REDUCTION OF HYDROELASTIC RESPONSE OF FLOATING PLATFORM IN WAVES

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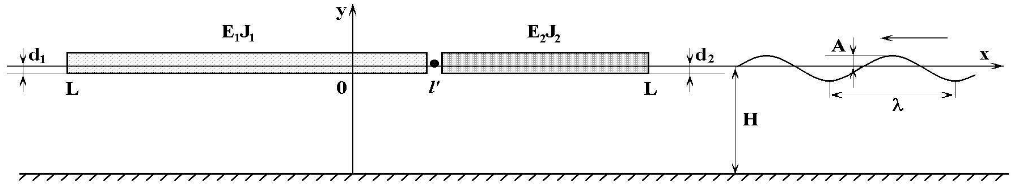
1. Introduction

A very large floating structures are considered as an alternative of such land-based large facilities as, for example, airport. A proposed design of floating airport has a thin plate configuration of large horizontal extend. Bending rigidity of such a floating plate is small, and wave-induced motion of the plate is significantly affected by its elastic deflection. Analysis of floating plate behaviour in waves is based on hydroelasticity, in which the coupled hydrodynamics and structural dynamics problems are solved simultaneously. A goal of the analysis is to predict accurately both the plate deflection and stresses in the plate and to find a way for their reduction. The latter is of great importance for securing safety and the structure performance. Reduction of the motion of an floating elastic plate in waves by surrounding it by an breakwater was studied numerically in [1,2]. It was shown that breakwaters effectively reduce the plate response for long waves but in the case of short waves the reduction is not well-pronounced. The idea to put a floating structure in the shadow of a breakwater for reduction of the structure response is clear and practical. However, the behaviour of a structure in restricted water might be affected by its hydrodynamic interaction with the breakwater and resonance phenomena might occur. Another way to reduce the floating plate response was suggested in [6] that is to adjust to the front side of the elastic plate a wave reflector – vertical submerged plate, the height of which is about three times less than the water depth – or a wave-breaking structure – multi-column floating structure of small extend. Experiments [6] revealed that both the wave reflector and the wave-breaking structure decrease deflections of the main structure in the case of short incident waves. However, for long incident waves which provide greater deflections of the main structure than short waves, the experiments did not detect well-pronounced effects of the additional structures. Both approaches [2,6] are based on the idea to protect (to shield) a floating elastic structure from the incident wave action, in order to reduce a part of the wave energy which can be absorbed by the structure.

In order to test possible approaches aimed to reduce floating plate response in waves, direct numerical simulations of hydroelastic behaviour of the plate are very attractive. Three-dimensional numerical simulation of the linear response of an elastic plate in waves is the most accurate approach. Three-dimensional numerical simulations of floating rectangular plate in waves were performed in [3,4]. However, at present these simulations are still time-consuming and expensive to use them at the design stage. At the very initial stage of design it looks reasonable to use the simplest models of floating plate behaviour, in order to discover main trends and to distinguish main features of the problem. If an effect is well-pronounced within a simple model, it is expected to be of importance also within more accurate models.

In this paper two approaches to reduce elastic deflection of floating plates are described within the two-dimensional linear theory. In the two-dimensional problem the plate is modeled by an Euler homogeneous beam. Developed method is applied also to the problem of cracked floating beam. The first approach is based on the concept of vibration absorber well-known in many engineering applications. Within the second approach the floating beam is connected to the sea bottom with a spring, rigidity of which can be adjusted in such a way that the beam deflection due to incident waves is reduced.

