TRANSIENT MOTION OF A VERTICAL CYLINDER: MEASUREMENTS AND COMPUTATIONS OF THE FREE SURFACE

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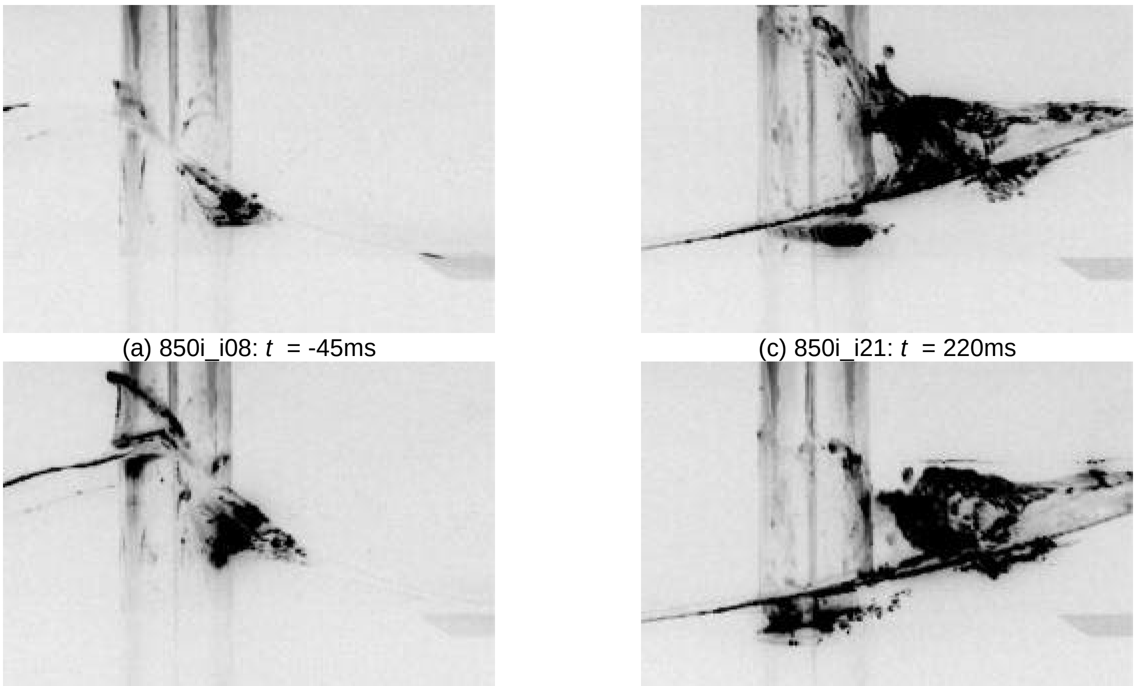
1. Introduction

In recent years there have been concerns in the offshore oil industry over the violent motions of the water surface produced around a vertical cylinder by relatively large steep waves (say height/diameter = 1, height/length =0.1). The concerns are threefold: (1) the water may reach the deck of the platform, as for example on Shell's Brent Bravo platform in the north Sea in January 1995; (2) the local pressures may damage the platform, as for example on BP's Schiehallion platform in the eastern Atlantic in November 1998; (3) a transient ringing vibration may be excited, as observed for example in the model tests of the Norwegian Draugen, Troll and Heidrun platforms in 1992. Figure 1 shows the water surface motion in question, as observed during earlier experiments (Chaplin *et al.*, 1997). The jet shown on the left (which is more pronounced if the wave is steeper) is thought to be associated with (1) and (2). The mound of water on the right that forms after the crest has passed appears to be associated with the 'secondary loading cycle' (see Chaplin *et al.*, 1997) which is an important feature of (3).

It was argued by Rainey (1997) that some of these features would also be seen in the simpler case of a vertical cylinder undergoing a transient motion in still water. The present experiments explore this suggestion. The experimental set-up, and initial results with harmonic cylinder motion, were reported in Chaplin *et al.* (1999).



