NONLINEAR WAVE GROUP EVOLUTION IN DEEP AND INTERMEDIATE-DEPTH WATER: EXPERIMENTS AND NUMERICAL SIMULATIONS

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Among model equations which describe evolution of nonlinear waves, the Zakharov (1968) equation and the Dysthe modified nonlinear Schrödinger (MNLS) equation (Dysthe 1979, Lo & Mei 1985) are considered to be the most general. Stiassnie (1984) showed that this MNLS equation, which is valid for higher wave steepness than the conventional cubic Schrödinger equation, could be derived from the Zakharov equation assuming narrow frequency spectrum approximation. The use of Zakharov equation to simulate the modification of propagating wave groups, hence, can be advantageous lifting the limitation of narrow band spectrum. Although those equations are known for decades, only in a limited number of studies the theoretical predictions based on those models are directly compared with the experimental results. Such a comparison is carried out here. We report on systematic and accurately controlled experiments in which evolution of the well-defined wave groups in deep and intermediate-depth water is studied along the tank. The spatial evolution of the same wave groups is studied numerically by solving properly discretized Zakharov equations as well as the Dysthe equation. The selection of this problem is due to two main reasons. First, transformation of wave groups propagating toward the beach has significant practical consequences, while being of considerable scientific importance for understanding the nonlinear dynamics of water waves. Second, in an earlier study of this problem (Shemer et al. 1998) it was demonstrated that the cubic Schrödinger equation is incapable to describe correctly the complicated pattern of the spatial evolution of nonlinear wave groups, in particular the asymmetric shape of the evolving group envelope. The application of more advanced models is thus warranted.

Experiments were carried out in an 18 m long, 1.2 m wide and 0.6 m deep wave tank. A computer-controlled paddle-type wavemaker is located at one end of the tank, while a wave-energy-absorbing beach is placed at the far end. A probe-supporting bar with four wave gauges is mounted on a separate carriage, which can be moved along the tank. The distance between the two consecutive probes is 0.4 m. Detailed measurements of instantaneous surface elevation are carried out at eight fixed measuring stations, thus covering 32 distances from the wavemaker. Wave groups with a Gaussian envelope have been studied. The following periodically repeated driving signal is applied to the wavemaker:

$$s(t) = a_0 \exp-(t/5T)^2 \cos(\omega_0 t), \quad \omega_0 = 2\pi/T, \ -16T < t < 16T \,. \tag{1}$$

This signal produces wave groups, which are widely separated and have a relatively broad discrete spectrum. Experiments are carried out for two carrier wave periods, T=0.7s and T=0.9s, satisfying nearly deep-water conditions, and for three values of the driving amplitude a_0, corresponding to a nearly linear, nonlinear, and strongly nonlinear regimes. In the vicinity of the wavemaker, the maximum values of wave steepness $\varepsilon = A_0 k_0$, in the group were 0.07, 0.14 and 0.21, respectively (A_0 and k_0 being the carrier wave amplitude and number).

The MNLS coupled system of equations, which describes the evolution of the envelope $A(\eta, \xi)$, where $\eta=\varepsilon^2 kx$, $\xi=\varepsilon\,\omega(x/c_g - t)$, and c_g is the group velocity, and of the potential of the induced mean current ϕ, in the fixed coordinates is:

$$A_\eta + i A_{\xi\xi} + i |A|^2 A + 8\varepsilon |A|^2 A_\xi + i\, 4\varepsilon A \phi_\xi |_{z=0} = 0 \tag{2}$$

$$4\phi_{\xi\xi} + \phi_{zz} = 0 \qquad (-h < z < 0) \tag{3}$$

with ϕ satisfying the following boundary conditions:

$$\phi_z = |A|^2{}_\xi, \quad z = 0, \qquad \phi_z = 0, \ z = -h. \tag{4a, b}$$

Following Lo and Mei (1985), equations (2) - (4) are solved using the pseudo-spectral method and the split-step Fourier method.