Four 2D-problems on hydroelastic behaviour of a floating beam in waves are considered, where the beam is (**i**) homogeneous, (**ii**) cracked, (**iii**) compound with an elastic connection between the parts of the beam, (**iv**) homogeneous and elastically connected to the sea bottom. The problems are treated by the common method described below. The formulation is given for the third problem on compound beam behaviour, which is the most general one. The scheme of the flow and the main notations are given in the figure



2. Formulation of the problem

The plane linear problem of a floating beam in waves is considered. The beam vibration is caused by periodic incident wave of frequency ω and small amplitude A. The beam consist of two parts (see figure) with their bending stiffnesses $E_i J_i$ and drafts d_i (i=1,2) being prescribed. The beam drafts are assumed much smaller than both the total beam length $2L$ and the liquid depth H. We shall determine the beam deflection and the stress distribution in the beams and study their dependence on characteristics of the beam parts and conditions of their connection.

Non-dimensional variables are used below: L is taken as the length scale, $1/\omega$ as the time scale, the amplitude of the incident wave A as the deflection scale, the product $\rho g A$, where ρ is the liquid density and g is the acceleration due to gravity, as the pressure scale, $2Ld\rho g$ as the scale of bending stresses, and the product $A\omega L$ as the scale of the velocity potential. Within the linear wave theory the non-dimensional hydrodynamic pressure $p(x,0,t)$ along the beam , $-1<x<1$, and the beam deflection $w(x,t)$ are given as $p(x,0,t)=\Re[e^{it}P(x)]$ and $w(x,t)=\Re[e^{it}W(x)]$, respectively. The new unknown complex-valued functions $P(x)$ and $W(x)$ satisfy the following equations and the boundary conditions:

$$P(x)+\frac{\gamma}{2\pi}\int_{-1}^{1}P(x_0)K(x-x_0)dx_0=e^{ikx}-W(x), \tag{1}$$

$$\beta(x)W^{IV}-\alpha(x)W=P(x) \qquad (-1<x<1), \tag{2}$$

$$W''(\pm 1)=0, \qquad W'''(-1)=0, \qquad W'''(+1)=k_lW(+1), \tag{3}$$

$$W(l-0)=W(l+0), \qquad \beta_1W''(l-0)=\beta_2W''(l+0), \qquad \beta_1W'''(l-0)=\beta_2W'''(l+0), \tag{4}$$

$$W''(l-0)+k_T[W'(l-0)-W'(l+0)]=0, \tag{5}$$

where
$$\alpha(x)=\begin{cases}\alpha_1 & \text{for } x\in[-1,l),\\ \alpha_2 & \text{for } x\in(l,1],\end{cases} \qquad \beta(x)=\begin{cases}\beta_1 & \text{for } x\in[-1,l),\\ \beta_2 & \text{for } x\in(l,1].\end{cases}$$

The problem (1) - (5) contains eight parameters:

$$k_l=K_lL^3/E_1J_1,\;\; k_T=K_TL/E_1J_1,\;\; \gamma=L\omega^2/g,\;\; \alpha_j=\gamma d_j/L,\;\; \beta_j=E_jJ_j/(\rho gL^4),\;\; (j=1,2)$$

and k which is the positive solution of the dispersion equation $k\tanh(kH_0)=\gamma,\ H_0=H/L$. The function $K(z)$ in (1) is given as

$$K(z)=-2\pi i\frac{ke^{-ik|z|}}{H_0(k^2-\gamma^2)+\gamma}+2\pi\sum_{j=1}^{\infty}\frac{s_je^{-s_j|z|}}{H_0(s_j^2+\gamma^2)-\gamma},$$

where $s_j=(\pi j-\delta_j)/H_0$ and δ_j is the solution of the equation $\delta_j=\arctan(\gamma H_0/(\pi j-\delta_j))$, $j\geq 1$.

The boundary-value problem (1)-(5) describes the hydroelastic behaviour of a free-free homogeneous beam in waves with $\alpha_1=\alpha_2,\ \beta_1=\beta_2,\ k_l=0$ and $k_T=\infty$, of a free-free cracked beam with $\alpha_1=\alpha_2,\ \beta_1=\beta_2,\ k_l=0$ and $k_T\geq 0$, of a free-free compound beam with $\alpha_1\neq\alpha_2,\ \beta_1\neq\beta_2,\ k_l=0$ and $k_T\geq 0$, and of an homogeneous beam connected elastically to the sea bottom, with $\alpha_1=\alpha_2,\ \beta_1=\beta_2,\ k_l\neq 0$ and $k_T=\infty$.