(a) 850i_i08: t = -45ms

(c) 850i_i21: t = 220ms

(b) 850i_i10: t = -5ms

(d) 850i_i23: t = 260ms

Figure 1. Negative images of a focused wave passing a cylinder of diameter 0.1m: height/diameter ≈ 1.8, height/length ≈ 0.1 (case 770 of Chaplin *et al.*, 1997). Images (a) and (b) show the jet on the front face of the cylinder. At its peak, the elevation of the top of the jet was almost one diameter higher than that shown in (b). Images (c) and (d) show the water surface rise at the rear.

2. The misleading nature of the small-time expansion scheme

In Rainey (1997) it is pointed out that the small-time expansion scheme predicts violent motion of the water surface around a cylinder moving from rest with a finite initial acceleration. In fact, the same is true if the motion starts arbitrarily smoothly – that is to say, with only the Nth derivative of cylinder velocity v being given a finite value initially (say $v = t^N$). This is because the lowest-order term in the velocity potential, say ϕt^M, must satisfy the boundary-condition ϕ = 0 on the horizontal plane of the still water surface. Otherwise the dynamic free-surface condition would produce a term proportional to t^{M-1} in the free-surface elevation, from which the kinematic free-surface condition would produce a term proportional to t^{M-2} in the velocity potential. This would contradict our assumption above that the lowest-order term was ϕt^M.

Moreover, the cylinder-surface boundary condition is a velocity proportional to t^N, given the cylinder velocity assumed above, so we deduce that $M = N$, for otherwise ϕ would be zero everywhere. Thus as well as having ϕ = 0 on the still water surface, ϕ gives unit cylinder velocity. It is therefore the velocity potential given in Rainey (1997), which has infinite vertical velocity at the waterline around the cylinder. By contrast, our experiments show no such violent water surface motion. The same observation, for the case of "impulsive accelerations", is made by Wang & Chwang (1989). It thus appears that the small-time expansion method is fundamentally misleading in this regard. It would be interesting to know if the same is true for the improved small-time expansion schemes, developed hitherto only in 2-D, by Joo et al (1990) and King & Needham (1994).

3. The linear solution for arbitrary transient motion of a vertical cylinder in still water

For the purpose of identifying non-linear behaviour in the observations, comparisons can be made with the linear time domain solution developed by McIver (1994). The case of sinusoidal motion is described in Chaplin *et al.* (1999). From an arbitrary velocity $V(t)$ imposed on the cylinder, McIver's solution is based on a theorem stated by Mei, (1983). This gives a velocity potential for the flow as

$$\phi = \Omega V(t) + \int_{-\infty}^{t} \Gamma(t-\tau)\, V(\tau)\, \mathrm{d}\tau \tag{1}$$

where $\Omega V(t)$ is the 'infinite frequency' potential and (as in the case of ϕt^M above) $\Omega = 0$ on the still water plane $z = 0$. For the purposes of computing the free surface elevation $\eta(r,\theta,t)$ from the dynamic boundary condition, the first term in (1) plays no part, and

$$\eta = -\int_{-\infty}^{t} \left.\frac{\partial \Gamma(t')}{\partial t'}\right|_{t'=t-\tau,\ z=0} V(\tau)\ \mathrm{d}\tau\,, \tag{2}$$

taking length and acceleration scales such that the cylinder's radius and acceleration due to gravity are both unity. The kernel in (2) is given by

$$\left.\frac{\partial \Gamma}{\partial t}\right|_{z=0} = -\frac{2\cos\theta}{\pi}\int_{0}^{\infty} \frac{\cos Wt\ \tanh qh\ C_1(qr)}{q\left|H_1'(q)\right|^2}\ \mathrm{d}q\ , \tag{3}$$

where h is the water depth, C_1 and H_1 are functions defined in McIver (1994), and $W^2 = q\ \tanh qh$.

4. Experimental results, compared with the linear solution.

The experiments were carried out in a tank which is equipped with a servo-controlled carriage, to which the 200mm diameter vertical test cylinder was mounted. Measurements are presented here for cases where the cylinder was accelerated horizontally from rest, driven at constant velocity, and then brought to rest again. The overall displacement in each test was equivalent to 75% of a diameter. In figure 2 measurements of the resulting water surface displacements obtained from wave gauges at the front and rear faces of the cylinder are compared with computed time-series. These were obtained as described above for the same points, using the velocity of the cylinder, and taking its cylindrical coordinates as moving with it. (Conventionally, the coordinates are taken as fixed, but the difference is the second order and thus theoretically immaterial.) Agreement deteriorates as the Froude number is increased, and in particular the linear theory overestimates the motion of the water surface on both sides of the cylinder during and after its deceleration. It is worth recalling that the predictions do not capture the effect of that part of the pressure distribution on the cylinder that is related to the square of its velocity. This is on the scale of the stagnation pressure, which, in steady flow in the absence of separation, would act on both the front and rear faces. Broken lines in figure 2 show the effect simply of adding the corresponding head to the water surface elevations at each point. The result is a marginal improvement in the agreement.