A modification of the Zakharov equation is necessary to describe the spatial evolution of the wave spectrum, in contrast to the temporal evolution described by the conventional version of this equation. The spatial discrete Zakharov mode-coupled equations describe evolution of the complex "amplitude" $B_j(x)$ of each free wave in the spectrum due to four-wave interactions in a unidirectional space domain, which satisfy the near resonant condition:

$$\omega_0 + \omega_1 - \omega_2 - \omega_3 = 0, \quad |k_0 + k_1 - k_2 - k_3| \leq O(\varepsilon^2 k_0). \tag{5}$$

Equation for the spatial evolution of each component can be written as (Agnon 1999):

$$i \cdot c_g \frac{\partial B_0}{\partial x} = \sum_{\omega_1 \omega_2 \omega_3} T_{0,1,2,3} B^*{}_1 B_2 B_3 \delta_{0+1-2-3} e^{-i(k_0+k_1-k_2-k_3)x}, \tag{6}$$

where $T_{0,1,2,3}$ is the interaction coefficient. Since the variable in (6) has the same dimensions as in the conventional Zakharov equation, the expressions for calculating the bound components and the kernels used e.g. in Stiassnie & Shemer (1984) are valid for (6) as well. After the number of free modes is chosen and the resonating quartets are selected, evolution equation is written for each free mode. The spatial evolution of the whole wave field along the tank is thus expressed as a set of mode-coupled nonlinear complex ordinary differential equations (ODEs), which can be solved using the modified Runge-Kutta method.

The development of asymmetry of the group envelope can be seen in the simulated surface elevations. In Fig. 1, the experimental results at x = 2.89 m and x = 8.67 m are presented in Figs. 1*a*, *b*, respectively. The corresponding MNLS model simulations at these two locations are presented in Figs. 1*c*, *d*, whereas the Zakharov model simulations are given in Figs. 1*e*, *f*. At x = 2.89 m, both these simulations are quite similar to the experimental result, the agreement being slightly better for the Zakharov model. This similarity between the simulated surface elevations in both models and experimental observations is retained away from wavemaker, at x = 8.67 m.

The instantaneous surface elevation at a given location in the Zakharov model is obtained as a superposition of all spectral modes, including free and bound waves. It thus seems natural to compare the evolution of the wave amplitude spectra measured at various locations along the tank, with the corresponding simulations based on the present version of the Zakharov model. Such a comparison is carried out in Fig. 2. The spectrum widening along the tank observed in the experiments (left column) is faithfully reproduced in the simulations (right column). The evolving along the tank spectra in Fig. 2 become gradually more asymmetric, resembling the spatial evolution of the envelope shape observed in Fig. 1.

The present study is thus characterized by experiments and numerical simulations carried out in an inter-related fashion. A convincing agreement between the simulations and the experimental results is obtained. In particular, impressive correspondence can be noted between the skewed wave group shapes at relatively remote locations in the experiments and the numerical simulations. The similar results obtained from two completely different mathematical models confirm the validity of the simulations according to both methods. Under present experimental conditions, both the MNLS and the Zakharov models perform quite well, with the Zakharov model representing the experiments somewhat more faithfully.