3. Method of solution

Problem (1) - (5) can be solved with the help of the normal mode method in the same manner as in [7]. This method reduces the integral equation (1) to infinite system of algebraic equations with respect to the principle coordinates of the pressure $P(x)$. However, the eigenfunctions of the compound beam are rather complicated and, moreover, they do not correspond to the features of the hydrodynamic pressure distribution along the beam. A main idea of the present study is to use different basic functions for the pressure and the beam deflection. Trigonometric functions are used as basic functions to present the pressure in the form

$$P(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}ac_n\cos\pi nx+\sum_{n=1}^{\infty}as_n\sin\pi nx. \tag{6}$$

Substitution of expansion (6) into equation (2) leads to the following expansion for the beam deflection

$$W(x)=\frac{1}{2}a_0wc_0(x)+\sum_{n=1}^{\infty}ac_nwc_n(x)+\sum_{n=1}^{\infty}as_nws_n(x). \tag{7}$$

The functions $wc_j(x)$ and $ws_j(x)$ satisfy conditions (3)-(5) and equation (2) with $P(x)$ being replaced by $\cos(j\pi x)$ and $\sin(j\pi x)$, respectively. The functions $wc_j(x)$ and $ws_j(x)$ are considered here as basic functions for the beam deflection. The integral equation (1) with account for expansions (6) and (7) leads to the infinite system of algebraic equations with respect to the coefficients ac_n and as_n.

$$(\mathbf{I}+\frac{\gamma}{2\pi}\mathbf{S}+\mathbf{A})\,\vec{a}=\vec{e}. \tag{8}$$

Here $\mathbf{I}=\text{diag}(2,1,1,...)$ is diagonal matrix, symmetric matrix $\mathbf{S}$ comes from the integral term in (1), symmetric matrix $\mathbf{A}$ comes from the term $W(x)$, and $\vec{a}=(ac_0/2,ac_1,ac_2,...ac_n,as_1,as_2,...as_n)^T$. The elements of the vector $\vec{e}$ are the coefficients in the expansion of $\exp(ikx)$ with respect to the trigonometric functions. All elements of the matrices $\mathbf{S}$ and $\mathbf{A}$ and those of the vector $\vec{e}$ are given by analytical formulae.

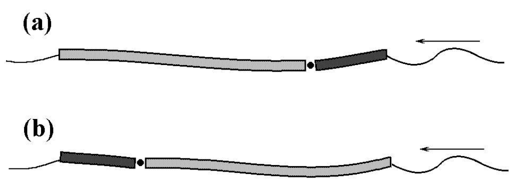
4. Free-free homogeneous beam

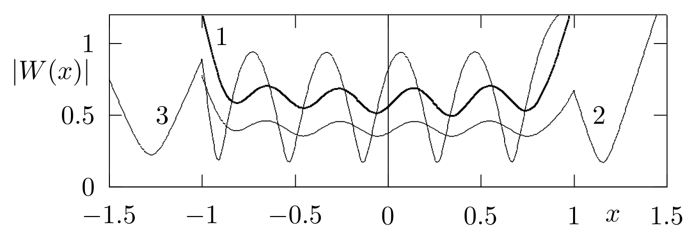
Problem (1) - (3) with $\alpha_1=\alpha_2,\ \beta_1=\beta_2$ and $k_l=0$ corresponds to that of hydroelastic behaviour of the homogeneous free-free beam in waves and was studied in [5] by the domain decomposition method, in [7] by the normal mode method and in [8] with the help of a combination of these methods. The obtained numerical results are in good agreement for low frequencies of incident waves but differ each other for high frequencies. The problem of high-frequency excitation of floating elastic plates is not solved yet. The low-frequency case is considered here only.

