AIR CUSHION UNDER FLOATING OFFSHORE STRUCTURE

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1 Introduction

In this paper, we present a new integral equation to describe the motion of an air cushion supported platform. These platform are studied as a design concept for floating airports. The amplitudes of motion of such body are expected smaller and a better repartition of pressure on the body reduces the mechanical structural loads. In this paper, we study the behavior of an air cushion supported floating platform exited by waves. The platform consists of a rigid body and an air cavity beneath it. We assume that there is no air leakage. For clarity, we restrict our theory to heave motion.
We assume the flow being potential reducing the problem to the determination of a potential Φ and use usual assumptions of linearized potential theory. The platform's boundary Σ is split into the boundary Σ_1 for the wetted part of the platform, and Σ_2 which is the free surface underneath the platform and submitted to air cushions pressure. An integral equation is then given for the determination of the potentials of diffraction and radiation. It is possible to extend the method to several air cushions, connected or not, and to take into account the pitch motion.

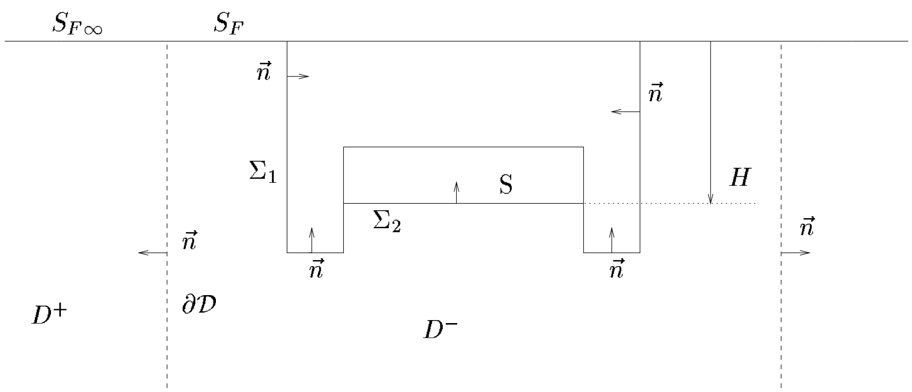

Figure 1: Definition of the geometry

2 Boundary conditions

We define η to be the surface elevation under the cavity, S the interface of this cavity at a distant of H meter from the free surface, V_{cs} and p_{cs} the volume and pressure in the cavity when the platform is at rest. The instantaneous pressure p_c is supposed to be uniform in the platform cavity. We have the following kinematic and dynamic condition at the interface Σ_2:

$$p_c = -\rho g H - \rho g \eta - \rho \frac{\partial \Phi}{\partial t} \qquad , \qquad \frac{\partial \eta}{\partial t} = \frac{\partial \Phi}{\partial z} \tag{1}$$

The change of pressure can be determined by the change of the volume of cavity. The air compression obeys the adiabatic law and we can write:

$$\frac{\Delta p}{p_{cs}} = -\gamma \frac{\Delta V}{V_{cs}} \qquad , \qquad \Delta V = -\iint\limits_{\Sigma_2} (\eta - \xi)\, dS \tag{2}$$

where ξ is the platform heave motion and $\gamma = 1.4$. Combining relations [1] and [2] leads to the boundary condition on Σ_2.

$$\frac{\partial^2 \Phi}{\partial t^2} + g\frac{\partial \Phi}{\partial z} - \frac{\gamma p_{cs}}{\rho V_{cs}}\frac{\partial \Delta V}{\partial t} = 0$$

$$\text{or} \qquad \frac{\partial^2 \Phi}{\partial t^2} + g\frac{\partial \Phi}{\partial z} + \frac{\gamma p_{cs}}{\rho V_{cs}}\iint\limits_{\Sigma_2}\frac{\partial \Phi}{\partial z}\,dS - \frac{\gamma p_{cs} S}{\rho V_{cs}}\frac{\partial \xi}{\partial t} = 0$$

$$\text{Let} \quad \alpha = \frac{\gamma p_{cs}}{\rho g V_{cs}}$$

αS, which is a non-dimensional number, represents the ratio of the force due to air compression by the buoyancy force.
In frequency domain, we write $\Phi(\underline{x}, t) = \phi(\underline{x})\, e^{-i\omega t}$ and we have then the boundary conditions:

$$\left\{ -\nu\phi + \frac{\partial \phi}{\partial z} + \alpha\iint\limits_{\Sigma_2}\frac{\partial \phi}{\partial z}\,dS + i\omega\alpha S\xi = 0 \right\}_{\Sigma_2} \quad ; \quad \left\{ \frac{\partial \phi}{\partial n} = -i\omega\xi n_z \right\}_{\Sigma_1} \tag{3}$$

We also add the usual linearized free surface condition at $z = 0$ and the Sommerfeld radiation condition at infinity.

