DESIGN OF THREE-DIMENSIONAL BODIES SUBJECT TO WATER IMPACT

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1. Introduction

In spite of increasing needs of offshore industry and shipbuilding, the three-dimensional impact problem is far to be well solved yet. However, within the classical assumptions of Wagner's theory [1] there exists a way to derive exact solutions of the 3D impact problem thus providing preliminary qualitative results. This can be done with the help of so-called inverse Wagner's problem [2].

Wagner's approach is formally valid for the initial stage of a blunt body impact onto the liquid free surface. During this stage the boundary conditions can be linearised and imposed on the initial position of the free surface. In the direct problem of impact the body shape and the velocity of its entry are given. For the most general configurations this problem is very complicated and difficulties are connected with the coupled resolution for both the liquid flow and the position of the contact region between the entering body and the liquid. The impact problem is non-linear even within the linearised Wagner's theory. The direct Wagner's problem has been effectively solved for both two-dimensional and axisymmetric cases [3]. Three-dimensional effects are handled at present mainly by using aspect-ratio correction factors.

The inverse problem offers an attractive alternative. Within this problem the body velocity and the contact region shape are prescribed at any instant and it is required to determine the liquid flow and to reconstruct the body shape. The inverse problem of impact is linear and, even in the most complicated cases, can be reduced to integral equations and quadratures. For elliptical contact regions the calculations can be performed analytically.

For various applications of industrial interest the case of elliptic contact regions is general enough to cover a wide rage of physical configurations [3]. Semi-axes of the contact region at any instant of time can be either given in advance (inverse Wagner's problem) or defined by additional constraints (optimization problem). It should be noted that the optimization problem includes the solution of the inverse Wagner's problem but can be much more complicated if the constraints are complex. In the present report a free fall of a blunt body onto an initially calm liquid free surface is considered. The mass of the body M is given, and the hydrodynamic force on the body $F(t)$, where t is the time, is prescribed for t being small enough. We shall reconstruct the body shape, entry of which provides the given resistance force. The problem is considered under the following assumptions: (i) the liquid is ideal and incompressible; (ii) the liquid flow is three-dimensional and irrotational; (iii) the contact region between the liquid and the body is elliptic; (iv) the eccentricity of the contact region e is independent of time; (v) the boundary conditions on the liquid surface can be linearised and imposed on its initial position. The considered problem can be reduced to the inverse Wagner's problem and effectively analyzed. The obtained theoretical results may be of help for preliminary design of bodies subject to water impact.

2. Formulation of the problem

Initially ($t = 0$) the resting liquid occupies the lower half-space, $z < 0$, and a body touches the liquid free surface, $z = 0$, at a single point which is taken as the origin of the Cartesian coordinate system $Oxyz$. The vertical velocity of the body $U(t)$ is governed by the second Newton's law:

$$MU_t = Mg - F(t) \qquad t > 0, \quad \text{and} \quad U(0) = U_0, \tag{1}$$

where g is the gravity and $F(t)$ is the hydrodynamic force acting on the entering body. According to hypotheses mentioned above, the velocity potential $\phi(x, y, z, t)$ satisfies the following equations:

$$\begin{cases} \phi_{xx} + \phi_{yy} + \phi_{zz} = 0 & z < 0 \\ \phi = 0 & z = 0, (x, y) \notin D(t) \\ \phi_z = -U(t) & z = 0, (x, y) \in D(t) \\ \phi \to 0 & x^2 + y^2 + z^2 \to \infty \end{cases} \tag{2}$$

where $D(t)$ is the contact region between the liquid and the moving body. The boundary of $D(t)$ is referred to as the contact line, position of which is convenient to describe in the implicit form: $t = t_c(x, y)$. It is clear that $t_c(0, 0) = 0$ and $D(t) = \{x, y \mid t_c(x, y) < t\}$. If the body position is given at any instant as $z = f(x, y) - h(t)$ –where the function $f(x, y)$ describes the body shape and the function $h(t)$ is the penetration depth, $h_t = U(t)$ and $h(0) = 0$– the matching condition on the contact line follows from the time integration of the kinematic free surface condition:

$$f(x, y) = h(t_c(x, y)) + \int_0^{t_c(x,y)} \frac{\partial \phi}{\partial z}(x, y, 0, \tau) d\tau \qquad (x, y) \in R^2. \tag{3}$$

The hydrodynamic force $F(t)$ (within the Wagner's approximation) is then introduced in equation (1) to give the velocity:

$$F(t) = \frac{d}{dt}[M_a(t)U(t)] \quad \Rightarrow \quad U(t) = \frac{M(U_0 + gt)}{M + M_a(t)} \tag{4}$$

where $M_a(t)$ is the added mass of the floating disk $D(t)$. It is worth noting that the force depends on the shape of the contact region and the body velocity but not directly on the body shape. If the function $t_c(x, y)$ is given, the shape of the contact region $D(t)$ is known at any instant of time. The added mass $M_a(t)$ is then calculated; equation (4) yields the entry velocity $U(t)$ finally providing the penetration depth $h(t)$. The vertical velocity of the liquid free surface $\phi_z(x, y, 0, t)$ follows from (2) and the function $f(x, y)$ is calculated from (3). The procedure is straightforward and provides exact solutions of the Wagner's problem. Inverse Wagner's problem for self-similar flows was discussed by Borodich [4].