Numerical calculations were performed for the conditions of the experiments carried out by Wu *et al* [8] for homogeneous narrow plate in a channel: d =8.36mm, H =1.1m, h =38mm, EJ =471kg m^3/s^2, L =5m. The frequency of incident wave is equal to 4.4s^{-1} (period of the wave $T = 1.429$s) and 2.2s^{-1} (period of the wave $T = 2.875$s). In this cases $\beta = 7.7 \cdot 10^{-5}, \alpha = 0.016$ and $\alpha = 0.004$, $\gamma = 9.85$ and $\gamma = 2.43$, $k = 10.1$ and $k = 3.654$, respectively, depending on the incident wave frequency. Convergence of the numerical algorithm was checked by changing the number of terms taken into account in each sum of (6) and (7). Ninety terms were used to plot the obtained numerical results. The present results for the homogeneous beam, are identical with those obtained in [5,7,8] by other methods. The amplitude of the beam deflection $|W(x)|$ is shown below only for $T = 1.429$s.

5. Free-free compound beam

The linear problem of two floating beams is considered. The beams are connected with the help of a torsional spring. Vibrations of the beams are caused by periodic incident wave of small amplitude. The longer beam is referred to as the main structure, characteristics of which are prescribed. The shorter beam is referred to as the auxiliary plate, length of which is given. Both characteristics of the auxiliary plate and the torsional spring stiffness which essentially reduce the vibration amplitude of the main structure are determined. The auxiliary plate can be adjacent either in front of the structure (case *a*) or at the rear side of it (case *b*).



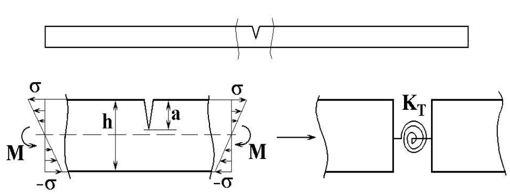


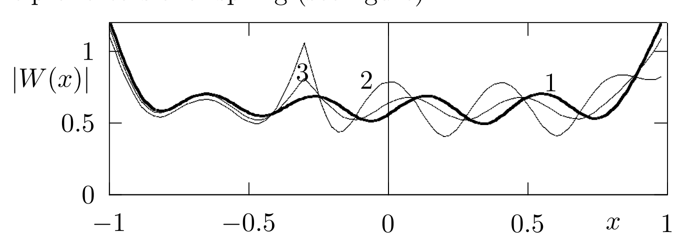
Calculations were performed for the conditions of experiments [8] for main plate. Period of the incident wave is equal to 1.429s (line 1 is for a single plate, line 2 is for case *a*, line 3 is for case *b*, length of the auxiliary plate is equal to 0.25 of the main plate length, $(EJ)_{aux} = 100(EJ)_{main}$). It was revealed that:
• auxiliary plates adjacent in front of the main structure (case *a*) decrease the structure vibrations;
• vibrations of the main structure are increased with auxiliary plates attached to its rear side (case *b*);
• reduction of the vibration is strongest if the plates are simply connected ($k_T = 0$);
• auxiliary plate of length 1.5m decrease the deflections by 20% (case *a*) and increase them by 10% (case *b*);
• essential reduction (35%) of the structure vibrations was obtained in the case of rigid auxiliary plates of length 2.5m simply connected in front of the main structure.

Roughly speaking, in order to reduce the floating plate vibrations, a rigid plate of smaller length has to be simply connected in front of the main structure.

6. Free-free cracked beam

In order to model the cracked beam problem, the method of matched asymptotic expansions is used. According to this method, the beam is divided into the 'inner' region which surrounds the crack, and the 'outer' region, where the transverse variation of the stresses is not important and the plate is modeled by an elementary homogeneous beam. In the leading order as $h/L \to 0$, reduction in stiffness of the beam due to the presence of a crack is modeled with the help of a torsional spring (see figure).





