The bow wave of a vertical surface-piercing circular cylinder in a steady current

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1. INTRODUCTION

The bow wave of a vertical surface-piercing cylinder in a steady current breaks at modest Froude numbers. A view of this flow is shown in figure 1, taken from experiments in which a 210mm diameter stainless steel cylinder was towed at constant speed through water initially at rest (Chaplin & Teigen, 2000). The bow wave is much like that of a blunt-bowed ship, where the resulting loss of momentum may represent a significant proportion of the wave resistance.

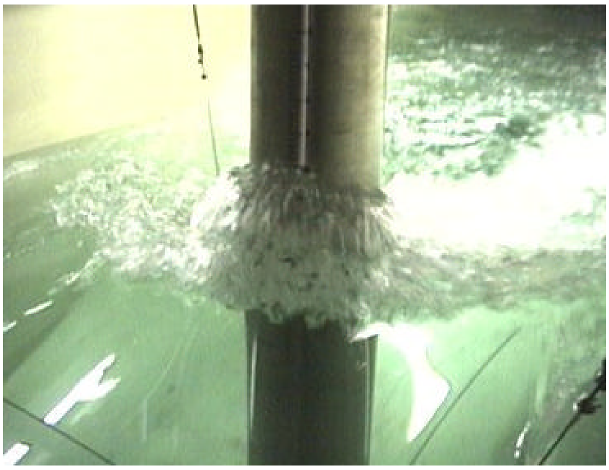

Figure 1 Flow upstream of a cylinder towed at Froude number $Fr = V/\sqrt{gd} = 1.64$, where V is the velocity and d the diameter, and Reynolds number $Re = Vd/\nu = 4.6\times10^5$

In the experiments, the wave resistance of the vertical cylinder was estimated from pressure measurements made at many points over its surface, and is plotted in figure 2 in the form of the equivalent loaded length ΔL. This is defined by

$$\text{(wave resistance)} = \Delta L \times \text{(drag per unit length far beneath the surface).} \tag{1}$$

The wave resistance increases rapidly once the Froude number has exceeded 0.5, and reaches a maximum at a Froude number of about 1. At the peak it is equivalent to the loading, at deeply submerged elevations, on a length of cylinder of about 0.8*d*.

In working towards an understanding of these results, this paper is concerned with the problem of predicting the Froude number at which the bow wave of a vertical cylinder will first break. It is assumed that until this happens the flow upstream of the cylinder is represented reasonably well by potential flow analysis, even though in practice the downstream region will be dominated by the cylinder's wake.

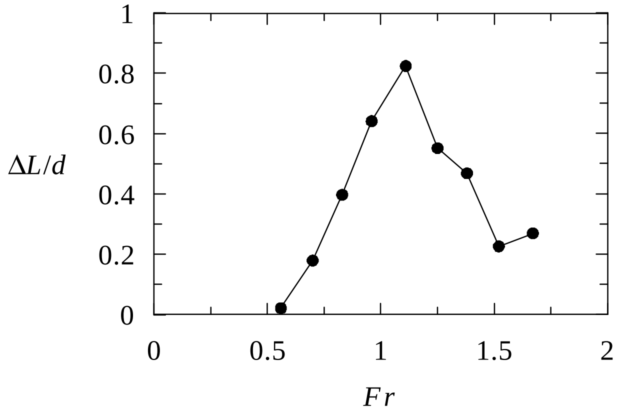

Figure 2. Wave resistance on a vertical surface-piercing cylinder expressed as the equivalent loaded length ΔL. $Re/Fr = 2.80 \times 10^5$.

In a reference frame (r, θ, z) fixed on the cylinder, the flow is steady. The origin is on the cylinder's axis at still water level; z is measured vertically upwards, and $\theta = 0$ is the direction of the incident flow. Velocities are normalised with respect to the incident velocity V, and lengths with respect to the cylinder's radius $d/2$. The free surface boundary conditions are

$$\eta = Fr^2(1 - v_r^2 - v_\theta^2 - v_z^2) \quad \text{and} \quad v_z = v_r \frac{\partial \eta}{\partial r} + v_\theta \frac{1}{r}\frac{\partial \eta}{\partial \theta} \ . \qquad (2), (3)$$

After describing an approximate model for the flow, a fully non-linear numerical solution is outlined, using the method of desingularised sources (Cao *et al.*, 1991). In the approach followed here, the boundary condition on the cylinder's surface is imposed by computing the three-dimensional image system associated with each source.

2. APPROXIMATE SOLUTION

As a first approximation, flow in any horizontal plane is assumed to be that corresponding to two-dimensional potential flow past a cylinder:

$$v_r = \cos\theta\,[1 - \frac{1}{r^2}]\ ,\ v_\theta = -\sin\theta\,[1 + \frac{1}{r^2}]\ . \qquad (4), (5)$$

and v_z^2 is neglected in (2). The vertical velocity follows from (3), and the vertical acceleration of a particle is

$$\frac{dv_z}{dt} = v_r \frac{\partial v_z}{\partial r} + v_\theta \frac{1}{r}\frac{\partial v_z}{\partial \theta} = \frac{8Fr^4}{r^{10}}[-4 + 9r^2 - 4r^4 + (-3 + 8r^2 - 9r^4)\cos 2\theta + 3r^6 \cos 4\theta] \qquad (6)$$

As the Froude number is increased, the expression on the right hand side of (6) first reaches a value of –1 (corresponding to a particle with a downwards acceleration of g) at a point on the surface of the cylinder r = 1, at 35.3° around from the stagnation point. This represent the conditions in which the water surface would first break, and occurs at a Froude number of 0.465.