Analysis of the described procedure indicates that a main difficulty in the inverse problem is to calculate the distribution of the velocity potential $\phi(x, y, 0, t)$ over the disk $D(t)$ from the boundary-value problem (2). Once the distribution has been found the computations are reduced to quadratures which can be evaluated with a given accuracy. Accuracy of the numerical solution of problem (2) is not easy to control; that justifies the importance of its exact analytical solutions.

3. Inverse problem for elliptic case

Analytical solution of problem (2) is known [5] for elliptic region $D(t)$. This solution can be derived as a limiting case of the well-known solution for an ellipsoid moving in an unbounded liquid. The boundary of the contact region is given by equation: $x^2/a^2(t) + y^2/b^2(t) = 1$. For given semi-axes $a(t)$ and $b(t)$, $b(t) > a(t)$, the function $t_c(x, y)$ is obtained by resolving this equation with respect to time t. The solution of (2) provides the distribution of the velocity potential over the contact region

$$\phi(x, y, 0, t) = -\frac{U(t)a(t)}{E(e)}\sqrt{1 - \frac{x^2}{a^2(t)} - \frac{y^2}{b^2(t)}}, \qquad e = \sqrt{1 - a^2/b^2}, \tag{5}$$

and the vertical velocity of the liquid free surface

$$\frac{\partial \phi}{\partial z}(x, y, 0, t) = -\frac{U(t)}{E(e)}\left[E(\arcsin\frac{b(t)}{\sqrt{\lambda + b^2(t)}}, e) - \sqrt{\frac{b^2(\lambda + a^2)}{\lambda(\lambda + b^2)}}\right], \qquad \frac{x^2}{a^2(t) + \lambda} + \frac{y^2}{b^2(t) + \lambda} - 1 = 0, \tag{6}$$

where $\lambda(x, y, t)$ is the positive root of the second equation, and $E(\theta, e)$ and $E(e)$ are the elliptic integrals of the second kind. The added mass of the elliptic disk is $M_a = (2\pi/3)\rho a^2 b/E(e)$. The entry velocity $U(t)$ is given by (4) and the vertical velocity of the free surface by (6) for any x and y. Evaluation of the integral in (3) finalizes the reconstruction of the entering body shape.

The case of constant entry velocity is more simple. In that case, it is shown [2] that the entry of elliptic paraboloid provides an elliptic contact region. Elliptic paraboloid with two parameters is a quite general shape to approximate almost any blunt body near the impact point. The obtained analytical solution can be used to evaluate the aspect-ratio correction factor and to analyse the accuracy of results given by the strip theory for elongated bodies.

4. Design of entering body shape

The problem of reconstruction of entering body shape which provides a prescribed history of the resistance force $F(t) = F_*\beta(t/T)$ is considered. Here F_* is the constant, T is the time scale and $\beta(t')$ is the non-dimensional function, $t' = t/T$. The entering body shape has to be determined for given function $\beta(t')$ and given constants M, U_0, F_*, T and e. Two cases are considered: (i) $\beta(t') = 1$ and (ii) $\beta(t') = t'\exp(-t')$. The first case is expected to provide the body shape, for which the entry velocity reduces in an optimal way without high acceleration. The second case roughly imitates a typical history of the impact force.

In the first case the body acceleration is constant, $U(t) = U_0(1 - c_0 t_1)$, and the semi-axis are:

$$b(t) = b_0[t_1/(1 - c_0 t_1)]^{\frac{1}{3}}, \quad a(t) = b(t)\sqrt{1 - e^2},$$

$$t_1 = \frac{gt}{U_0}, \quad c_0 = \frac{F_*}{Mg} - 1, \quad b_0^3 = \left(\frac{3}{2\pi}\frac{E(e)}{1 - e^2}\right)\frac{F_*}{g\rho}$$

It is seen that $b(t) = O(t^{\frac{1}{3}})$ as $t \to 0$, which indicates that $f(x, y) = O([x^2 + y^2]^{\frac{3}{2}})$ close to the impact point. With known semi-axes of the contact region, the inverse Wagner's problem is solved according to the procedure described in section 3.