The equivalent torsional spring stiffness K_T for a single-sided crack is assumed known as a function of the beam parameters and the crack length a. The 'outer' solution for the floating free-free beam which is divided by the torsional spring into two parts, provides the bending stresses outside the crack region. Therefore the 'outer' solution is described by the problem considered in Section 5, where $\alpha_1 = \alpha_2,\ \beta_1 = \beta_2,\ k_l = 0,\ k_T \geq 0$.

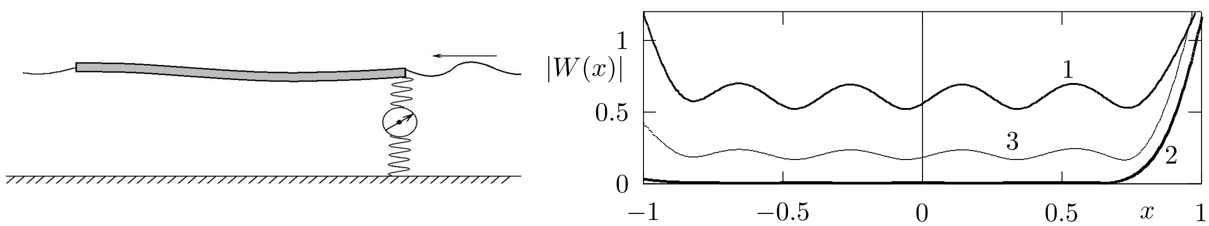
As a result of numerical calculations, the distribution of beam deflections and bending moments were obtained outside the crack region for different positions of the crack and its length. Analysis of numerical calculations gives:

- Presence of a crack changes the distributions of both the plate deflections and stresses, if the crack is longer than a half of the plate thickness. The longer the crack, the more pronounced are the changes.
- Local maximum of the deflections and local minimum of the bending stresses occur at the crack position.
- These changes are much more pronounced if a crack is located at the points of maximum bending stresses of the equivalent homogeneous plate.

Calculations were performed for the conditions of the experiments [8]. The results are depicted for homogeneous beam ($a = 0$, curve 1), broken beam ($a = h$, $l = -0.3$, curve 2) and cracked beam ($a/h = 0.8$, $l = -0.3$, curve 3). It is seen that the presence of the crack increases locally the deflections but decreases the stresses in the beam. The 'outer' solution gives necessary data to evaluate the stress intensity factor at the crack tip and to predict the evolution of the crack length in time.

7. Floating beam with its edge being elastically connected to the sea bottom

Numerical solution of problem (1)-(5) with $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$, $k_T = \infty$ and $k_l > 0$ revealed that elastic connection of the front edge of the floating beam to the bottom can essentially reduce the beam deflections in the main part of the beam. Rigidity of the elastic connector can be adjusted in an optimal way for a given frequency of incident wave.



In the figure the amplitudes of the beam deflections are shown for the free-free beam ($k_l = 0$, curve 1) and for elastically connected beam ($k_l = 1000$, curve 2 and $k_l = 700$, curve 3). Parameters of numerical calculations are given in Section 4, $T = 1.429$s. The curve with $k_l = 1300$, is similar to the curve with $k_l = 700$. It is seen that the dimensional rigidity of the elastic connector $K_l \approx 3800 kg/s^2$ can be considered as optimal for the conditions of experiment [8]. For another frequency of incident wave the connector rigidity has to be changed, which can be done with an active control system.

8. Conclusion

The method of numerical solution of the floating beam problem is based on expansions of the hydrodynamic pressure and the beam deflection with respect to different basic functions. This makes it possible to simplify the treatment of the hydrodynamic part of the problem and at the same time to satisfy accurately the beam boundary conditions. Two approaches aimed to reduce the beam vibrations are described. The effect of the vibration reduction is well pronounced and can be utilized at the design stage. Combination of the presented approaches is expected to be perspective.

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