In figure 3, images of the observed water surface are shown above those obtained from linear predictions for the same motion and at the same times.

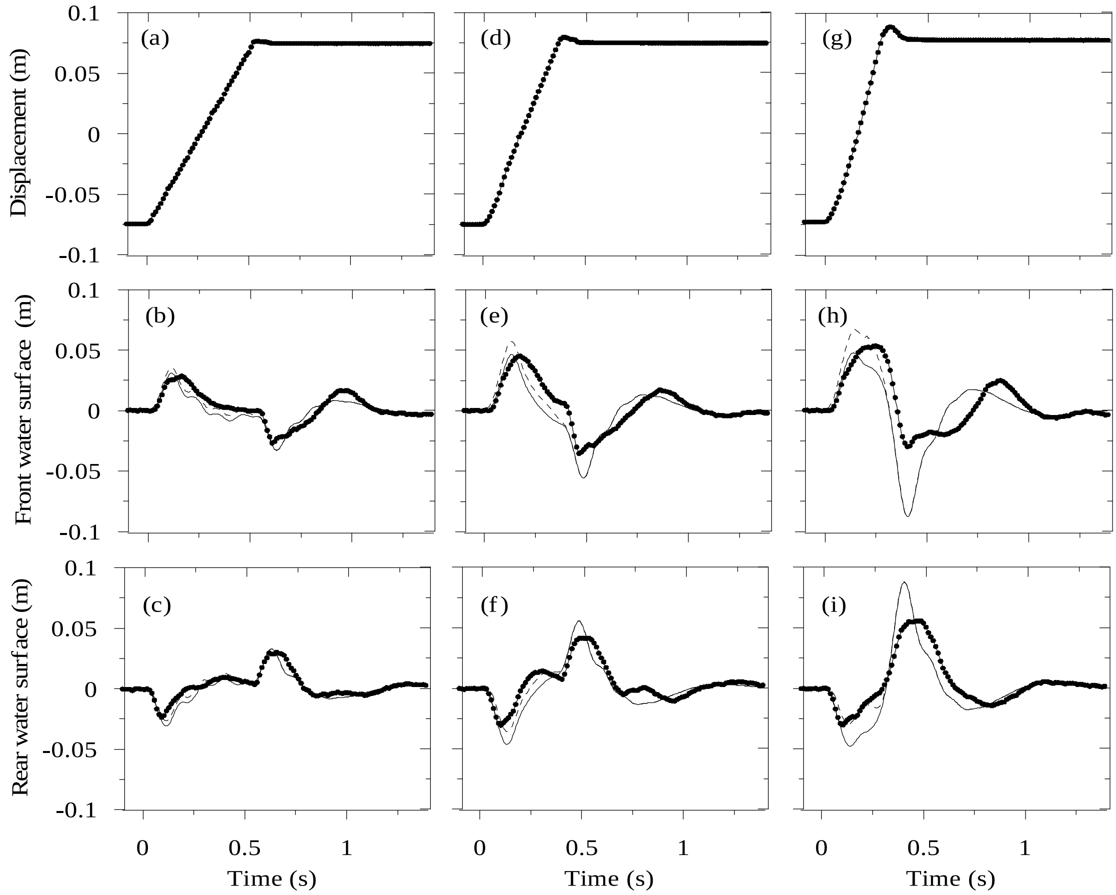


Figure 2. Measurements (points) and predictions (lines) of water surface elevations at the front (b,e,h) and the back (c,f,i) of the cylinder undergoing the displacements shown in (a,d,g). Froude numbers were 0.20, 0.28, 0.36 for cases in the left hand column, in the centre, and on the right respectively.

