q;t)G_2(P,Q)dS + \frac{1}{4\pi}\phi_0\frac{\sigma(P;t)}{\gamma(P)} \tag{6}$$

$$+\int_0^t d\tau \int_{\bar{S}_b} \sigma(Q;\tau)F(P,Q;t-\tau)dS$$

where $\gamma(P)$ is the source distribution on $\bar{S}_b$ which makes the body surface an equipotential surface of potential ϕ_0 and satisfies the homogeneous integral equation

$$\gamma(P) = \int_{\bar{S}_b} \gamma(Q)\frac{\partial K(P,Q)}{\partial n(P)}dS \tag{7}$$

Equation (7) can be desingularized in a similar way as Eq.(5), and $\gamma(P)$ can be solved by finding the eigenfunction of $\partial K(P,Q)/\partial n(P)$ with the eigenvalue equal to 1, where $K(P,Q) = 1/(2\pi)(1/r + 1/r_1)$. The potential, ϕ_0, is constant in the interior of the equipotential surface. It can be computed at the origin by $\phi_0 = -\int_{\bar{S}_b}\gamma(Q)(1/|Q| + 1/|Q'|)dS$, where $|Q|$ and $|Q'|$ denote distances between Q and the origin, and Q' and the origin, respectively.

3. NUMERICAL IMPLEMENTATION

It is assumed that N_p patches are used to describe a body surface. Each patch can be represented by a NURBS surface (Farin, 1991). Let $P(x(u,v), y(u,v), z(u,v))$ be a point on a patch; x, y and z denote the Cartesian coordinates; and u and v are two parameters for the surface definition. On a NURBS surface, $P(u,v)$ can be defined as follows:

$$P(u,v) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{m} w_{ij}C_{i,j}N_{i,p}(u)N_{j,q}(v)}{\sum_{i=0}^{n}\sum_{j=0}^{m} w_{ij}N_{i,p}(u)N_{j,q}(v)} \tag{8}$$

where w_{ij} are the weights; $C_{i,j}$ form a network of control points; and $N_{i,p}(u)$ and $N_{j,q}(v)$ are the normalized B-splines basis functions of degrees p and q in the u and v directions, respectively.

Since Eq.(5) is singularity free, it can be discretized by directly applying the Gaussian quadrature and the trapezoidal time integration scheme. The Gaussian quadrature points are arranged in the computational space, rs, then their corresponding coordinates, normals, Jacobian in the physical space can be obtained based on Eq.(8). Therefore, the Eq.(5) can be written as

$$\begin{aligned}
V_n(P_i;t) = -\sigma(P_i;t) &+ \sum_{j=1}^{N_p}\sum_{r=1}^{N_j}\sum_{s=1}^{M_j} w_r w_s \left[\sigma(Q_j^{rs};t)\nabla_P G_1(P_i,Q_j^{rs})\cdot \mathbf{n}_{p_i} + \sigma(P_i;t)\nabla_P G_1(P_i,Q_j^{rs})\cdot \mathbf{n}_{q_j^{rs}}\right] J_j^{rs} \\
&+ 2\sum_{j=1}^{N_p}\sum_{r=1}^{N_j}\sum_{s=1}^{M_j} w_r w_s \sigma(Q_j^{rs};t)\nabla_P G_2(P_i,Q_j^{rs})\cdot \mathbf{n}_{p_i} J_j^{rs} \\
+\Delta t &\left[\frac{1}{2}\sum_{j=1}^{N_p}\sum_{r=1}^{N_j}\sum_{s=1}^{M_j} w_r w_s \sigma(Q_j^{rs};0)\frac{\partial F(P_i,Q_j^{rs};t)}{\partial n}J_j^{rs} + \sum_{k=1}^{k_t-1}\sum_{j=1}^{N_p}\sum_{r=1}^{N_j}\sum_{s=1}^{M_j} w_r w_s \frac{\partial F(P_i,Q_j^{rs};t-t_k)}{\partial n}\sigma(Q_j^{rs};t_k)J_j^{rs}\right], \\
& i = 1,2,\ldots,N_p
\end{aligned} \tag{9}$$

where N_j and M_j are the number of Gaussian quadrature points in the u- and v-directions on the jth patch. $P_i = P_i(u_n, v_m), n = 1, \ldots N_i, m = 1, \ldots M_i$ and $Q_j^{rs} = Q_j(u_r, v_s)$ are the position vectors of Gaussian quadrature points on the ith and jth patches in the physical space, respectively; $\mathbf{n}_{p_i}$ and $\mathbf{n}_{q_j^{rs}}$ are the corresponding unit normals; w_r and w_s are the weighting coefficients in the u and v directions; J_j^{rs} is the Jacobian of Q_j^{rs}; t is the time; and Δt is the time step, $t_k = k\Delta t$ and $t = k_t\Delta t$, where k and k_t are the time constants at any instant and for the total time, respectively. It can be seen that the algorithm is only controlled by the number of Gaussian quadrature points.