3 Boundary value problem

The fluid domain is split in two regions, separated arbitrary by an interface ∂D. The platforms stays in the region D^- and the region towards infinity is defined as D^+. The potential function in D^+ is written as the superposition of the incident wave potential and a diffracted wave potential as follows

$$\phi(\underline{x}) = \phi^{inc}(\underline{x}) + \phi^+(\underline{x})$$

In D^+, the total potential is denoted as $\phi^-(\underline{x})$. At the dividing surface ∂D we require continuity of the total potential and its normal derivative.
We introduce the Green's function $\mathcal{G}(\underline{x}, \underline{\xi})$ that fulfills $\Delta\mathcal{G} = 4\pi\delta(\underline{x} - \underline{\xi})$, the free surface and the radiation condition.
Applying Green's theorem for ϕ^+ and ϕ^- leads to the following formula:

for x $\in D^-$:

$$0 = -\iint\limits_{S_{F\infty}\cup\partial\mathcal{D}} (\phi^+\frac{\partial \mathcal{G}}{\partial n} - \mathcal{G}\frac{\partial \phi^+}{\partial n})\,dS$$

$$4\pi\phi^- = \iint\limits_{S_F\cup\Sigma\cup\partial\mathcal{D}} (\phi^-\frac{\partial \mathcal{G}}{\partial n} - \mathcal{G}\frac{\partial \phi^-}{\partial n})\,dS \tag{4}$$

The integrals over $S_{F\infty}$ and S_F become zero, due to the free surface condition for $\mathcal{G}$, ϕ^+ and ϕ^-. Adding up the two expressions in [4], leads to:

$$4\pi\phi^- = \iint\limits_{\Sigma} (\phi^-\frac{\partial \mathcal{G}}{\partial n} - \mathcal{G}\frac{\partial \phi^-}{\partial n})\,dS + \iint\limits_{\partial\mathcal{D}} ([\phi]\frac{\partial \mathcal{G}}{\partial n} - \mathcal{G}\,[\frac{\partial \phi^-}{\partial n}])\,dS \qquad \text{for x} \in D^-$$

$$\text{or} \qquad 4\pi\phi^- = \iint\limits_{\Sigma} (\phi^-\frac{\partial \mathcal{G}}{\partial n} - \mathcal{G}\frac{\partial \phi^-}{\partial n})\,dS + 4\pi\phi^{inc}$$

When $\underline{x}$ tends to Σ, we have then

$$2\pi\phi^- = 4\pi\phi^{inc} + \iint_{\Sigma_1\cup\Sigma_2} (\phi^- \frac{\partial\mathcal{G}}{\partial n} - \mathcal{G}\frac{\partial\phi^-}{\partial n})\, dS \qquad (5)$$

We decompose the potential into a potential of diffraction and a potential of radiation

$$\phi = \phi^D - i\omega\xi\phi^R$$

diffraction:
From [3], we write the boundary conditions for ϕ^D:

$$\Big\{ -\nu\phi^D + \frac{\partial\phi^D}{\partial z} + \alpha\iint_{\Sigma_2} \frac{\partial\phi^D}{\partial z}\, dS = 0\Big\}_{\Sigma_2} \qquad \Big\{\frac{\partial\phi^D}{\partial n} = 0\Big\}_{\Sigma_1}$$

$$\text{let} \quad \tau_D = \iint_{\Sigma_2} \frac{\partial\phi^D}{\partial z}\, dS$$

Integrating [3] on Σ_2 and re injecting the result in the equation, we obtain:

$$\tau_D = \frac{\nu}{1+\alpha S}\iint_{\Sigma_2} \phi^D\, dS \qquad \text{and} \qquad \frac{\partial\phi^D}{\partial n} = \nu\phi^D - \frac{\nu\alpha}{1+\alpha S}\iint_{\Sigma_2}\phi^D\, dS$$

Following Noblesse, we can write:

$$(\frac{\partial\mathcal{G}}{\partial n} - \nu\mathcal{G})_{\Sigma_2} = \mathcal{H}(r, r_1) = -(\frac{1}{r} + \frac{1}{r_1})_\zeta + \nu(\frac{1}{r} - \frac{1}{r_1})$$