3. FULLY NON-LINEAR SOLUTION

The fully non-linear solution for the steady bow wave flow uses a desingularised Eulerian approach, in which the velocity potential is represented partly by the sum of a large number of point sources distributed over the free surface. The sources are placed around the cylinder at regular radial and azimuthal intervals. As the solution develops, each source is moved vertically, maintaining a constant vertical offset from the free surface. A collocation point is placed on the free surface directly below each source.

At this point it would normally be necessary to introduce additional sources inside the cylinder to maintain its surface as a boundary. In two dimensions this problem can be solved explicitly by means of the circle theorem, but in three dimensions it is likely to involve the inclusion of a large number of extra sources and their corresponding collocation points. However, the aim of this work was to experiment with the idea of generating for each external source a three-dimensional image system which automatically preserves the boundary condition on the cylinder's surface.

The velocity potential of the three-dimensional image system associated with a single point source outside the cylinder can be constructed by using the method set out by Affes & Conlisk (1993) for the case of a cylinder in the neighbourhood of a vortex filament in free space. If ϕ_S is the potential due to the source, then the potential of the image system is

$$\phi_I = -\frac{1}{4\pi^2} \sum_{m=-\infty}^{\infty} e^{im\theta} \int_{-\infty}^{\infty} \left. \frac{\partial \hat{\phi}_S}{\partial r} \right|_{r=1} \frac{K_m(|k|r)}{|k| K'_m(|k|)} e^{ikz}\, dz \tag{7}$$

where K_m is the modified Bessel function of order *m*, and $\hat{\phi}_S$ is the double Fourier transform of the source potential,

$$\hat{\phi}_S = \int_{-\infty}^{\infty} \int_{-\pi}^{\pi} \phi_S \ \exp(-ikz - im\theta)\ d\theta\ dz\ . \tag{8}$$

The transform in the *z* direction can be performed analytically to give

$$\left. \frac{\partial \hat{\phi}_S}{\partial r} \right|_{r=1} = \int_{-\pi}^{\pi} e^{-im\theta} \frac{2k(R\cos\theta - 1)}{\sqrt{R^2 - 2R\cos\theta + 1}} K_1\left(k\sqrt{R^2 - 2R\cos\theta + 1} \right) d\theta \tag{9}$$

for a source of unit strength located at (*R*, 0, 0). The total velocity potential consists of the sum of the potentials due to the sources directly (making use of the symmetry of the flow about $\theta = 0$), the image systems of these sources, and the two-dimensional flow of which the velocity components are (4) and (5).

The solution proceeds iteratively, starting with a trial surface profile $\eta(r,\theta)$ obtained from (2), (4), (5), with $v_z = 0$. The source strengths are then obtained from the linear system (3), and each iteration is completed by updating the surface elevations from (2). Placing each source and collocation point on a fixed vertical line allows much of the computation involved in (7) to be carried out just once for a given mesh layout, and several of the subsequent operations can be done by using FFT routines.

Some results are shown in figure 3. At *Fr* = 0.36 and *r* = 1.2, only the vertical velocity differs markedly from the simple model. Since the computed vertical velocity is significantly reduced, it suggests that the critical Froude number will be rather greater than the figure of 0.465 mentioned above.

As described here the computational method for dealing with the boundary condition on the cylinder is not very efficient, since it requires more computer storage and time than the alternative of placing sources inside the cylinder. However, it does have some attractions, and there is scope for significant improvements.

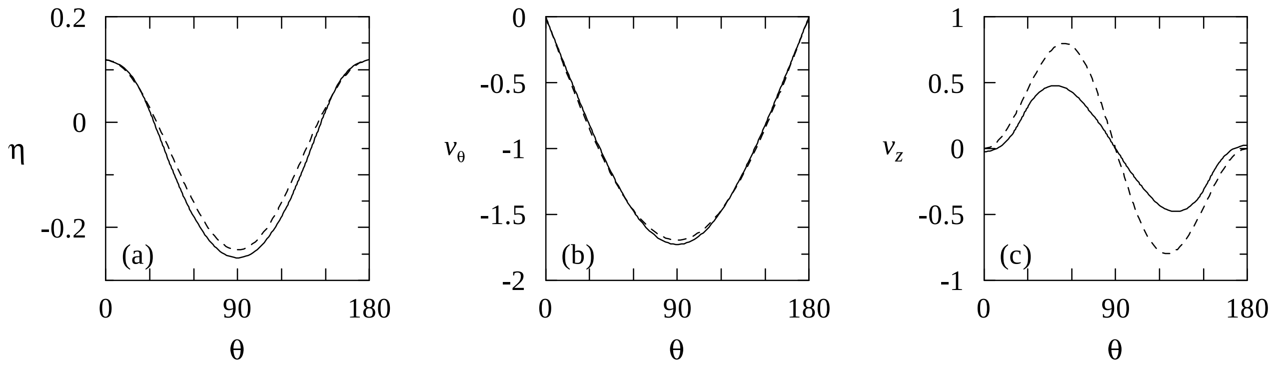

Figure 3. Approximate (broken line) and computed (continuous line) results at *Fr* = 0.36, and *r* = 1.2: (a) the water surface elevation; (b) the azimuthal velocity; (c) the vertical velocity. The upstream stagnation point is at θ = 180°.

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