4. NUMERICAL RESULTS

Since the singularity occurs only in the $1/r$ term, it is important to validate the desingularization of the integral equation with the $1/r$ term only before it is applied to the time-domain integration. The numerical scheme was applied to the problem of uniform flow ($U = -1.0$) past a sphere surface ($R = 1.0$). Due to the symmetry, only one-half of the surface was considered. In Figure 1, the dashed lines represent the control net of NURBS with 5x5 control points on one of patches ($N_p = 2$). The solid meshes are the surface of one-quarter of the sphere generated by the control net. The disturbed velocity potentials at the Gaussian quadrature points were computed using both NURBS and analytical expressions of the surface. The numerical testing was also conducted to investigate the convergence of numerical solution to the number of Gaussian quadrature points (NxN) over the hemisphere. The root-mean-squared (RMS) errors of computed velocity potentials based on the analytical expression and the NURBS representation of the surface are shown in Figure 2. It is shown that the computed velocity potentials converge to the analytical solution as the number of Gaussian quadrature points increased. The RMS error of the solution based on the NURBS representation is less than 1% when 10x10 Gaussian quadrature points are applied.

The panel-free method was applied to compute the response function for a hemisphere (R=5.0m) in heave. Figure 3 shows the nondimensional response function, $K_{33}(t)/(\rho\nabla)\sqrt{R/g}$, versus nondimensional time, $t\sqrt{g/R}$, for different Gaussian quadrature points used on the hemisphere, where R is the radius of the sphere and ∇ is the volume displacement. The time step was chosen as 0.05 second. The circle points give the analytic solution of Barakat (1962) obtained by the Fourier transformation from his frequency-domain results.

The nondimensional response function for the heaving hemisphere was also computed using different time steps with 16x16 Gaussian quadrature points. As shown in Figure 4, PFM is insensitive to the size of time steps. Figure 5 and Figure 6 present the added-mass and damping coefficients versus the nondimensional frequency for the hemisphere in heave. The numerical results were obtained by Fourier transformation from the response function using 16x16 Gaussian points as shown in Figure 3, and the analytical results were from Barakat (1962). Also in these figures, the frequency, the added-mass and the damping coefficients are nondimensionalized as $\omega^2 R/g$, $A_{33}/(\rho\frac{2}{3}\pi R^3)$ and $B_{33}/(\omega\rho\frac{2}{3}\pi R^3)$, respectively.

5. CONCLUSIONS

The panel-free method has been developed to solve the radiation problem of a hemisphere at zero speed in the time domain. The boundary integral equation in terms of source strength distribution is desingularized so that the Gaussian Quadrature can be directly applied to the exact body surface. Compared with the panel method, the advantages of PFM are: a) less numerical manipulating, since panelization of a body surface is not needed; b) more accurate, since the assumption for the configuration of source strength distribution as in the panel method is not needed and no approximation of surface geometry is involved; c) the Gaussian quadrature points, and their respective Jacobian and normals on the surface can be accurately computed from the NURBS expression; and d) the accuracy of the solution is controlled only by the number of Gaussian quadrature points. This method is currently being applied to the computation of ship motions in the time domain. The wave diffraction effect is also considered.

ACKNOWLEDGMENTS

This work was supported by the Natural Science and Engineering Research Council, Canada. Useful discussions with Dr. J.M. Chuang are also appreciated.

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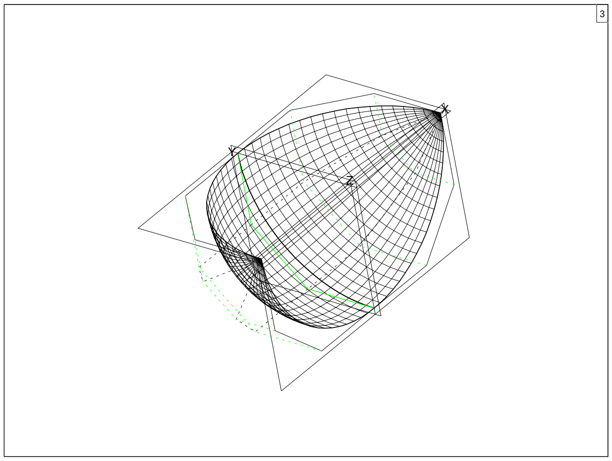