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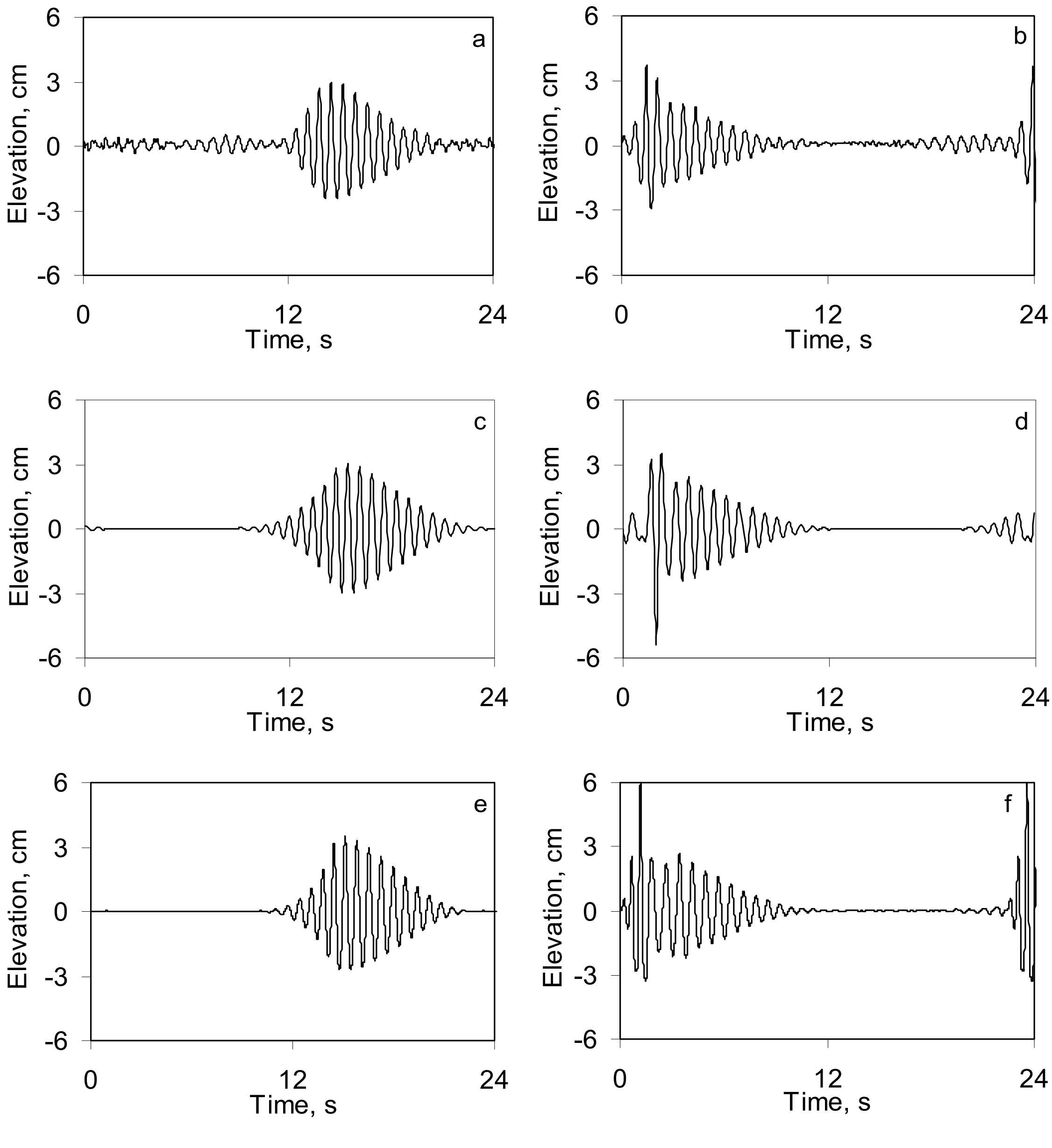

Figure 1 (a) & (b) Experimental results for $\varepsilon = 0.21$, T = 0.7 s at x = 2.89 m and x = 8.67 m; (c) & (d) MNLS simulations at the corresponding locations; (e) & (f) Zakharov simulations with 39 free modes.