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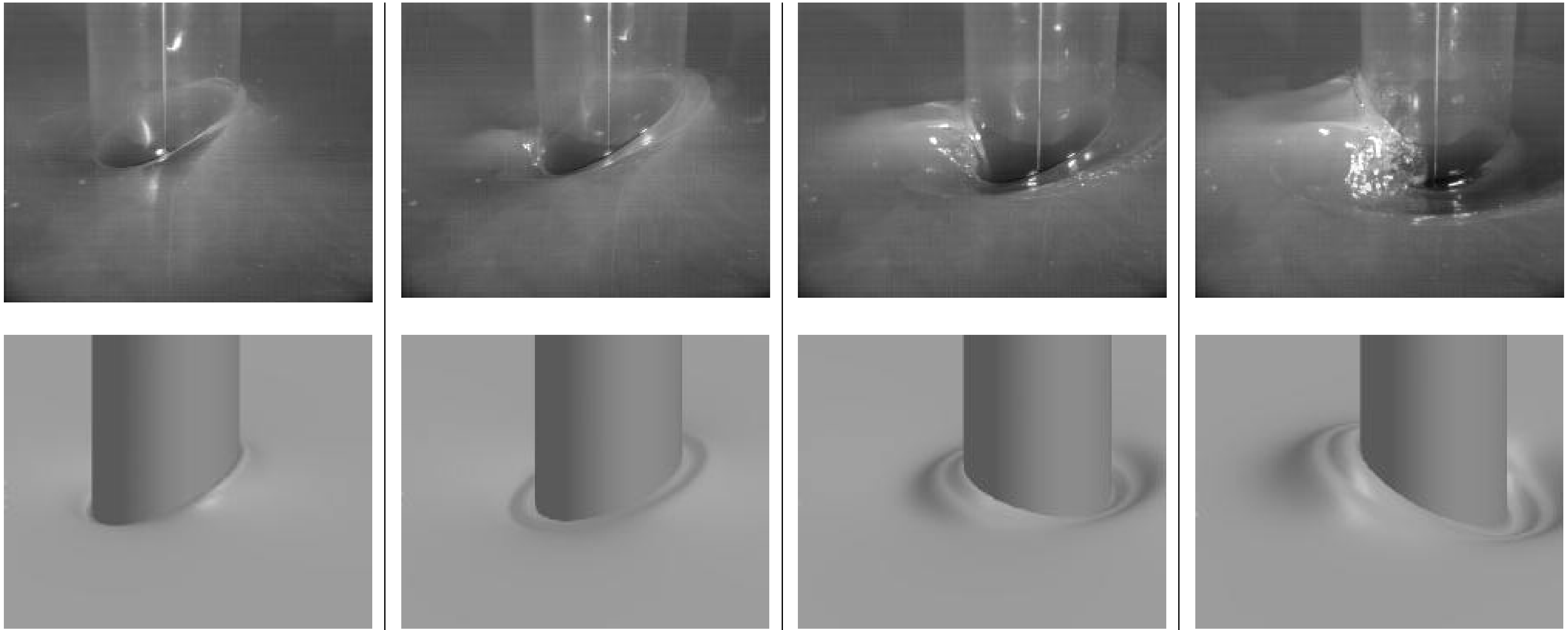

(a) 100ms. There is no sign of the violent surface motion predicted by the small-time expansion. Instead, the surface motion is well-behaved and closely follows the linear solution in accordance with the conclusions of Wang & Chwang (1989).

(b) 200ms. The surface is more elevated at the front of the cylinder than in the linear prediction, and shows the first signs of breaking. At the back of the cylinder, a steep localised mound begins to form, similar to that seen in figure 1. This is absent in the linear prediction.

(c) 300ms. The cylinder has just stopped. At the front, the breaking crest is well separated from the cylinder's surface. At the back, the mound is pronounced and has formed two plunging jets. In front of the cylinder the water surface remains close to the still water level, and does not follow the steep descent of the linear solution.

(d) 400ms. At the front of the cylinder, the breaking crest has moved about one radius away from the cylinder surface. At the back of the cylinder, the jets have broken, and are travelling forwards circumferentially. A vertical jet has started to form at the cylinder surface.

Figure 3. Images obtained from high speed video recordings (in the top row) and the linear solution (below) for the case shown in the right hand column of figure 2. Times are measured from the start of the motion.