Figure 1: Sphere surface and control points

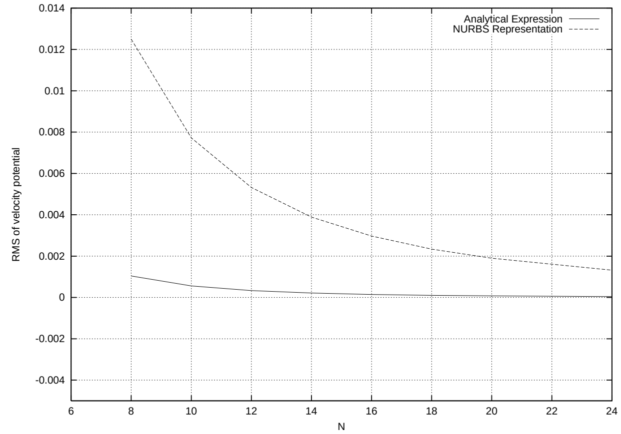

Figure 2: Convergence of numerical solution to the number of Gaussian quadrature points

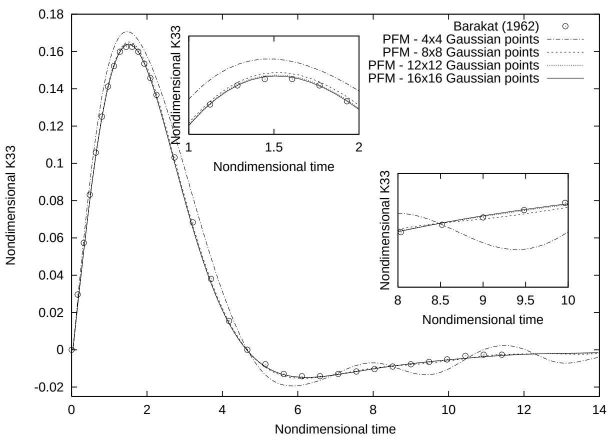

Figure 3: Nondimensional heave response function on a hemisphere versus nondimensional time ($\Delta t = 0.05s$)

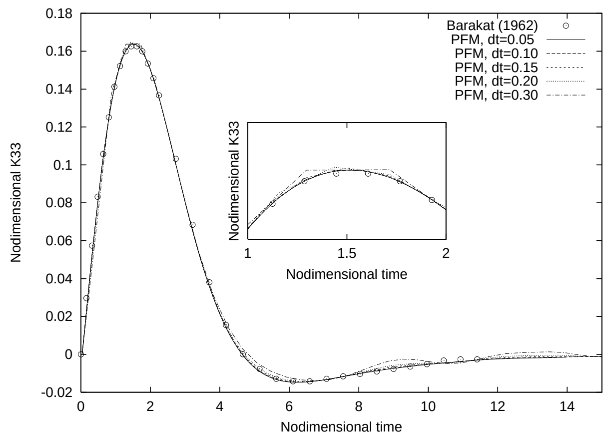

Figure 4: Nondimensional heave response function on a hemisphere versus nondimensional time (16x16 Gaussian points)

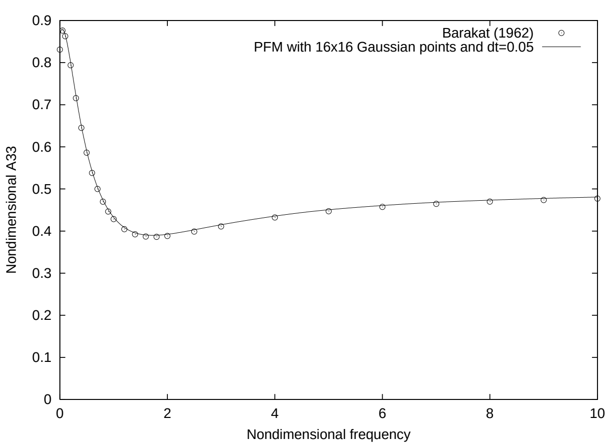

Figure 5: Nondimensional added-mass for a hemisphere in heave versus nondimensional frequency ($\Delta t = 0.05s$, 16x16 Gaussian points)

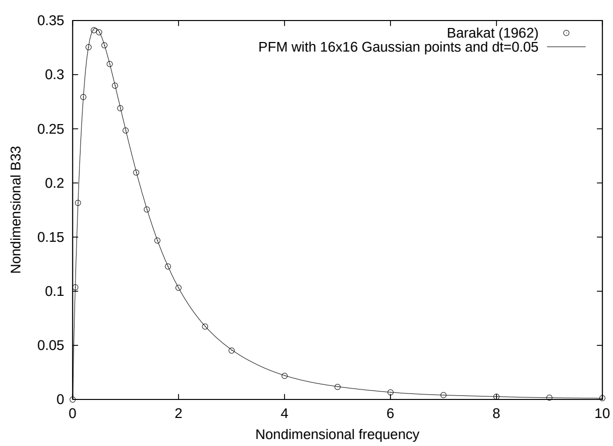

Figure 6: Nondimensional damping coefficient for a hemisphere in heave versus nondimensional frequency ($\Delta t = 0.05s$, 16x16 Gaussian points)