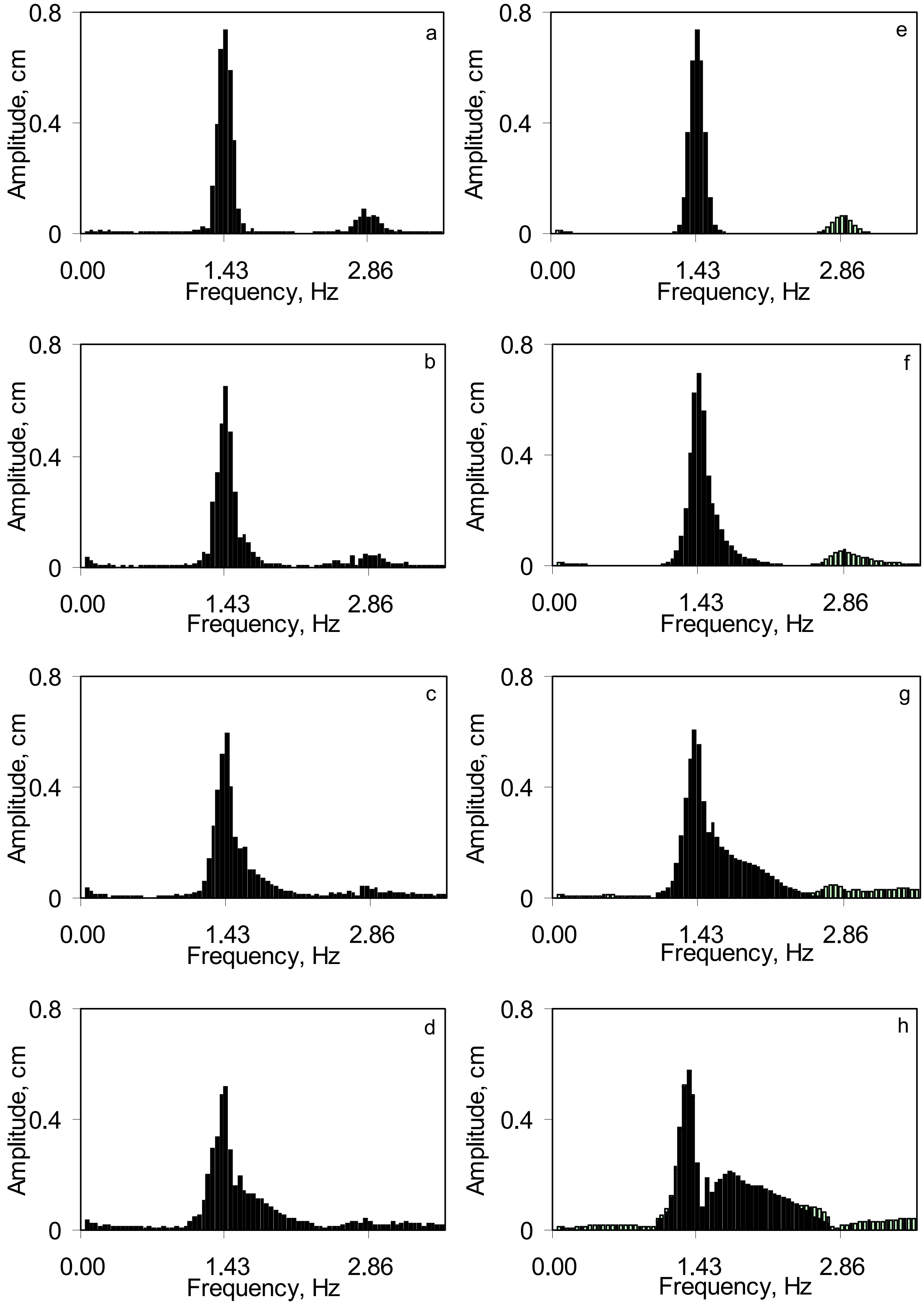

Figure 2. (a - d) Amplitude spectra of measured surface elevation for the conditions of Figure 1. Locations along the tank: (a) $x = 0.24$ m, (b) $x = 2.89$ m, (c) $x = 5.78$ m, and (d) $x = 8.67$ m; (e - h) Simulated amplitude spectra using Zakharov model with 39 free modes at the corresponding locations.