The potential ϕ^D is then found to be solution of the integral equation:

$$2\pi\phi^D - \iint_{\Sigma_1} \phi^D\frac{\partial\mathcal{G}}{\partial n}\, dS - \iint_{\Sigma_2}\mathcal{H}\,\phi^D\, dS - \frac{\alpha\nu}{1+\alpha S}\iint_{\Sigma_2}\mathcal{G}\, dS \times \iint_{\Sigma_2}\phi^D\, dS = 4\pi\phi^{inc} \qquad (6)$$

radiation:
We apply the same procedure for the radiated potential. The boundary equations read:

$$\Big\{ -\nu\phi^R + \frac{\partial\phi^R}{\partial z} + \alpha\iint_{\Sigma_2}\frac{\partial\phi^R}{\partial z}\, dS - S\alpha = 0\Big\}_{\Sigma_2} \qquad \Big\{\frac{\partial\phi^R}{\partial n} = n_z\Big\}_{\Sigma_1}$$

and we find:

$$\tau_R = \frac{1}{1+\alpha S}\{\nu\iint_{\Sigma_2}\phi^R\, dS + S^2\alpha\} \qquad \frac{\partial\phi^R}{\partial n} = \nu\phi^R - \frac{\nu\alpha}{1+\alpha S}\iint_{\Sigma_2}\phi^R\, dS + \frac{\alpha S}{1+\alpha S}$$

We obtain the integral equation for ϕ^R :

$$2\pi\phi^R - \iint_{\Sigma_1}\phi^R\frac{\partial\mathcal{G}}{\partial n}\, dS - \iint_{\Sigma_2}\mathcal{H}\,\phi^R\, dS - \frac{\alpha\nu}{1+\alpha S}\iint_{\Sigma_2}\mathcal{G}\, dS \times \iint_{\Sigma_2}\phi^R\, dS$$
$$= -\frac{\alpha S}{1+\alpha S}\iint_{\Sigma_2}\mathcal{G}\, dS - \iint_{\Sigma_1}\mathcal{G}n_z\, dS \qquad (7)$$

4 Numerical results

We apply our model to a 250m long and 78m wide rectangular platform. The total height is 15m and the water free surface in the air cushion is at distance of 10m from the mean sea level. The vertical walls thickness, surrounding the air cavity, is 4m for the 250m long side walls , and 6m for the 78m long end walls. With these values, we find $\alpha = 2.4310^{-5}$. In Figure [2] and [3], we compute the added mass and damping coefficients for heave motion. In figure [4] we compare the amplitude of the platform elevation for a unit height incoming head wave with the experimental results of Pinkster. The agreement is good. In figure [5], we compute the wave elevation amplitude in the air cushion. We check that, in agreement with our assumptions, no resonant mode will generate waves that hit the horizontal deck of the air cushion platform.
We first note that we obtain a negative added mass for a large range of frequency and also with discontinuities. This is due to the small width/length ratio. For wider platform this phenomenon never occurs.

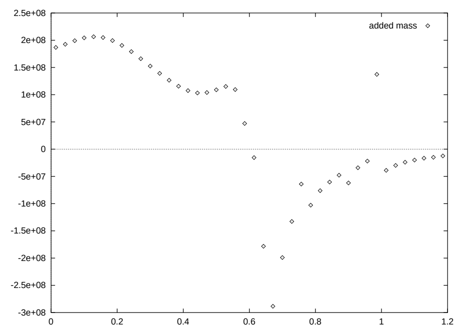

Figure 2: Added mass

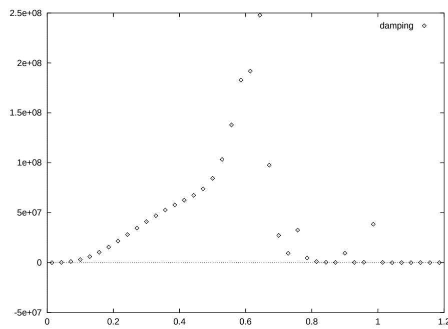

Figure 3: Damping

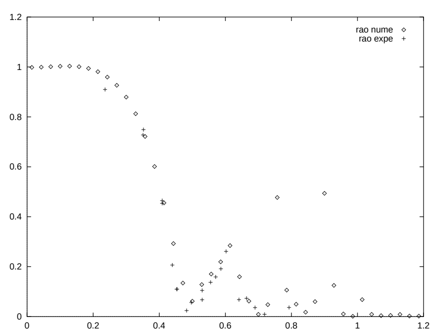

Figure 4: RAO

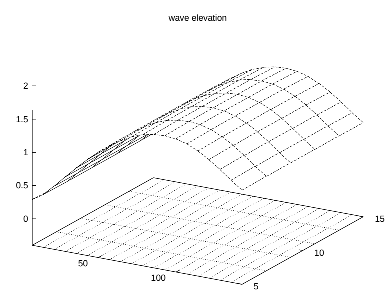

Figure 5: Wave elevation

References

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