In the second case the semi-axes are given by the formulae $a(t) = b(t)\sqrt{1 - e^2}$,

$$b(t) = b_0\delta^{\frac{1}{3}}\left[\frac{\gamma(t_1/\delta)}{1 + t_1 - \delta(c_0 + 1)\gamma(t_1/\delta)}\right]^{\frac{1}{3}}, \quad \delta = \frac{gT}{U_0}, \quad \gamma(\xi) = 1 - (1 + \xi)\exp(-\xi).$$

Now $f(x, y) = O([x^2 + y^2]^{\frac{3}{4}})$ close to the impact point. Calculations were performed for $U_0 = 4.43$m/s, $F_* = 29333$N, $T = 0.0113$s and $M = 100$kg, which correspond to the falling height of 1m and the maximum deceleration of $10g$. Two cases were considered: $e = 0.1$ and $e = 0.9$. The time variations of the body acceleration and velocity are illustrated in figure 1a. The time growths of the semi-axes $a(t)$ and $b(t)$ are depicted in figure 1b. It should be noted that the penetration depth $h(t)$ is always smaller than the characteristic lengths of the contact region, which is in accordance with the hypothesis of linearization of the Wagner's theory. The calculated body shapes are plotted in figures 2. In addition the free surface shape is drawn up to its contact with the body. The contact line for $e = 0.9$ is clearly a three-dimensional curve. The deformations of the free surface for a 3D configuration vanish more rapidly than for the 2D case as the distance increases from the body. Side views show the wetted parts of the body surface; there the horizontal lines present the positions of the contact line for different times with the time step $\delta t = 0.0048s$. The calculated shapes can be used in drop experiments to justify the presented approach.

5. References

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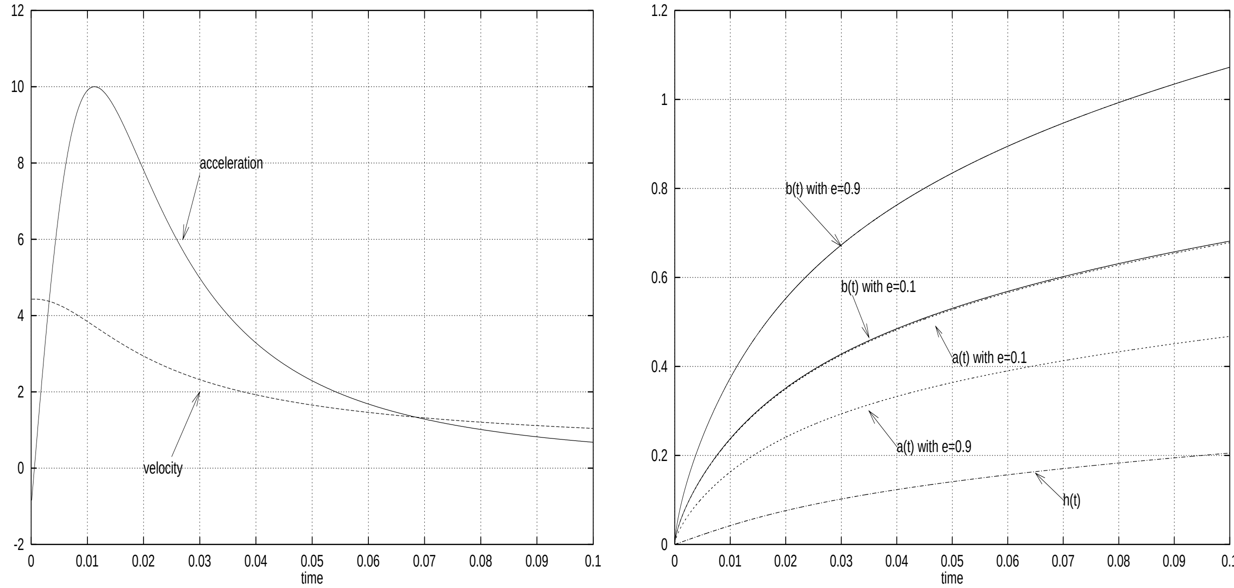

Figure 1: a) Time variations of the velocity and non-dimensional acceleration (in absolute values); b) time variations of the semi-axes: $a(t)$ and $b(t)$ and penetration depth $h(t)$ for two eccentricities: $e = 0.1$ and $e = 0.9$.

Figure 2: *left*: generated shapes with eccentricities $e = 0.1$ and $e = 0.9$ drawn at instant $t = 0.1s$; *center and right*: upper and side views of the shape under the undisturbed free surface; the vertical size is stretched with respect to the horizontal dimensions (factor 3:left, 4:right).